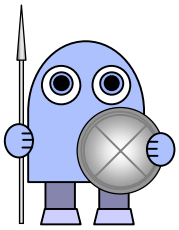


# MATHS CHALLENGE CARDS SET C

# ANSWER BOOK

MATHS CHALLENGE CARDS SET D

MATHS

four winds maths

**1** In most questions of this kind, you're looking for a simple rule or pattern. This one's unusual because there are two different rules which apply, one for an odd number of small squares and one for an even number of small squares :

If you've got an even number of small squares, then obviously just divide by 2 to get the number of black squares.

If you've got an odd number of small squares, you have to add 1 to the total and then divide by 2 to get the number of black squares.

**a** 11 x 11 grid : 61 black squares **b** 12 x 12 grid : 72 black squares
**c** 17 x 17 grid : 145 black squares **d** 100 x 100 grid : 5,000 black squares

**2** You'll probably have spotted that :

$$3, 5 \rightarrow 15 \quad (4^2 - 1)$$

$$7, 9 \rightarrow 63 \quad (8^2 - 1)$$

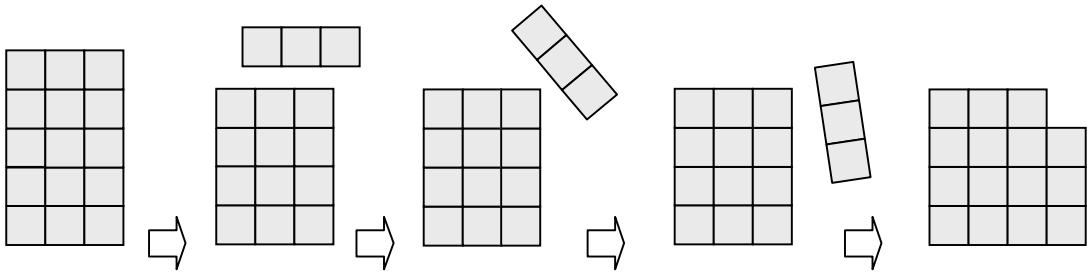
and then with the larger numbers :

$$19, 21 \rightarrow 399 \quad (20^2 - 1)$$

$$29, 31 \rightarrow 899 \quad (30^2 - 1)$$

**a** It's not that easy to put this pattern into words. You could say that if you have a pair of numbers which are 2 apart from each other, and you multiply them together, the number you get is always 1 less than the number you get by squaring the number between them. Perhaps a better way of explaining the rule is to say that if you've got three consecutive numbers, the square of the middle number is always 1 more than the product of the other two.

**b** This diagram shows how things work for 3 and 5 :



**3** Pulling two sticks out of the bag could give you any of these possible results :

| | | | |
|---|---|---|---|
| 3 + 4 (**7**) | 3 + 5 (**8**) | 3 + 6 (**9**) | 3 + 7 (**10**) |
| | 4 + 5 (**9**) | 4 + 6 (**10**) | 4 + 7 (**11**) |
| | | 5 + 6 (**11**) | 5 + 7 (**12**) |
| | | | 6 + 7 (**13**) |

As you can see, there are 10 different combinations – and just two of them give you a total length of 9cm. So we can say :

$$\text{prob (total of 9)} = \frac{2}{10} = \frac{1}{5}$$

**4** **a** 150 km (6 X 15 = 90 and 2½ x 24 = 60)

**b** The half-way point is 75 km from the start, so Hans reached it during the faster part of his journey. At 15 km/hr it would have taken him 5 hours to cover this distance, so he would have reached the half-way point at 1pm.

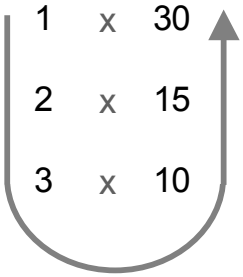
The probability of taking out three good eggs is the same as the probability of leaving behind one bad egg, and that's ¼ !

**5** The problem is just the same if you think of two different lorries setting off at the same time, one travelling from town A at 20 km/hr and one travelling from town B at 40 km/hr. Obviously there will be a place where the two lorries pass each other (going in opposite directions) but where exactly is this? The lorries are travelling at different speeds of course but as they move towards each other, they are certainly eating up the distance between them at a rate of 60 km/hr. In other words, after 2 hours there'll be no distance between them! In 2 hours the first lorry will have travelled 40 km from town A – and that's exactly where you'll find Marcel's café!

