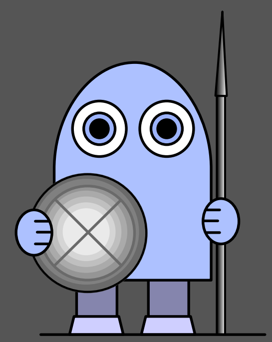
no problem ! book 1

50 maths problems with answers

four winds



four winds maths

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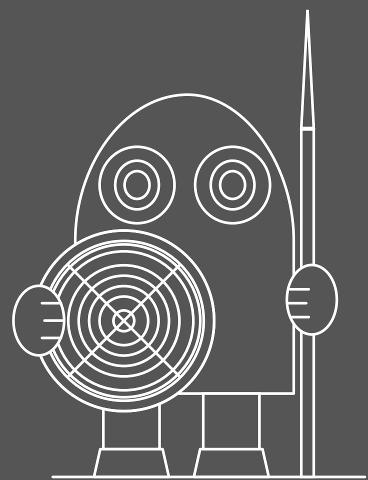


list of contents

1 your questions answered

Instead of a normal 'introduction' to the book, we've listed here answers to many of the questions you might ask at the beginning.

2 list of problems

This is a listing of all the problems, colour-coded to show you which area of the subject each problem focuses on.

3 problems 1 - 50

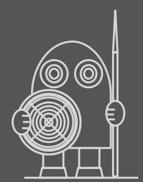
This is the main part of the book : here are the problems themselves, starting with a few easier ones and then going on to the harder challenges, before finishing with a small number of really quite difficult ones.

4 how to get started

This section has some helpful suggestions for those times when you're really stuck and just can't get going on a problem. Here you'll find a number of different ideas you can try, plus an easy way to remember them. Read this section before you start – or if you'd prefer, just dive straight in to the problems and come back to this when you need to.

5 answers 1 - 50

Here are full answers and explanations for all the problems. There's no right way of solving any of them and so for many we show you different ways of tackling them. This is purely for your interest of course ~ we know that for any of these problems, you might well have a better way (one which works well for you).



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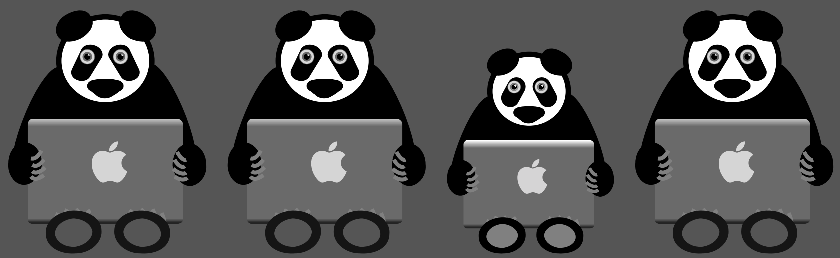
your questions answered . . .

where do all the problems come from ?

Here at Four Winds we have a dedicated team, working night and day to produce maths books – books of problems, books about maths etc. The team members live on bamboo shoots and take a short break each day to do the Times Crossword. They live happily together in the Four Winds HQ on the edge of the Berwyn Hills in Wales. Below is a picture of the four team-members who have worked on this book :



These four have quite different jobs : The poser loves making up interesting problems, whilst the checker likes to check the answers and the explanations. The artist (called El Nepalo) does all the graphics. And last of all, the team leader puts everything together . . .

Are the questions all of the same difficulty?

The first ten or so are a little easier (to get you started) and the last ten are rather more challenging.

What are the problems about ?

Well, as you'd expect, there are all kinds of problems about number. Then there are logic problems, as well as problems about probability and statistics. And of course, there are problems about shape. The book gives you a good selection of all these topics.

Is there one definite answer to each problem ?

Some of the problems have a definite answer, while others have a number of right answers. One thing is for certain, though – there's no one right way of doing any of them! Your way might be quite different from anything we've put in the answers section. But that's quite ok !

Do you have to do these questions on your own?

Some problem-solvers say they prefer working alone. Others say they prefer working in pairs because it makes them feel more confident. So, you choose! Work on your own or work with someone else – do what suits you best !

Do you have to do the questions in order ?

It's completely up to you !

How long should you spend on a problem ?

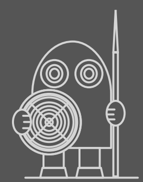
Again, it's up to you! If you enjoy problem-solving, then just keep going until you've found a solution. But if you've done lots of working-out and you're still lost, just leave the problem alone for the time being and come back to it later. Or perhaps try explaining the problem to someone else to see if they have any ideas. And if you just don't know where to start on a problem, do have a look at the 'how to get started' section : it has some useful suggestions.

Is it a good idea to get help if you're really struggling ?

If you've been working hard on a problem and you're getting nowhere, it's a good idea to explain the problem to someone else (mum, dad, brother, sister or whoever) to see whether perhaps they can come up with something new. BUT it's important to tell them that you really want to solve the problem using the maths you already know. Suppose, for example, someone says to you, 'I would use algebra for this problem'. The best thing in this case is for you to say, 'thanks, but I need to do it using the maths I already know.' And surprisingly, it often happens that while you're explaining a problem to someone else, you suddenly realise what it is you need to do to solve it.

Do you need algebra for any of these problems ?

No, you really don't, not for any of them! You can do all the problems using the maths you already know.



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Have these questions been tried out on anyone ?

Yes, of course! A number of willing volunteers cheerfully worked their way through the questions and afterwards shared their impressions with the Four Winds team. The suggestions they came up wtih helped us to make improvements to some of the questions and answers.

If you've got the right answer, is there any point reading the explanations ?

Yes, it's always good to see how someone else has tackled the problem, in case you come across new ideas. However, we don't ever want to give you the impression that our way of solving a problem is the only proper way, or even the best way. If you were going to drive from London to Manchester, you might look at a map and see that there are several different routes you could take (motorway, country roads or whatever). You'd choose a route which suited you. If someone then told you that a particular route was 'the only way' or 'the right way', you'd think they were either stupid or mad (or both). The same thing applies here.

Did the volunteers who worked through the questions have any advice for others ?

Yes, actually. They had various suggestions but the one thing they all agreed on was 'have a go!' – by which they meant don't be afraid to tackle any of the questions and even if at first you don't seem to be getting anywhere, do keep trying. Good advice!

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list of problems

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PTO

KEY

problems on logic, sets, combinations, permutations, probability, statistics

miscellaneous number problems (based on pre-algebra skills)

problems involving various aspects of shape (2-D and 3-D)

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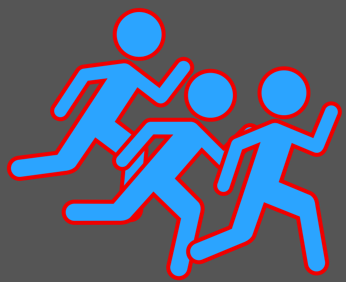
Sports Day

On Sports Day, the runners in the juniors' 200m race were Anne, Jeremy, Kate, Mary and Sanjit. This is how they finished :



- Kate came in second
- Mary was just behind Sanjit
- Anne was last

Write a list showing exactly who came where in the race.

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2 time's up!

Navya arrives home late one evening after a birthday visit to the cinema. Tired but happy, she brushes her teeth and heads for her bedroom. As she jumps into bed, Navya notices that the digital clock on the bedside table is showing 23:56

'That's a lucky sign,' she says, 'The four separate digits add up to 16 and today happens to be my 16th birthday!'

$$2 + 3 + 5 + 6 = 16$$

Thinking just of the 24-hour digital clock,

- Write down all the times where the digits add up to 24
- Write down all the times where the digits add up to 23
- Write down all the times where the digits add up to 22
- Write down all the times where the digits add up to 21
- Without writing them down, say how many digital times would give a total of 20

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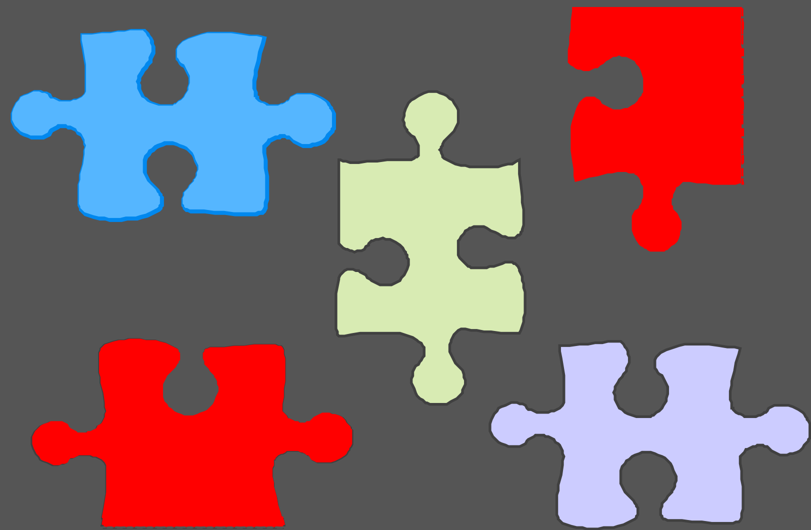
3 an edgy problem

Syed sits down one rainy day to solve a jigsaw puzzle which his uncle has given him. The finished puzzle is in the shape of a rectangle and all the pieces are roughly the same size. On the long side of the finished puzzle, you would count 54 pieces and on the short side of the finished puzzle, you would count 27 pieces.



1. How many pieces are there altogether in Syed's jigsaw?

2. As you can see, the corner pieces and the pieces along the edges have either one or two straight sides; these pieces are called 'edge pieces'. But there are also pieces which don't have any straight sides at all; these pieces are called 'inner pieces'. Just how many inner pieces are there in Syed's puzzle?