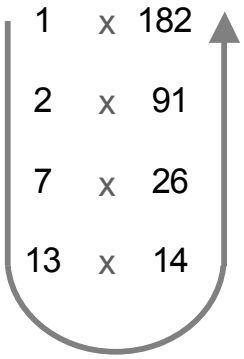
**6** **a** The volume of the standard chocolate box is 8 x 4 x 3 = 96 $cm^3$.The volume of each chocolate is just 3 x 2 x 1 = 6 $cm^3$, so you must get 16 of them in a standard box.

**b** This time each chocolate has a volume of 3 x 2 x 1.5 = 9 $cm^3$. 24 of them will take up 24 x 9 = 216 $cm^3$. of space, so that's the volume of the box you need. But 216 is a cube number, so the special shape for the box must be a cube (a 6 cm x 6 cm x 6 cm cube, in fact).

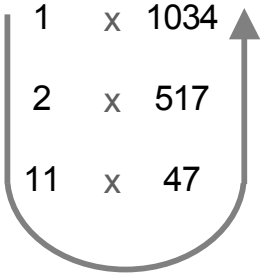
**7** The first stage for each number is to find all the 'factor pairs' – and this means finding numbers which divide into them, so try 2, 3, 5, 7, 11 etc. The next stage is to write the factor pairs in order and then make your list of factors :




factors : 1, 2, 3, 10, 15, 30

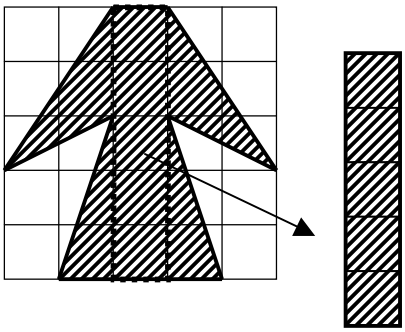



factors : 1, 2, 7, 13, 14, 26, 91, 182

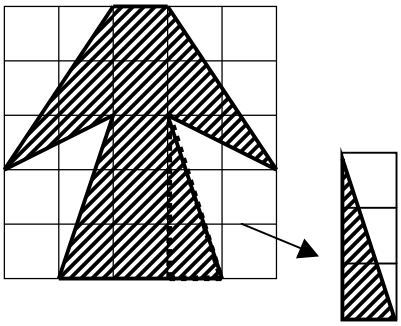



factors : 1, 2, 11, 47, 517, 1034

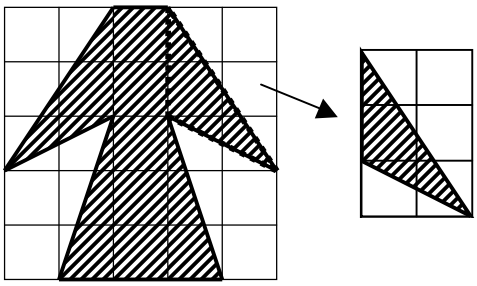
**8** Here's one way of finding the total area :



area = 5 x 1 rectangle
= 5 $cm^2$



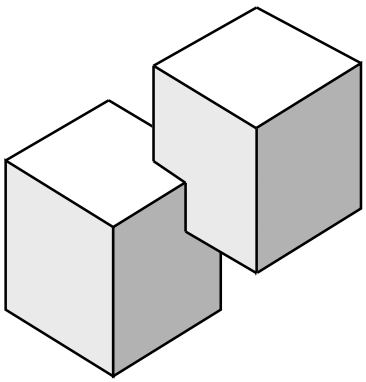
area = ½ of 3 x 1 rectangle
= 1½ $cm^2$

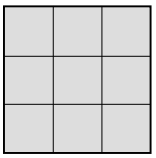


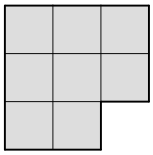
area = ½ of 3 x 2 rectangle less small triangle
= 3 - 1 = 2 $cm^2$

So, area of whole shape = 5 + (2 x 1½) + (2 x 2) = 12 $cm^2$

**9** One way here is to add up the areas of all the separate faces; of course, there are two different shapes of face to think about :



6 x  = 54

6 x  = 48

total = 102

Another way is to think of two 3 cm cubes (total surface area 108 $cm^2$) and then take away the 6 $cm^2$ you lose by taking a 1cm cube away and combining like this. Again, you get 102 $cm^2$.

**10** This question seems rather odd at first, because you aren't told how many customers there were on these two days! Without this information, it's hard to see how you can start to work things out. The surprising thing is that you can use any numbers you like for the customers and it doesn't make any difference to the answer.