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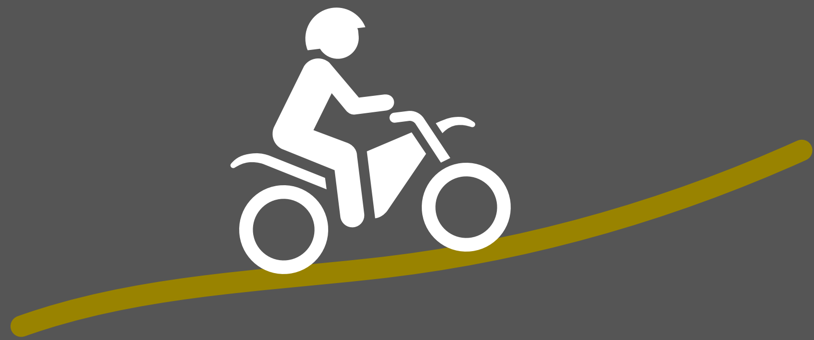
4 easy rider

Jamie works in a car factory in Detroit (that's in Michigan, USA); he likes to spend his weekends riding his scrambler bike in various off-road events around Detroit. Jamie's own bike is a Herald Rambler 250 but what he'd like to own one day is a Ducati Scrambler.



Jamie has never actually won an event but this season he's been doing pretty well. In the last seven club events his finishing positions have been as follows :

3rd, 2nd, 3rd, 4th, 2nd, 3rd, 4th

There are eight events in the season, so Jamie is busy training for the all-important last one. If he can end the season with an average finishing position of 3rd or better, he'll get a cash prize of $1200. This could go towards buying a Ducati ! What position must Jamie achieve in this last race if he's to end up with 3rd as his seasonal average ?

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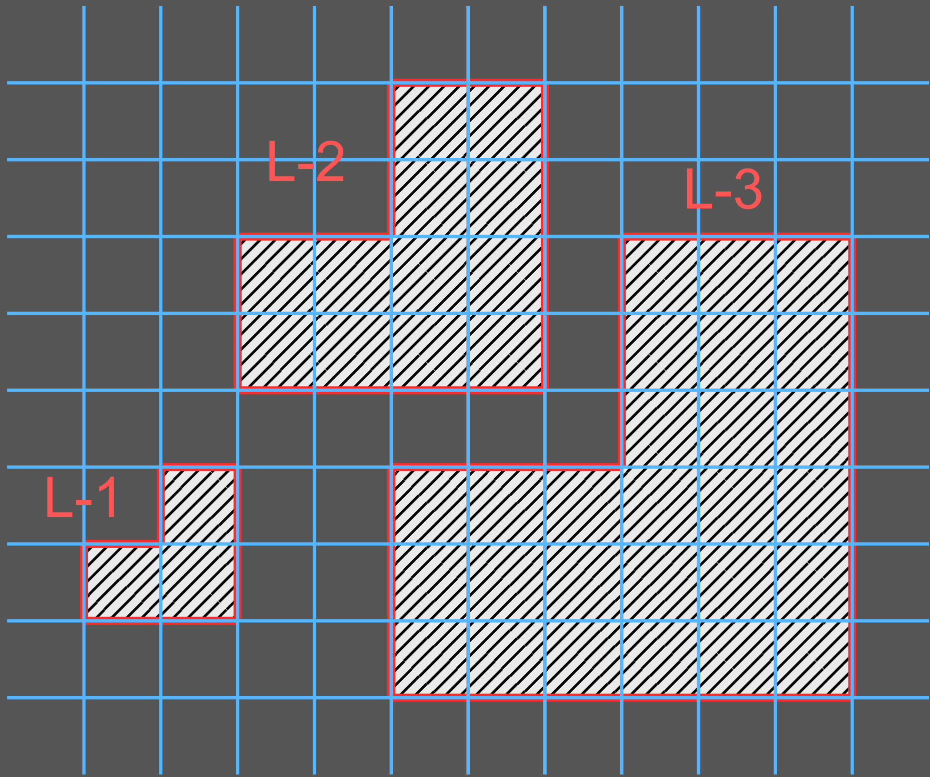
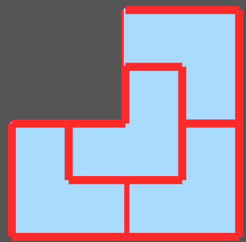
5 L is for learner

You'll need squared paper for this one . . .



Look at the figures on the right. You've got the same shape but in different sizes. Think of the smallest L-shape as a tile. If you have a number of these L-1 tiles you can completely fill a larger L-shape with them; of course, some of the tiles will need to be rotated or flipped over but it can be done.

Here's how for shape L-2 :

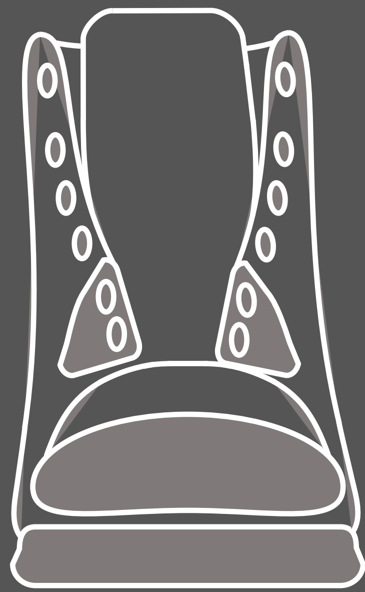


- Now try to fill the largest L-shape (shape L-3) with small tiles. Can you find more than one way of doing this?

- How many tiles were needed for shape L-2? – or for shape L-3? – and how about shape L-4 ? Describe the pattern you've found.

6 Jake's leaflets

Jake is a student and he's also a bit hard up. To make some money, he recently took a job delivering leaflets. It's not a great job : you have to deliver an awful lot of leaflets to earn just a little money but at least it's better than nothing! Or so Jake thought . . .



On his first day, Jake's feet hurt terribly. Before long, one of his toes was bleeding and his ankle was raw. But he kept going and kept going . . . until he'd delivered exactly half of the leaflets.

At this point Jake had some lunch (a quick sandwich in his case) and then bravely started off again. But bad luck was coming his way! When he'd delivered just 75 more leaflets, it started to rain, his boots began to leak – and really that was the end for Jake! He gave up (even though in his sack he still had 400 leaflets).

How many leaflets did Jake have at the start of his day?

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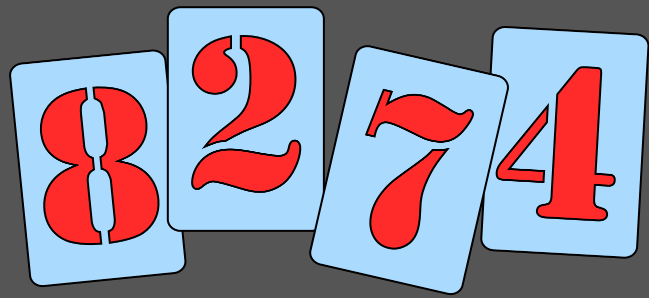
7 getting warmer . . .

Hassan's teacher gives him four number cards : 8, 2, 7 and 4, just like the ones pictured here. The teacher then gives Hassan a challenge : he has to use all four cards to make a number as close as possible to 7500. Hassan shuffles the cards about and finally settles on 7428. 'You can get nearer than that,' says the teacher – and straight away Hassan realises he needs to shuffle two of the cards around. You get the idea . . .



Using the same four cards as Hassan, write down the numbers you can make which come nearest to these :

- 7200
- 2900
- 5000
- 6250
- 4350
- 9270
- 8650
- 8351

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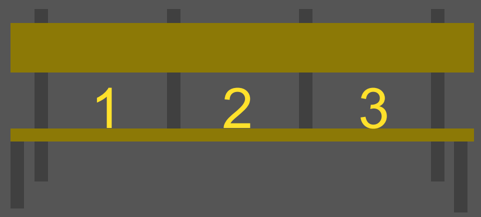
8 in the hot seat . . .

Rosie, Sam and Tom are sitting on the short bench outside the headmaster's room. They are in trouble - all three have been late getting to their lessons. There are 3 seats on the bench :



The children have just plonked themselves down randomly. What's the probability that Rosie is sitting in place number 1?

* not everyone has come across probability, so just before the answer to this question, you'll find a simple explanation . . .

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9 overlapping rings

Look carefully at the shape on the right. It's made up of three coloured rings linked together in a certain way. Imagine the rings are made of card and that you're looking at them from above.

- Suppose someone cuts the blue ring and removes it. Will the green and yellow rings still be linked?

Now look at the shape on the left. This one is made up of four coloured rings linked together in a certain way. Again, imagine the rings are made of card and that you're looking at them from above.

- Suppose someone cuts the blue ring and removes it. What will happen to the remaining three rings?
- Suppose instead someone cuts the blue ring and the red ring and then takes them both away. What will happen to the remaining two rings?