For example, say there were 10 customers on Saturday and 5 customers on Sunday. Then we'd have :

| | no of customers | average 'spend' | total spent |
|---|---|---|---|
| Saturday | 10 | £4 | £40 |
| Sunday | 5 | £7 | £35 |
| totals : | 15 | | £75 |

£75 spent by 15 customers over the two days → average = £5

Now suppose we say there were 6 customers on Saturday and just 3 customers on Sunday. This time we'd have :

| | no of customers | average 'spend' | total spent |
|---|---|---|---|
| Saturday | 6 | £4 | £24 |
| Sunday | 3 | £7 | £21 |
| totals : | 9 | | £45 |

£45 spent by 9 customers over the two days → average = £5

So, however many customers there were, the average (mean) amount spent over these two days was £5.

**11** In a 3cm x 3cm x 3cm cube you can find :

1cm cubes : 27

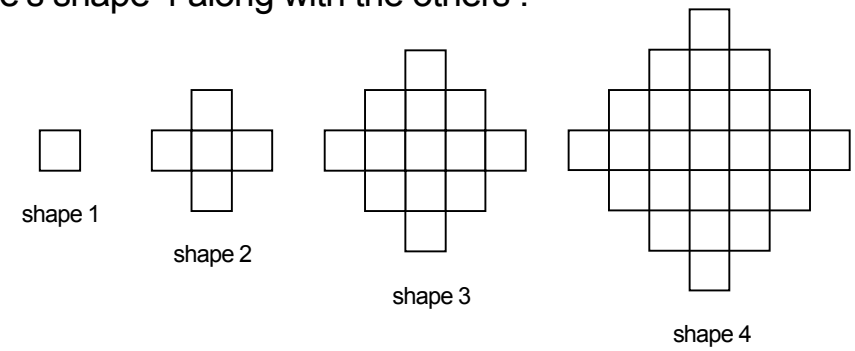
2cm cubes : 8

3cm cubes : 1

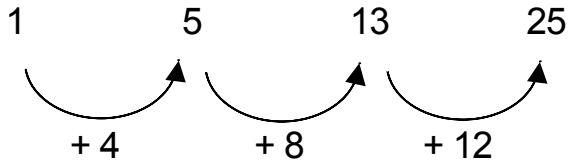
– which gives you 36 cubes altogether.

John is 12 and Alice is 9.

**12** Here's shape 4 along with the others :

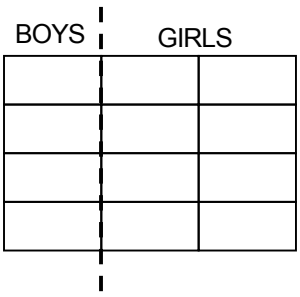



There is a pattern in the number of small squares you use, as you can see here :

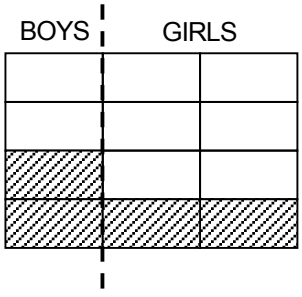



To find how many squares in pattern 5, you'll obviously have to add on 16, which will give you 41. If you then add 20, then 24 and then 28, you'll get the number of squares for pattern 8, which is 113.

**13** A diagram makes things easier here. To begin with, as there are twice as many girls as boys, you can say that boys are one third of the class :

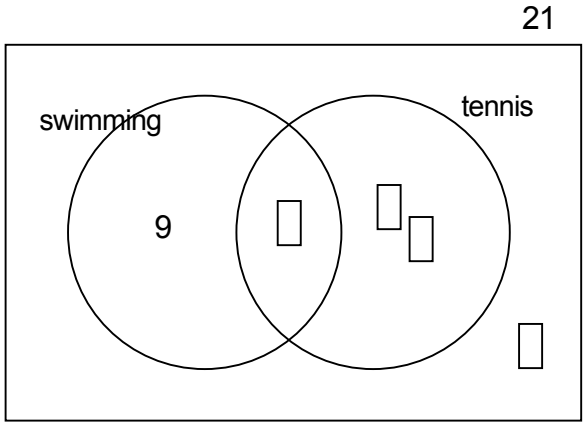



Now shade in the computer games enthusiasts – that's half the boys and one quarter of the girls :




You can see that shading now covers 4 out of 12 sections – and that's $^1/_3$ of the diagram, so the answer is $^1/_3$ of the class.