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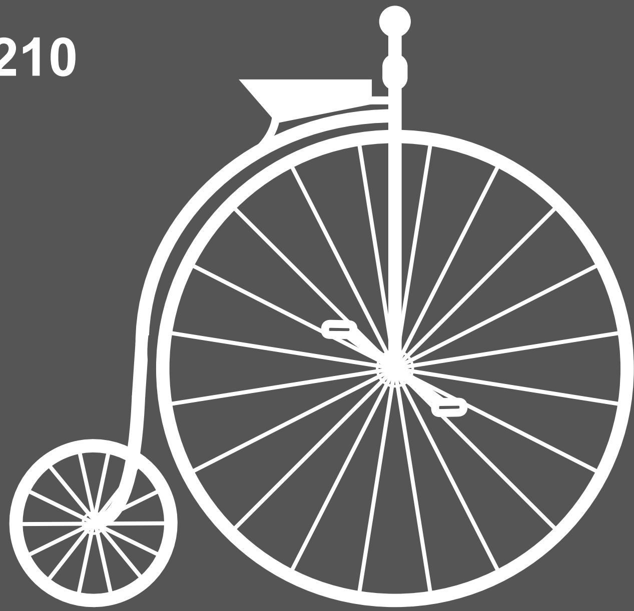
10 The Pedalman

The Maths Detectives are on the trail of a wicked bicycle thief they call The Pedalman. They have tracked him down to a street in London, called Old Montague Street, but they don't know which number house he's staying at. They do know for sure that it's one of these nine numbers :

23 49 62 87 105 121 169 188 210



They're also sure of these important facts :

- the number is a not a square number
- the number is not a prime number
- the number is not a multiple of 7
- the number is a multiple of 3

Your problem : What's the number of the house where The Pedalman is staying?

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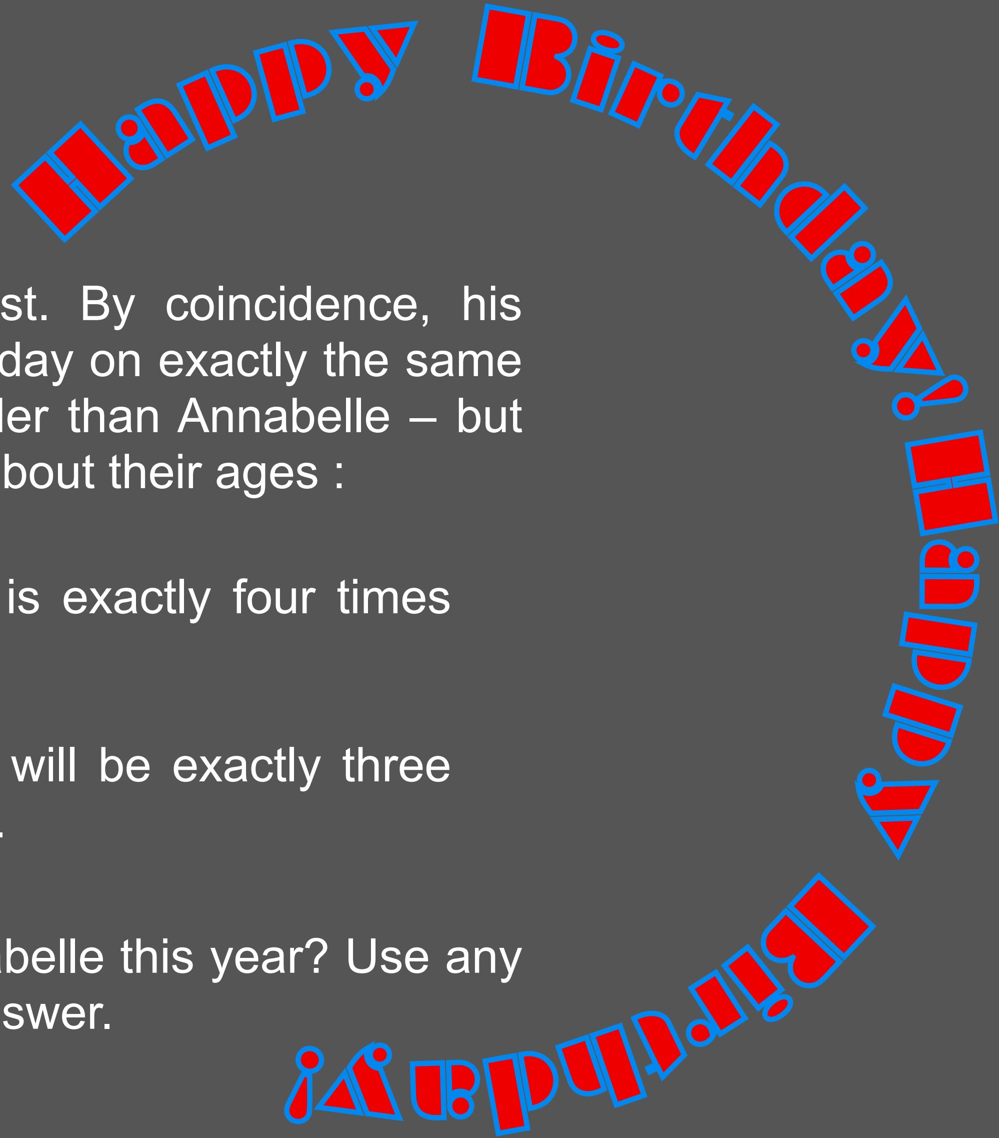
happy birthday, Ben !



Ben's birthday is on June 1st. By coincidence, his cousin Annabelle has her birthday on exactly the same day. As it happens, Ben is older than Annabelle – but here are two interesting facts about their ages :

- This year, Ben's age is exactly four times Annabelle's age.
- Next year, Ben's age will be exactly three times Annabelle's age.

So, how old are Ben and Annabelle this year? Use any method you like to get to an answer.

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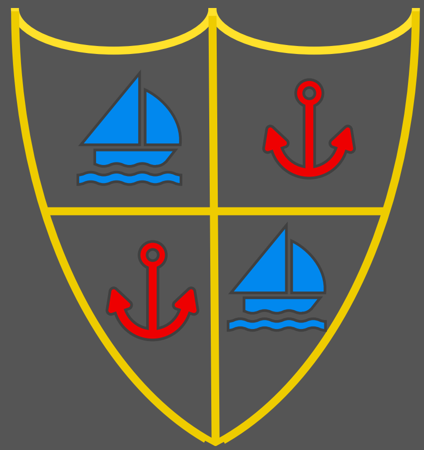
12 sailing by . . .

On the right is the new badge of the Lymington Sailing Club. As you can see, the design uses just three different colours. The Club Committee members like the new design but they're not sure about the colour scheme. They are quite sure that :



- The two boats must be the same colour as each other.
- The two anchors must be the same colour as each other.
- The trim must be yellow.
- There must be three different colours in the final design.

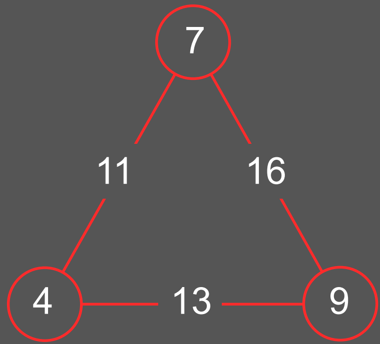
Here are the colours you're allowed to choose from :

How many different colour arrangements are possible ?

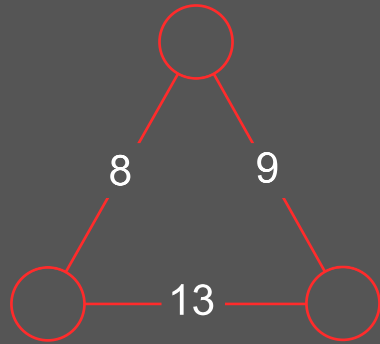
special note :
We're counting eg boats blue combined with anchors red as a completely different arrangement from boats red combined with anchors blue . . .

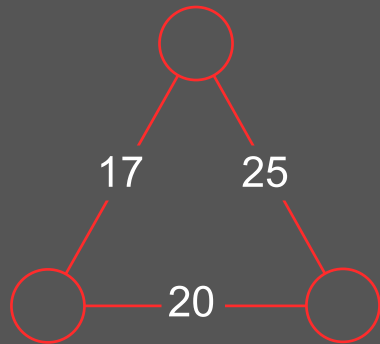
13 number triangles

Look at this 'number triangle' : as you can see, the numbers at the end of each line add together to give you the number between them.

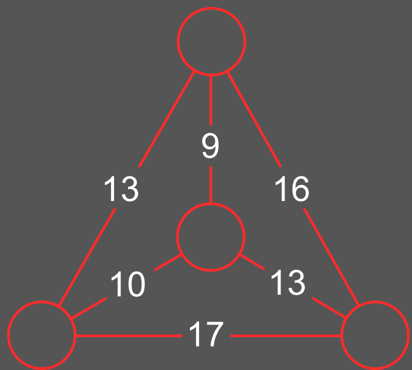


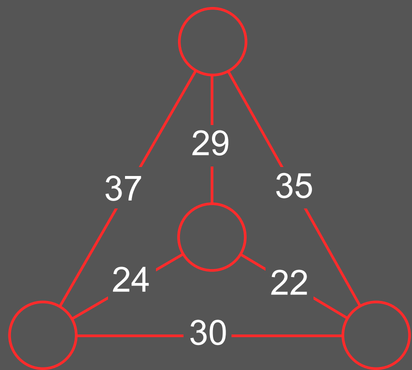
Now here are two more number triangles. This time, the 'totals in-between' are there but the numbers at the end of each line are missing. Try to work out what these missing numbers must be.

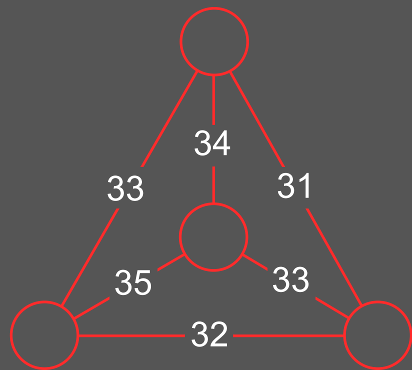




And to follow, something a bit harder! Try to work out what numbers should go into the circles.







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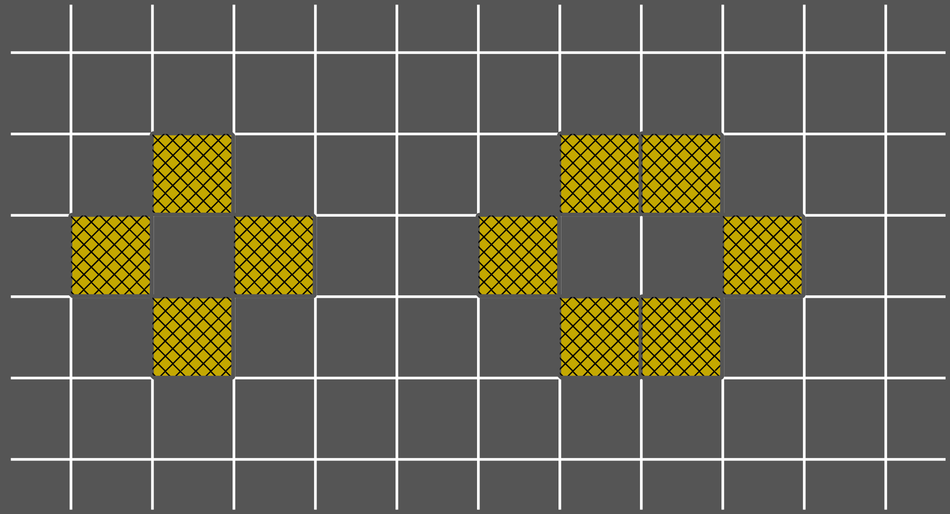
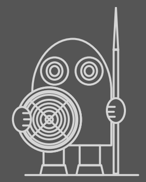
14 penning sheep

On MacHenry's Farm they sometimes need to make temporary pens for holding sheep. The sheep pens are made using square bales of hay. There are just two ways allowed for putting hay-bales together : they can be joined all along one side or they can be joined corner-to-corner, as in the diagram on the right.



As you can see from these two arrangemens, 4 hay-bales will let you enclose an area of 1 square and 6 hay-bales will let you enclose an area of 2 squares . . .

Now for your problem (and remember, you **must** follow the MacHenry's Farm rules): what's the **maximum** area you can enclose if you've got 9 bales of hay?



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15 parcels-to-go

There are two parcel delivery firms operating in the North-West of England : there's 'ParcelDrop' and then there's 'Parcels-to-Go'. These two firms are in fierce competition but they don't quite go for the same size of parcel; you can tell this from their delivery charges, which are as follows :

ParcelDrop . . . £2 basic charge plus 50p per kg

Parcels-to-Go . . . 75p per kg (no basic charge)

- Which of the delivery firms is cheaper for small parcels? (Take 'small' to mean just 1 or 2 kilograms.)

- There's a certain size of parcel which costs the same to send whichever of these two firms you decide to use. What size of parcel is this?

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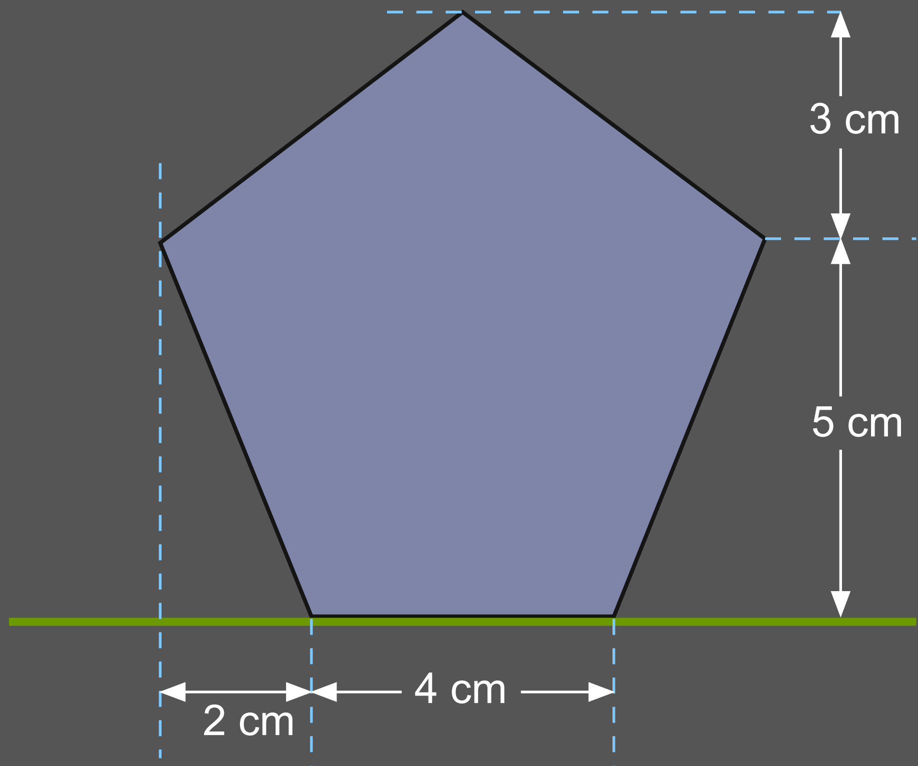
16 five-a-side

Look at the pentagon on the right. It's not a **regular** pentagon, as the angles are not all the same - but it has got **bilateral symmetry** (that's to say its left and right sides are mirror-images of each other). Here's your problem :

Using any method you like, find the exact area of this pentagon.



** You might find it easier to do this problem if you first copy the pentagon onto squared paper.*

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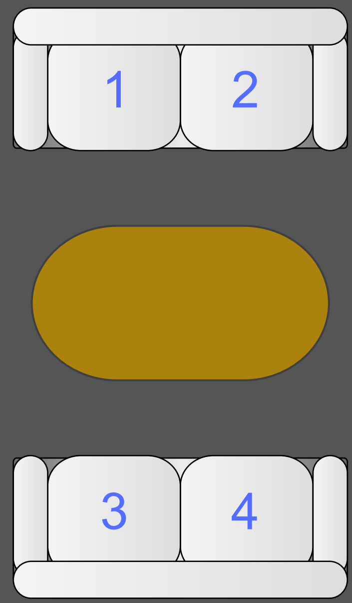
17 Alice's party

It's the first day of the school holidays and Alice has decided to invite three of her friends round to drink coffee and talk about the cycling trip they are planning. Mum says they may use the small sitting room, where there are two sofas on either side of a coffee-table. Alice knows she must think hard about who sits where if the afternoon is to go well. She draws a simple diagram, like the one on the right, with the places numbered from 1 to 4. There are four in the group (Alice, Ben, Charlie and Debbie) and these are the conditions which Alice knows she has to remember :



1 *Debbie must have seat number 4 as it's nearest to the telephone and she's expecting a call.*

2 *Alice must never be seated next to Charlie, as they always quarrel.*

3 *Debbie will happily sit next to Alice or next to Ben but she will not sit next to Charlie.*

4 *Charlie must not sit facing Alice as he always pulls faces and makes her giggle.*

Your problem : who sits where ?

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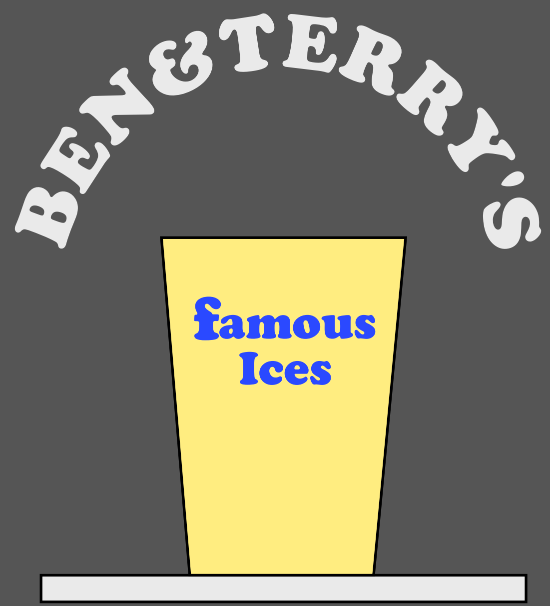
18 Ben and Terry . . .

This question is about a father-and-son ice-cream business based in the seaside town of Eastbourne. At one end of the beach, Ben is selling his ice-creams at the rate of one every 10 minutes. At the other end of the beach, his son Terry is selling ice-creams at the rate of one every 5 minutes. Between the two of them, how many ice-creams per hour are they selling? Working this out is proving to be a hard problem for Ben and Terry . . .