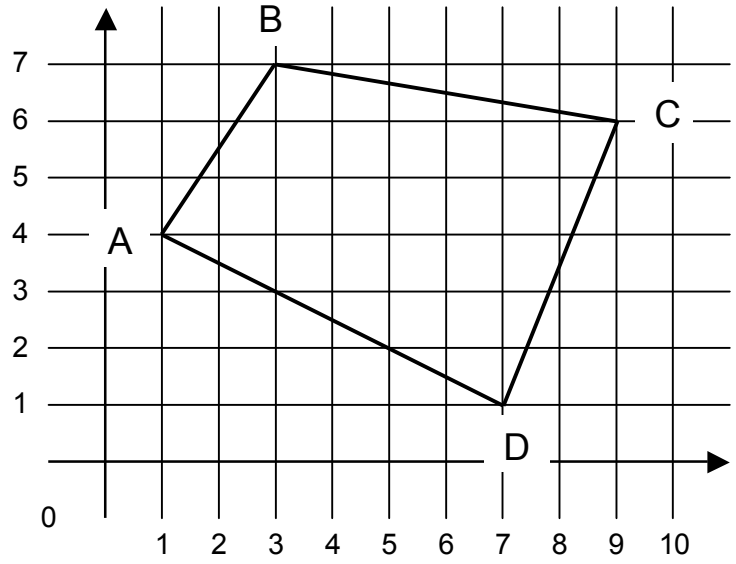
**14** Here's the information on a Venn diagram :



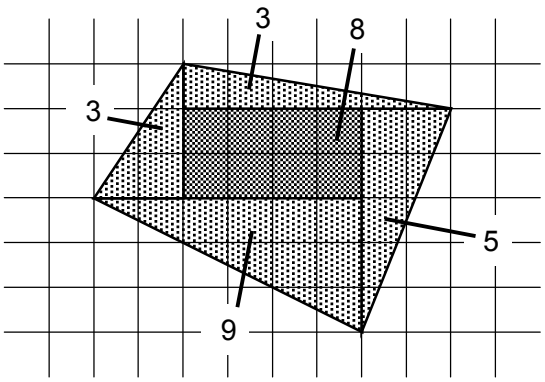

We've used □ to stand for the number of children who do both activities. With 21 children altogether in the group and 9 accounted for (those who swim but who don't play tennis), the remaining 12 must be divided equally into the four lots of □, which means that □ = 3. So the number of children who do both activities is 3.

**15** The wrappers from Toby's first 50 bars entitled him to 10 free bars. After eating these, he had 10 more wrappers to trade in and this gave him 2 more bars. This means that Toby was able to claim 12 free bars altogether !

**16** Your scalene quadrilateral should look like this :

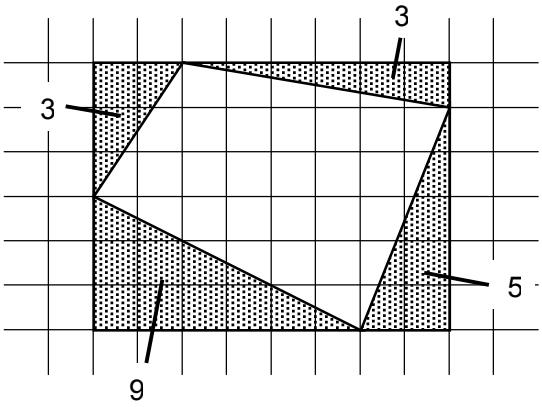



To find the area of this shape, you can divide it up into a rectangle and four right-angled triangles and just add the separate areas – or if you prefer you can draw a rectangle around the shape and then find the area by subtraction :




area = 3 + 3 + 5 + 9 + 8

= 28 $cm^2$




area of rectangle = 48

area of triangles = 20

area of quad = 48 - 20

= 28 $cm^2$

**17** Exactly one quarter !

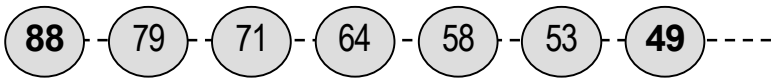
**18** From the chart you can see that altogether 20 cars crossed the bridge during this period. There were 10 cars with just 1 person in each, 5 cars with 2 people in each . . . and so on, making a total of 40 people. So, we have 40 people travelling in 20 cars and this gives you a mean (average) of 2 people per car.

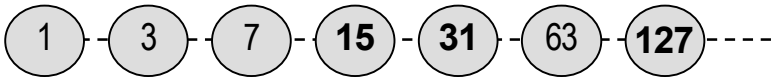
**19** The answer is no! You can't fit these rectangles into a 10cm x 10cm square, no matter how hard you try. The reason is simple : the rectangles you're given have a total area of 110$cm^2$ and the square you're thinking of fitting them into has an area of just 100$cm^2$.

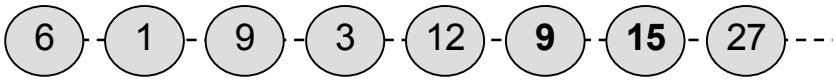
**20**

3 - 4 - 7 - 11 - 18 - **29** - **47** - - -

625 - 125 - **25** - **5** - 1 - 0.2 - - -




**21** Here are the sums of these three sequences :

$$1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{15}{8}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{31}{16}$$

The pattern in these answers is not hard to spot : the denominator is always the same as the last one in your sequence and you just double this number and subtract 1 to get the numerator. So we can simply write down :

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} = \frac{255}{128}$$

**22** Have a look at what happens when you add together the first few odd numbers :

$$1 = 1$$

$$1 + 3 = 4$$

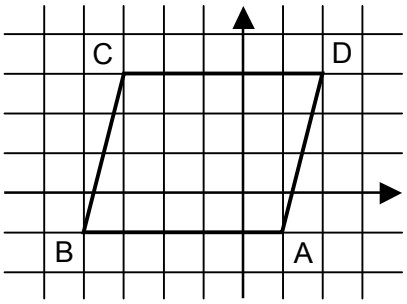
$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

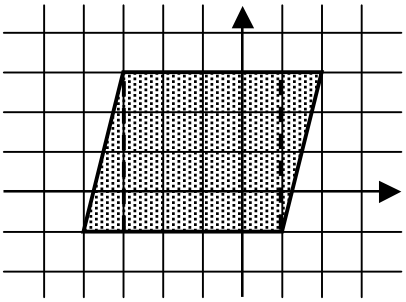
As you can see, the first odd number is the same as $1^2$, the first two odd numbers add up to $2^2$, the first three odd numbers add up to $3^2$ and the first four odd numbers add up to $4^2$. So if Edward gets a total of $10^2$, he must have added the first ten odd numbers – and that's from 1 to 19. So Edward lives at 19 Laburnum Avenue.

**23** **a** See the diagram beneath for the correct positions of the points A, B and C.

**b** Once you've plotted points A, B and C, you'll find that there are three different positions for D which would make ABCD a parallelogram; the one you're after is of course (2,3) since this is the only one with positive co-ordinates :




**c** The area of the parallelogram is 20 $m^2$ (or 20 square units if you're not using a 1cm grid).



**24** This does seem odd : the signposts at St Aubin and at Hautot both give you the definite idea that the two towns are 21 km apart – and yet the sign along the road seems to show that the towns are only 20 km apart. The clue to the mystery is the information given in brackets *road-signs in Normandy give distances to the nearest km.* The actual distances could be something like this :


The total distance of 20.7 km would be rounded up to 21 km whilst the 8.4 km and the 12.3 km would be rounded down to 8 km and 12 km (giving 20 km when you add them together).

**25** **a** John's guess was 110 kg and his wife's guess was 90 kg.

**b** No, it's true, at least not if you think in terms of kilograms rather than percentages. John's guess was 11 kg out (10% of 110) whilst his wife's guess was only 9 kg out (10% of 90).

26 **a.** $80 = 4^2 \times 5$

**b.** $92 = 2^2 \times 23$

**c.** $245 = 7^2 \times 5$

**d.** 40 can't be written in this way – the only squares which you can try are 4, 9, 16 and 36 and none of these is any good. 50 can be written as 52 x 2. But 60 can't be done – again the only squares under 60 itself are 4, 9, 16, 25, 36 and 49 and none of these is any good.

---

27 Each circuit for Ronnie is 140m, whereas each circuit for Reggie is 100m. The smallest number which both 100 and 140 both divide into is 700, so 700m is the minimum distance the rabbits' training run could be.

---

28 The rule is that to see how many odd numbers you've got in your series, you just add 1 to the last number and halve the result. This answer is also the number you're squaring. For example, look at :

$$1 + 3 + 5 + 7 + 9 + 11$$

The last number in this series is 11. Add 1 to 11 and halve the result and you get 6 – and you've got 6 odd numbers in the series. The sum of the series is 36, which equals $6^2$. So the answers to the questions are :

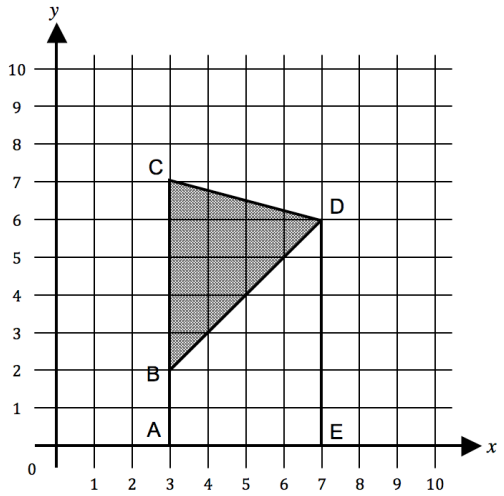
**a.** To get 16 x 16, we must have 16 odd numbers in our series, which means the last odd number must be 31.