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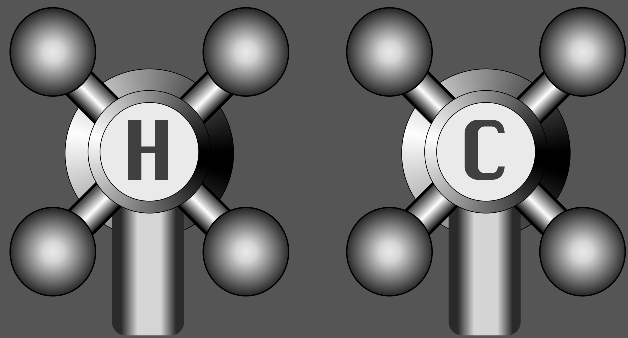
19 . . . and Sally

The last problem was about Ben and Terry. . . whereas this problem is about a girl called Sally, who loves to measure things. One day she times how long it takes for her bath to fill using just the hot tap; she finds it takes 12 minutes. When she uses just the cold tap to fill the bath, it takes only 6 minutes. If she were to switch on both taps together, how long would it take her to fill the bath?



hint *: There's a link between this question and the last one. Take a hard look at how you solved the last question and then ask yourself : does that question suggest a different way of looking at the facts you're given here?*

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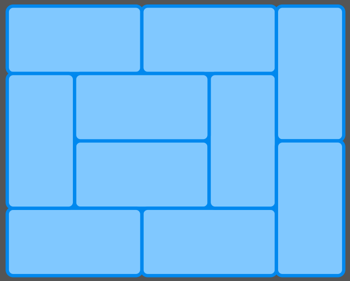
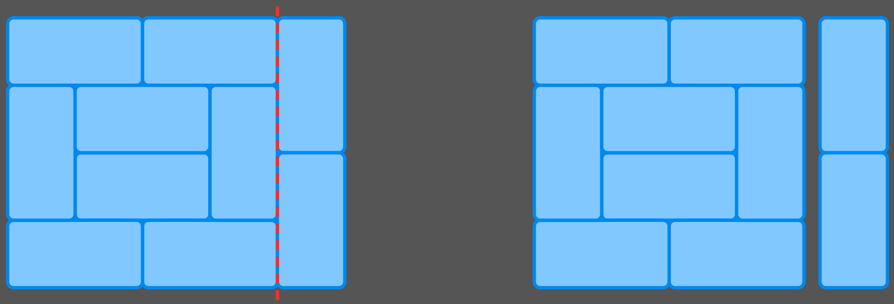
20 whose fault is it?

Suppose you start off with a 5 x 4 rectangle and you want to tile it completely with 2 x 1 tiles. You'll obviously need 10 tiles and you'll probably find different ways of doing it. Here's one way :



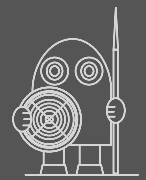
Something you might notice about this way of tiling the large rectangle is that it has a *fault line* in it. We'll show this a bit more clearly here :



A *fault line* is a line which runs right across the rectangle from one side to the other. In a way the fault line divides the rectangle into two separate parts.

Mathematicians have proved that you just can't tile a 5 x 4 rectangle with 2 x 1 tiles without getting a fault line somewhere. But you can tile a 6 x 5 rectangle completely with 2 x 1 tiles without getting a fault line. Can you find a way of doing this?

special note : You'll find turned-over dominoes make good plain 2 x 1 tiles if you want to solve this problem in a practical way – which is probably the best way of doing it.



21 mr owl ate my metal worm

You know what a *palindrome* is : it's a word or a number which reads the same way from left to right and from right to left. In the world of numbers, 757 is an example of a *palindrome number* and 48084 is another one; and if it's years you're thinking of, then 1331 is an example of a *palindrome year*.

As you might know, there are *palindrome words*, like radar, rotor or reviver (though of course they don't all have to start with an *r)*. And there are even *palindrome sentences* where the letters are the same left to right and right to left; the title of this problem is one example (although here the grouping of letters is not symmetrical).

- Write down all the palindrome numbers between 100 and 2000 where the digits of the number add up to 8.

- 2002 was a palindrome year. When will the next one be ? And when was the last palindrome year before 2002 ?

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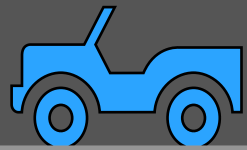
22 Ravi's mean question



Ravi's teacher asks him to think of three numbers, 'Any three numbers', says the teacher, 'but the average of the three numbers must be 10 exactly!' Ravi does as the teacher asks, and he then gives the class these three pieces of information about his numbers :

- The mean of the three numbers is 10. (We knew that already!)
- The smallest number is 10 less than the largest number.
- The largest number is exactly double the middle number.

What are Ravi's three numbers?

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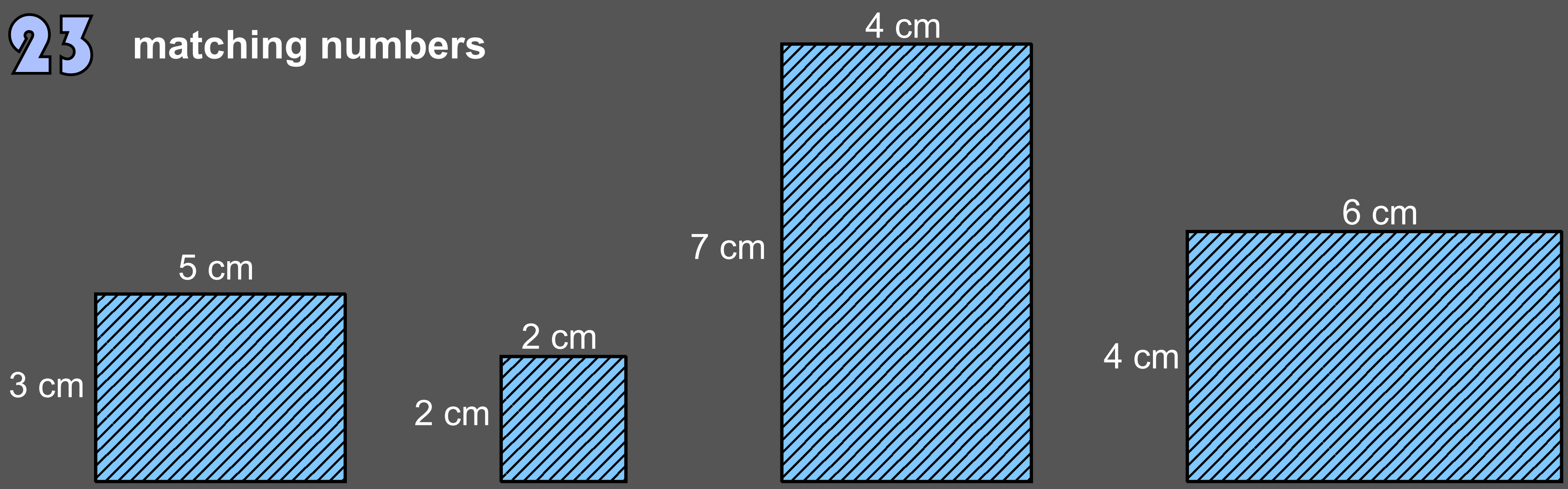
23 matching numbers



area = 15 cm^2
perim = 16 cm

area = 4 cm^2
perim = 8 cm

area = 28 cm^2
perim = 22 cm

area = 24 cm^2
perim = 20 cm

If you look at the rectangles above, you'll notice that for each rectangle the figure for the area and the figure for the perimeter are different. But there are rectangles where the area and the perimeter show the same number! Try to find two different rectangles where this happens. Keep to whole numbers for the lengths of the sides.

hint : Perhaps one of the rectangles you're looking for is a square. Remember, a square is a kind of rectangle.

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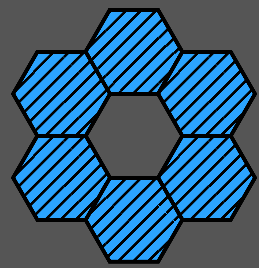
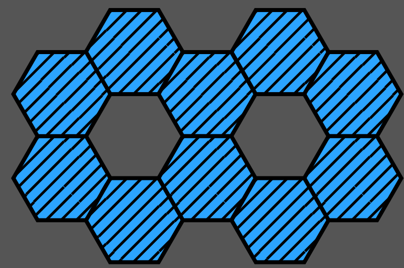
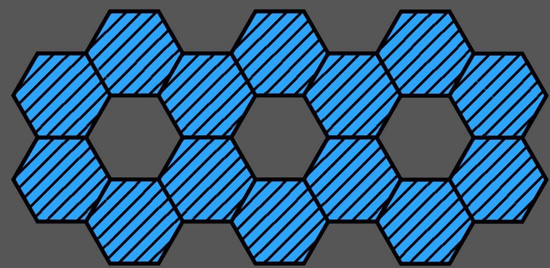
24 six-a-side !

Mark has made the shapes on the right by putting together hexagon tiles. The tiles are completely symmetrical and each of their sides is exactly 1cm.







- How many tiles has Mark used in the first shape (call it the 'one-hole' shape)? How many has he used in the next one (the 'two-hole' shape)? And how many has he used in the 'three-hole' shape?

- Without drawing it, work out how many tiles Mark would use for a four-hole shape.

- Mark has a large box of these hexagon tiles and he spends some time making a shape like the others but longer. He uses 150 tiles. By looking at your previous answers, work out how many holes this new long shape must have.