**b.** To get 50 x 50, the last number must be 99.

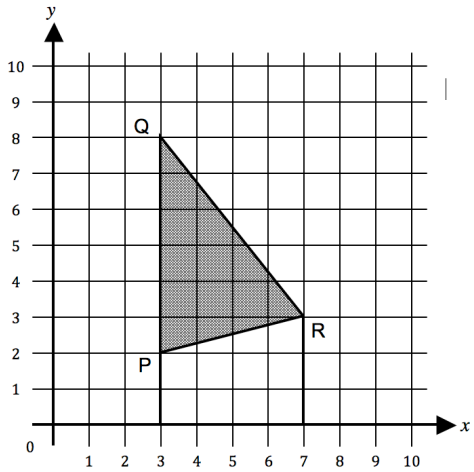
**c.** If we add 1 to 119 and halve the result we get 60, so there must be 60 odd numbers in this series.

---

29



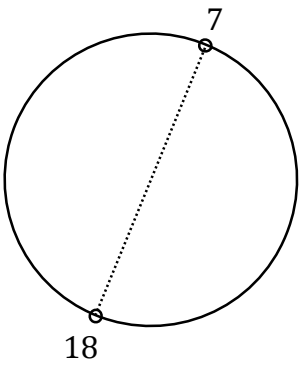

From the diagram you can see that trapezium ACDE has an area of 26 squares and trapezium ABDE has an area of 16 squares. By taking the one away from the other, you get 10 squares for the area of triangle BCD.



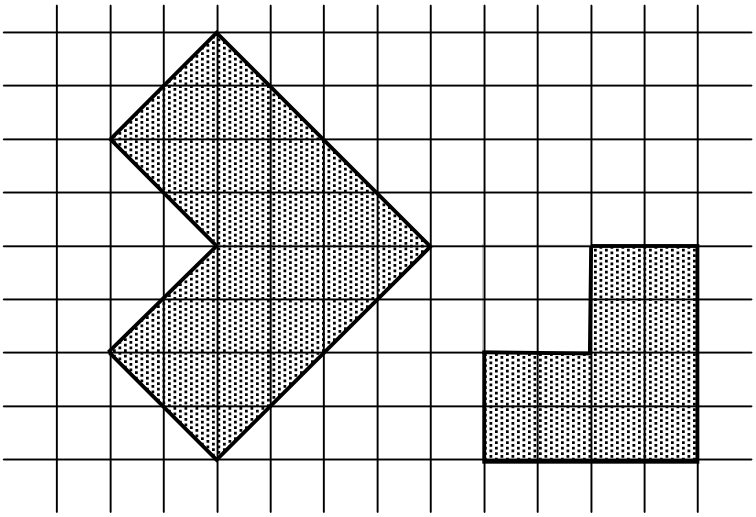

Using the same method, we get area of triangle PQR = 22 squares – 10 squares = 12 squares.

---

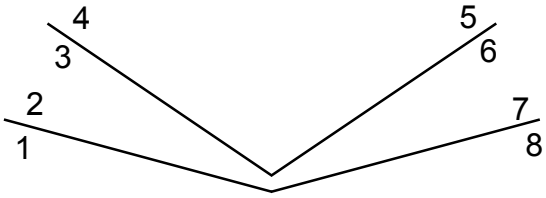
30 7 is directly opposite 18. Around the right-hand side of the circle there are 10 numbers between the 7 and the 18. (8, 9, 10 . . . 16, 17). So there must be 10 numbers around the left-hand side of the circle between the 7 and the 18. They will be 19, 20, 21 . . . 27, 28. So there are 28 children in the circle.