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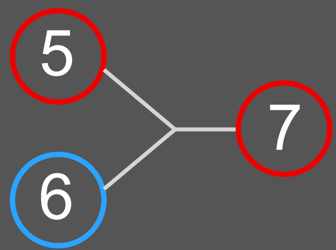
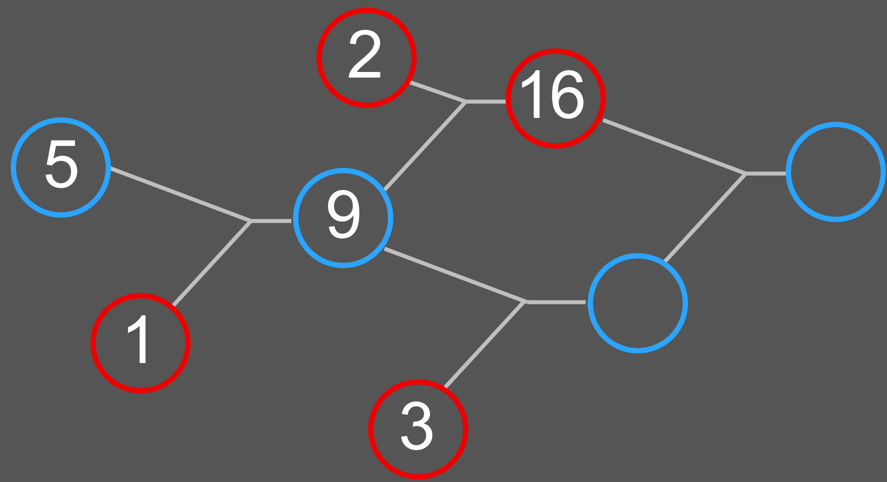
25 mapping webs 1

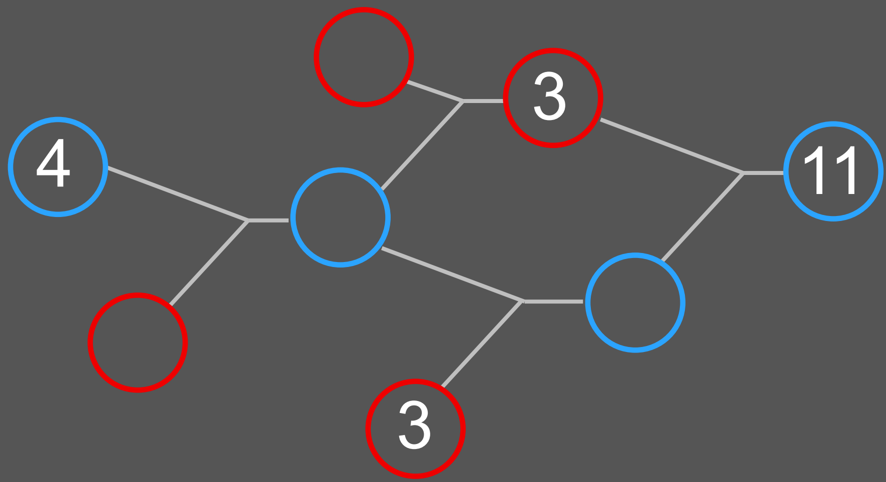
Perhaps you know what a **mapping web** is : you have a certain rule for combining numbers and using this rule you can build up a web of numbers. Starting with different rules and with different numbers, you get different mapping webs.

Now to explain the mapping web for this problem : wherever you see lines coming from a blue circle and a red circle and linking to another circle on their right, you just double the number in the blue circle and then subtract the number in the red circle, putting your answer in the circle on the right.

Here's an example :







Your problem : make a quick copy of these two mapping webs and work out how to fill in the blanks.

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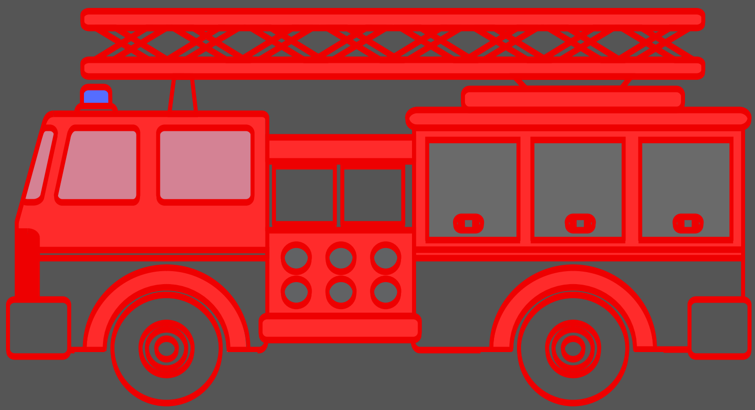
26 three brothers

Simon, John and Peter are brothers. One of them is a teacher, one is a fireman and the third brother is a builder. One of the brothers lives at the seaside, one lives in the country and the other one lives in town. Here’s some more information about them :

- John is not a teacher
- Simon does not live at the seaside
- Peter is not a fireman
- the teacher lives in town
- the fireman does not live in the country
- Simon is not a builder
- the teacher is not Peter



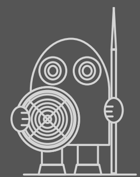
Try to work out which brother does which job and where each brother lives and then answer these questions :

1 Who is the fireman?

2 Where does the builder live?

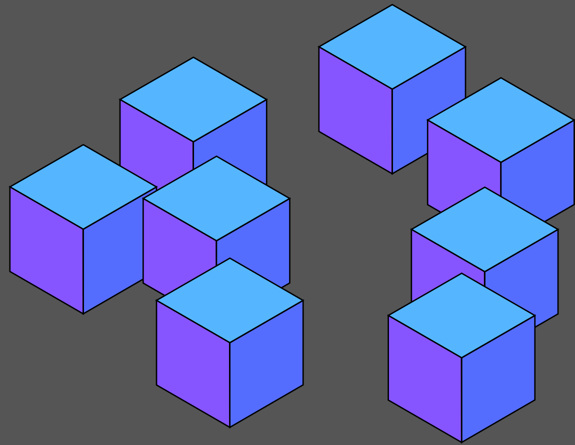
3 What is Simon's job?

4 Which brother lives by the sea?



27 cube surfaces

Sajid has some brightly coloured 1cm unit cubes. He finds that there are three different shapes of cuboid which he can make by joining eight of the unit cubes face to face.



- Make a sketch of each of Sajid's three cuboids.
- Carefully work out the surface area of each of the cuboids.
- What's the total surface area of the eight unit cubes before they're glued together?

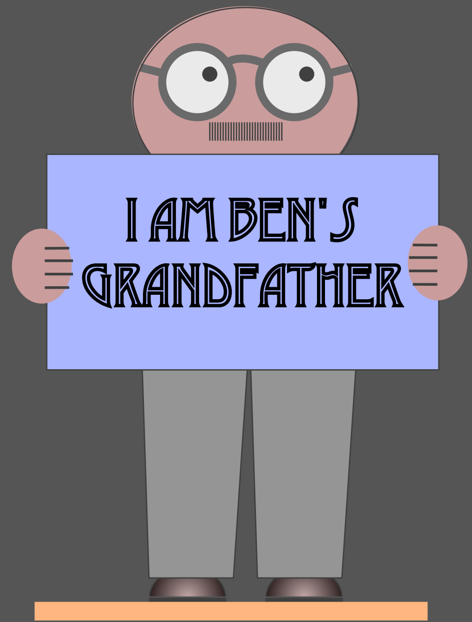
** A **cuboid** (just in case you're not sure) is a rectangular 3-D shape. You can think of it as a cube that's been stretched. Most cardboard boxes are cuboid in shape.*

** You might not have come across **surface area** before now – but it's not a difficult idea : To work out the surface area of a solid shape like a cube or a cuboid, you just find the area of each separate face – and then add all these areas together. And that's surface area!*

28 a grandson called Ben . . .

Ben has a grandfather whose house is just a short walk away from Ben's house. Here are some interesting facts about the grandfather's age :

- It's a prime number.
- When you add together the digits of this number, you get 8.
- John's parents are both aged 45.



Ben doesn't know how old his grandfather is. 'He could be any age,' says Ben. But of course, that's not true, is it? Using the information above, work out how old Ben's grandfather is most likely to be.

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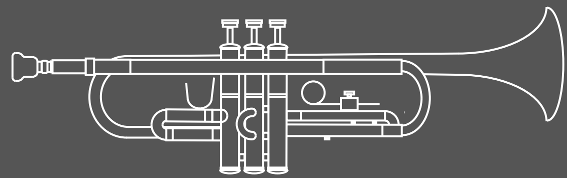
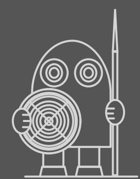
29 brass notes

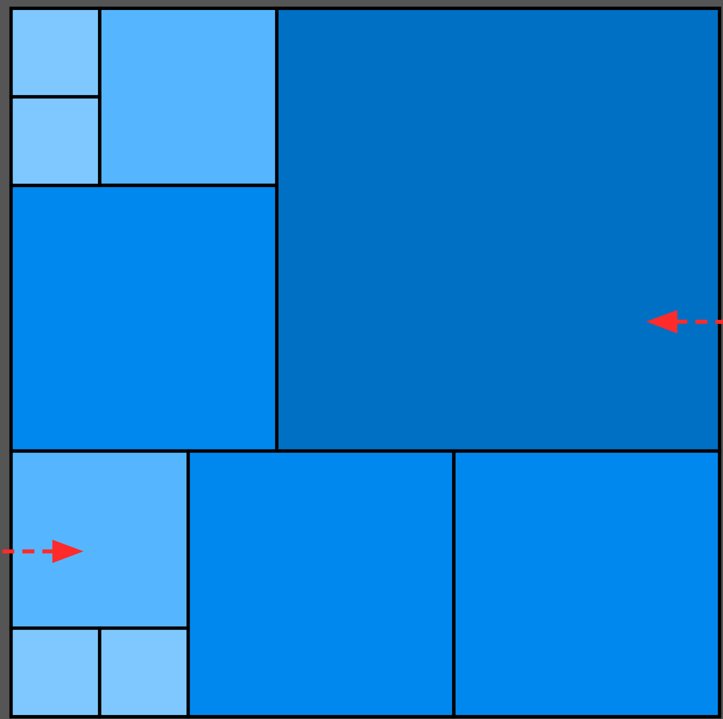


The trumpet has 3 main keys, or 'valves' as trumpeters call them. Each valve can be either up or down. So, how many different positions can you find for these three valves to be in? In other words, how many different arrangements (up or down) of these three valves can you find? Decide on your own way of listing the different possibilities – but do explain it!



30 all square

The larger square shown here is made up of ten smaller squares – and these smaller squares are of four different sizes :



. . . what's the area of this square ?

problem :

if this square has an area of 36 square units . . .

NOTE : squares with the same shade of blue have the same area as each other

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31 Mr Average

In the Olympic Village (men's quarters) there's a small lift which takes athletes from the ground floor up to the shower rooms. There's a sign in the lift which says,'Max. Weight 360 kg'. Usually only one or two athletes at a time get into the lift but one morning four men at once get into the lift. This is a maths question, so let's call these four athletes Mr A, Mr B, Mr C and Mr D. Mr A is a shot put champion, Mr B is a long-distance runner, Mr C is a sumo wrestler and Mr D is a hurdles champion. Here's some information about what they weigh :

- Mr A weighs exactly the average (mean) for the group
- Mr B weighs 40 kg (and that's 80% of D's weight)
- Mr C weighs exactly double the group (mean) average

Your two problems are these : Firstly, what does Mr D weigh? And secondly, do the four men together exceed the maximum permitted (safe) weight for this small lift?

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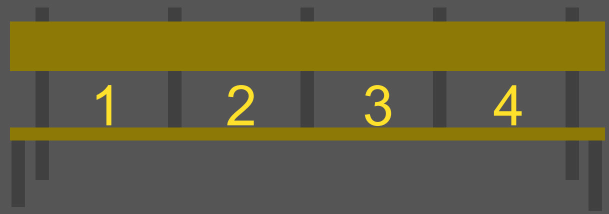
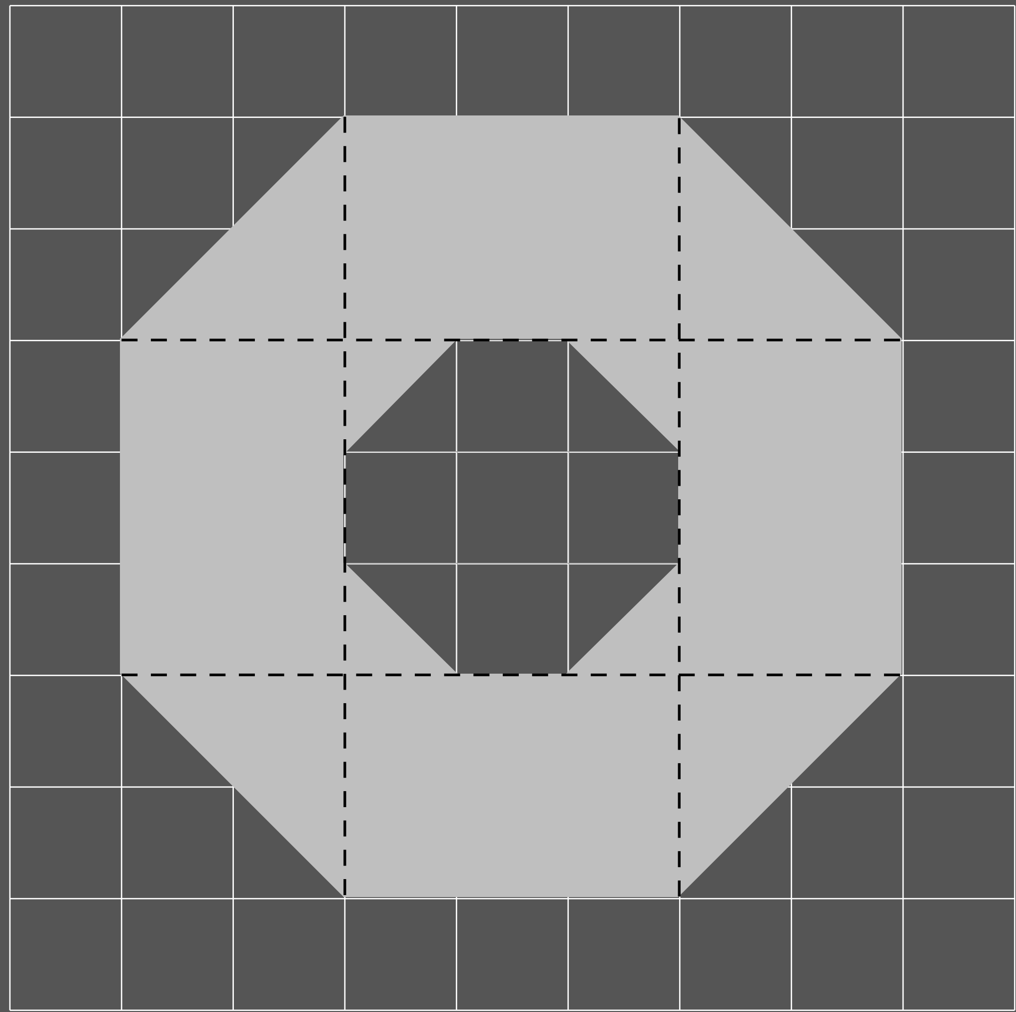
32 the long bench

Rosie, Sam and Tom are sitting on the long bench outside the headmaster's room. They are in trouble once again - this time for bringing pet mice to school. There are 4 seats on this bench :



There's already someone sitting at one end of the bench but the children just plonk themselves down randomly on the remaining three seats. Assuming that seats 1 and 4 are equally likely to be occupied already, what's the probability that Tom sits down in place number 1?

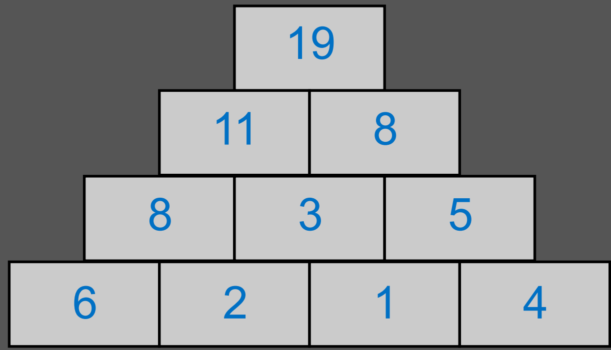
33 an octagon ring . . .



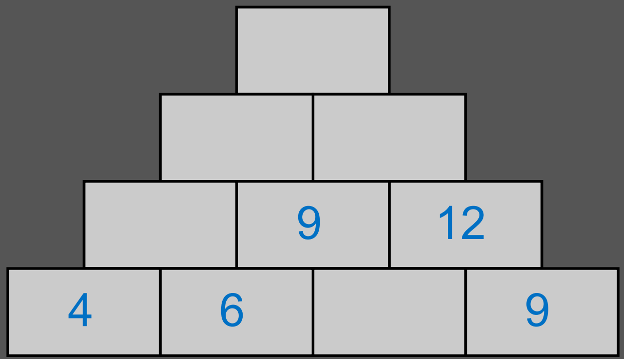
Look carefully at this shape, which has been drawn on a square grid. It's a kind of ring (you could call it an 'octagon ring') and all the corners fall exactly on grid points. Use any method you like to work out the area of this ring. Your answer will be an exact number of squares.

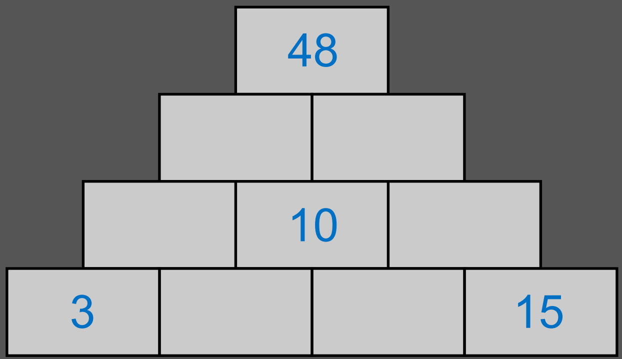
34 you're my number wall !

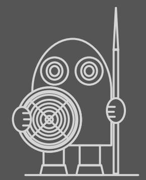
Perhaps you've come across **number walls**? The way a number wall works is this : when you have two bricks next to each other, you add their numbers together – and this total is the number for the brick resting on them. You'll see what this means as soon as you look at the number wall on the right. Easy number wall questions usually give you the numbers on the bottom row and some others above – and then ask you to fill in the gaps. It gets a little harder when there are numbers missing from the bottom row – and it becomes more difficult altogether when most of the mumbers are missing.



Here are your two problems. There's an easy one to begin with! You'll soon work out what the missing numbers must be. The second problem is harder. As you can see, you're given just four numbers and you have to find all the others. At first it looks impossible but – with a little hard thinking – it can be done!







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35 remainders

If you divide 11 by 2, you get a remainder of 1; divide 11 by 3, you get a remainder of 2; divide 11 by 4, you get a remainder of 3.

Exactly the same thing happens with 23 : divide it by 2, you get a remainder of 1; divide it by 3, you get a remainder of 2; divide it by 4, you get a remainder of 3.

You'll find you get the same pattern in the remainders if you divide 47 by 2, by 3 and by 4. Just try it!

Now here's the challenge. There is a number under 65 which is even more remarkable. If you divide it by 2, you get remainder 1; if you divide it by 3, you get remainder 2; if you divide it by 4, you get remainder 3; and if you divide it by 5, you get remainder 4.

Can you find this special number?

rem

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36 parking mad . . .

There are three cars parked outside Dr Brown’s surgery. One of the cars is a sports car, one is a family saloon and one is an estate car. One of the cars belongs to Dr Brown, one belongs to Mr Smith and one belongs to Miss Green. Here’s some information about them :

- the sports car is blue
- the red car is a family saloon
- one of the cars is yellow
- Dr Brown’s car is not yellow and it’s not a sports car
- Miss Green’s car is not blue

And your problem is : Who owns the sports car?