31 The area of the given shape is 24 squares. Here's the same shape but with half the area :



32 To get an idea of what's going on in this kind of arrangement, let's look at a simpler case. Imagine a newspaper made up of just two double sheets. This is how the pages will work out :



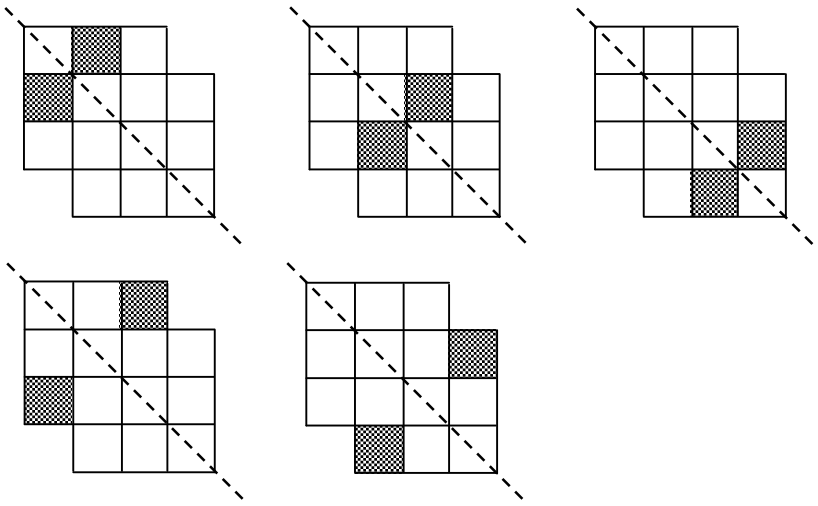

Notice that the centre spread has 4 + 5 = 9 and that every other time you separate out a double page you'll see the page numbers facing you add up to 9. Looking at the outer double page, you'll see that we have 1 + 8 = 9. The last page in the newspaper is 8 – and because there are four pages on each double sheet, there are just two double sheets in all.

Now we apply the same thinking to the questions on the card :

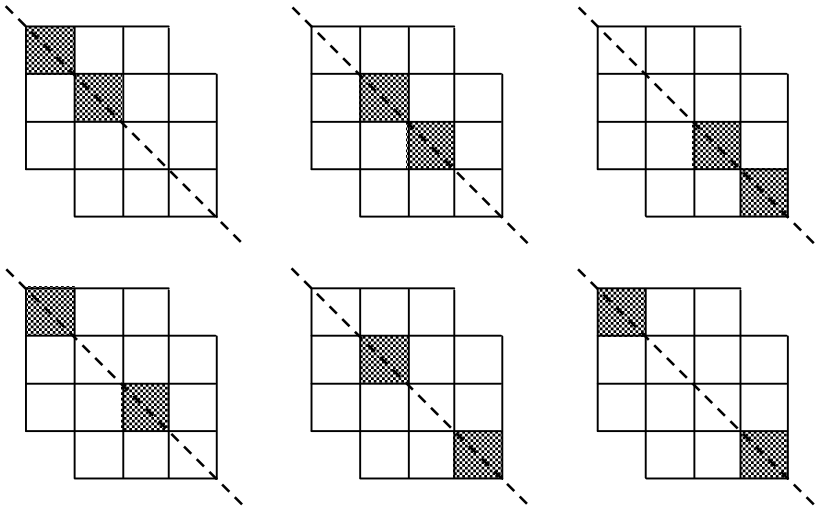
**a.** The centre pages are 16 and 17. These add up to 33, so the first and last pages must also add up to 33. The first page is number 1, so the last page must be number 32.

**b.** With 32 pages in all and four pages to each double sheet, there must be 8 double sheets.

33 **a.**



**b.**



34 **a.** 36 : 5 + 31, 7 + 29, 13 + 23, 17 + 19

**b.** 50 : 3 + 47, 7 + 43, 13 + 37, 19 + 31

35 **a.** 2, 2 **b.** 1, 4 or 4, 2

36 **a.** 5 **b.** 1 **c.** 1 (by putting in 1 at the beginning) and 25 (by putting in 7 at the beginning). **d.** no, if you double and take 1 away, you'll always get an odd number.

37 **a.** There are 75 apples altogether.

**b.** Alfred and Chris each need to give 5 apples to Bella.

---

38 **a.** The cuboid measures 3cm x 4cm x 10cm.

**b.** The volume of the cuboid is 120 $cm^3$.

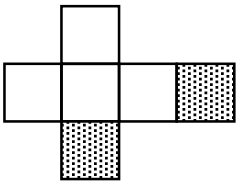
**c.** The smaller cuboid has volume 30 $cm^3$. Its dimensions could be (in cm) : 1 x 1 x 30, 1 x 2 x 15, 1 x 3 x 10, 1 x 5 x 6 or 2 x 3 x 5.

---

39 (**a**) 14 squares (**b**) 30 squares (**c**) 55 squares

---

40 This one will have the two shaded sides next to each other :



---

41 **a.** 92, **84**, 77, 71, 66, 62, **59**

(subtract 8, subtract 7, subtract 6 . . . )

**b.** **2**, 5, 11, 23, 47, **95**, 191

(double each term and add 1 to get the next)

**c.** 6.3, **5.7**, 5.1, 4.5, 3.9, 3.3, **2.7**

(keep subtracting 0.6)

**d.** 195, 168, **143**, 120, 99, 80, **63**

(these are just the square numbers with 1 subtracted – or if you prefer, you can think of the sequence as a progressive ie subtract 27, subtract 25, subtract 23 . . . )

---

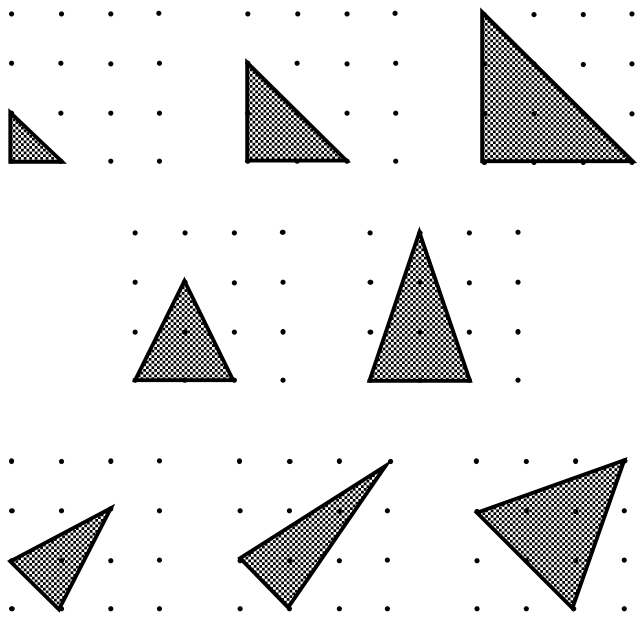
**e.** 2, 3, **5**, 7, 11, **13**, 17, 19

(these are just the first few prime numbers)

**f.** 64, **32**, 16, 8, 4, 2, 1, ½
(keep halving)

---

42



---

43 **a.** 90 m / sec = 5400 m / min = 5.4 km / min = 324 km / hr

**b.** 250 m / sec = 15,000 m / min = 15 km / min = 900 km / hr

---

44 (**a**) 15 and 2 (**b**) 25 and 4 (**c**) 41 and 8

---

45 I have 16 pencils and 3 jars.

---

46 The first counter has 7 on one side and 8 on the other; the second counter has 8 on one side and 6 on the other.

---

47 All that really matters is the time it takes Edward to complete the journey – and that's obviously half an hour. Running non-stop for half an hour, the dog will have covered 5 km.

---

48 **a.** You need to halve the 38 and add 1 to the result, to get our total = $20^2$ – 20, which is 400 – 20 = 380

**b.** This time we're looking at one more than half of 98, which is 50. So our total = $50^2$ – 50, which is 2500 – 50 = 2450

**c.** We're initially after a number where the number squared minus the number itself gives 870. It doesn't take long to see that 30 is the number we're after. Next we need to take 1 away from the 30 and double the answer, which gives 58. So now we know that 2 + 4 + 6 + . . . + 58 = 870, ie $x$ = 58.

---

49 ○ *method 1 :* By trying different likely measurements for the width of the upright and the length of the cross-pieces, you will soon find that the uprights are 0.25m in width and the cross-pieces are exactly 1m long. The strip we're interested in has 3 uprights and 2 cross-piece sections, so it must measure 2.75m.

○ *method 2 :* If we put together the two strips we're given, we'll get a strip with 6 uprights and 4 cross-piece sections. This will measure 5.5m from end to end – but this is exactly double what we're after, so our unknown strip must measure 2.75m.

---

50 **a.** For the first 10 days the 6 ferrets will eat 3 kg per day; that's 30 kg in all consumed and 15 kg remaining. Obviously the ferrets eat 500 g of ferret food each per day, so when there are only 5 ferrets left, they'll be eating 2½ kg per day among them. Divide 15 by 2½ and you get 6, or in other words the remaining 15 kg of food will last 6 days. So that's 10 + 6 = 16 days in all that Sally's supply of ferret food will last.

**b.** It takes 3 men 3 days to paint 3 rooms – so it would take 1 man 3 days to paint just 1 room. This means that 1 man working for 1 day can paint only 1/3 of a room.

---