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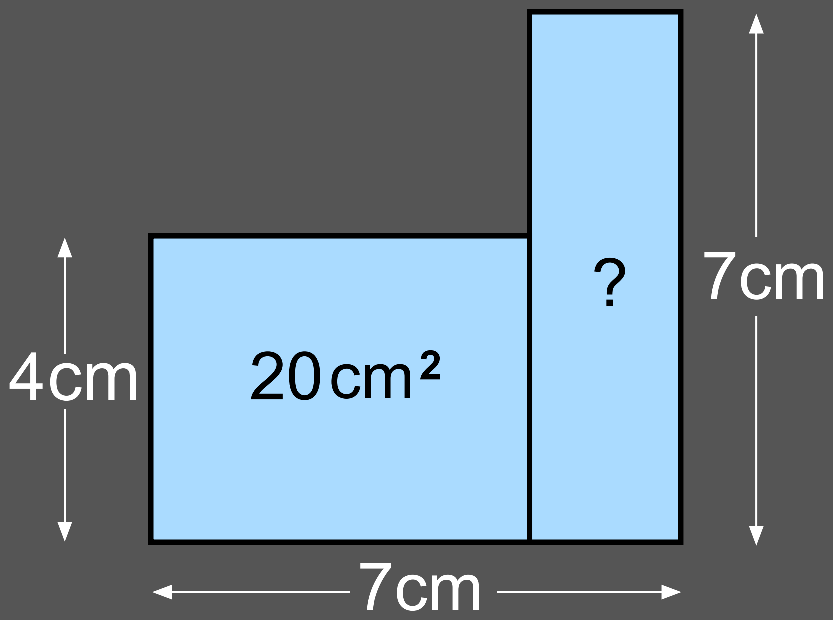
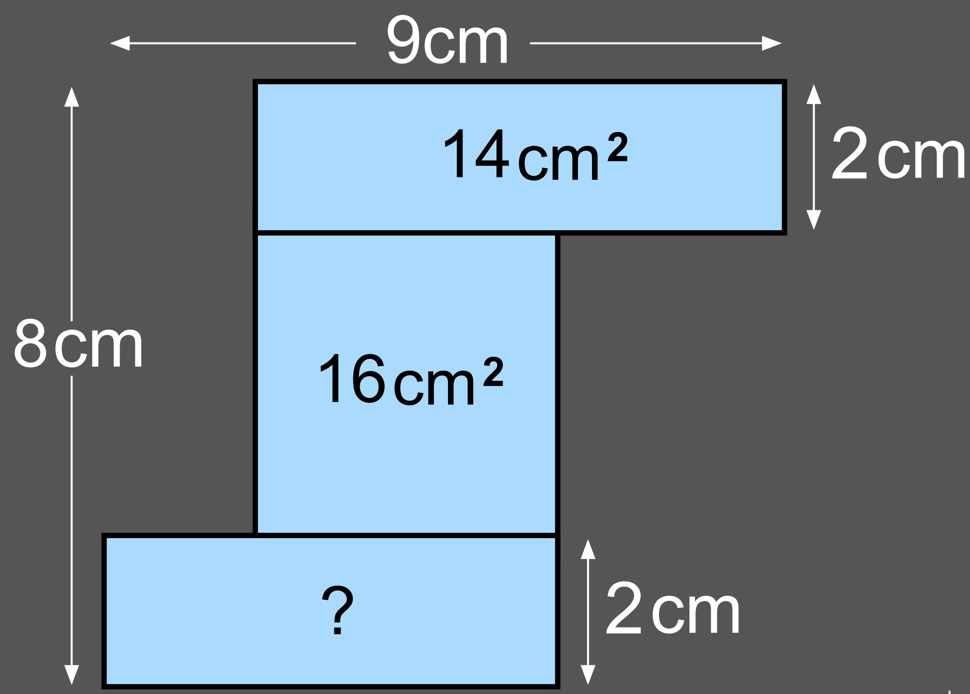
37 area mazes 1 & 2

Look at the two figures below. The one on the left is made up of two rectangles and the one on the right is made up of three rectangles.

Some lengths are shown on these figures and so are some areas. For each of these figures, you have to to work out the unknown area. With each figure, you will need to use the lengths and areas you're given in order to work out other lengths, until finally you have enough information to find the area of the last rectangle.





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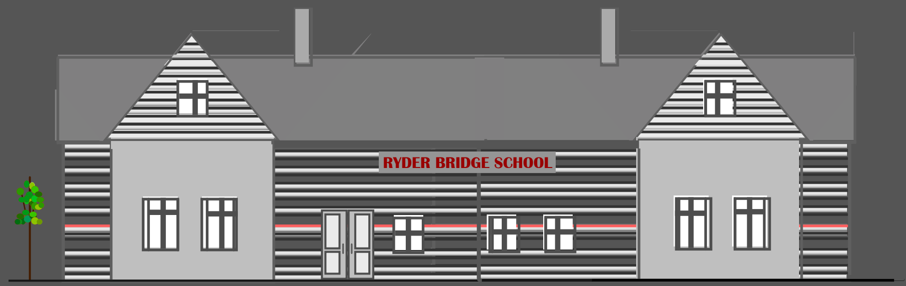
38 teaching equality

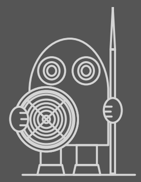
Of the 36 teachers who work at Ryder Bridge School, exactly half are men and half are women. The School is open every weekday (that's ten half-days in all) and on any half-day three-quarters of the teachers will be present.



- Mr Gladstone, the Schools Inspector, turns up at Ryder Bridge one thursday afternoon. Can he be sure there will be a female teacher present ?

- What's the largest number of female teachers who might be present on any particular half-day?

- What's the smallest number of female teachers who might be present on any particular half-day?



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39 DIY magic square

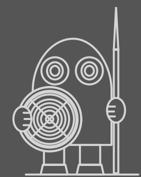
PROBLEM 1 : You've probably come across 'magic squares' before. As you know, the numbers along each row and down each column, and along each diagonal, must add up to the same total, which we call the 'magic total'. The square on the right is a magic square but some of the numbers have been rubbed out. Your problem is to fill in the gaps.

		10
15		1
		13

Once you've completed the magic square, just answer this simple question : what's the connection between the number in the centre and the magic total? The connection you've spotted here is actually the same for all 3 x 3 magic squares !

PROBLEM 2 : This problem is a little harder. First, draw yourself an empty square grid like the one on the left. Now try to make up a magic square of your own, using each of the numbers 0 to 8 once and once only. Use any method you like – and remember, if it gets you to a magic square which works, then it's a good method!

hint : if you've spent a long time on this one and you're getting nowhere, look again at the interesting connection you found at the end of problem 1 . . . it might just be useful . . .



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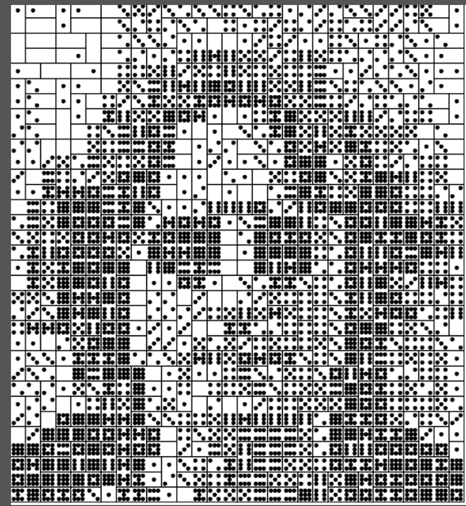
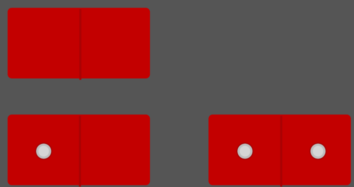
40 domino faces

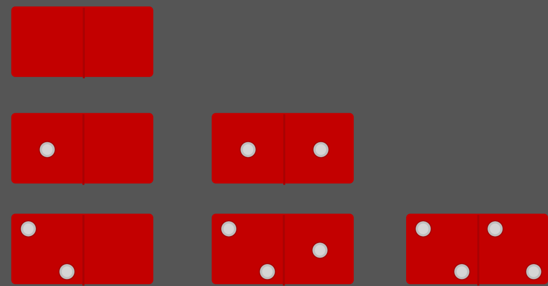


You probably know that an ordinary set of dominoes (sometimes called '6-spot dominoes') has 28 dominoes in it. There are many different games you can play with a set of 6-spot dominoes; you'll find them all on the internet. Of course, 6-spot dominoes aren't the only kind you can buy; some people like to play with 9-spot dominoes. On the right you can see a portrait of John Lennon which mathematics man Robert Bosch made using nine sets of 9-spot dominoes. When you have a few moments to spare, take a look at his website : *DominoArtwork.com*

Here's the set of 1-spot dominoes. As you can see, this set has just three dominoes in it.



And here's the set of 2-spot dominoes. As you can see, this set has six dominoes in it.



- How many dominoes are there in the 0-spot set?
- Make a quick drawing of the set of 3-spot dominoes.
- How many dominoes are there in the 5-spot set? (Don't draw them!)
- Without drawing them, work out how many dominoes there are in the 9-spot set.

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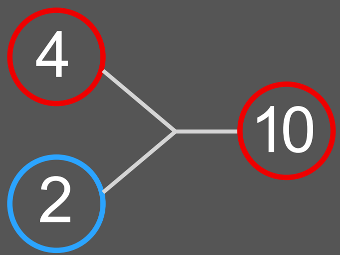
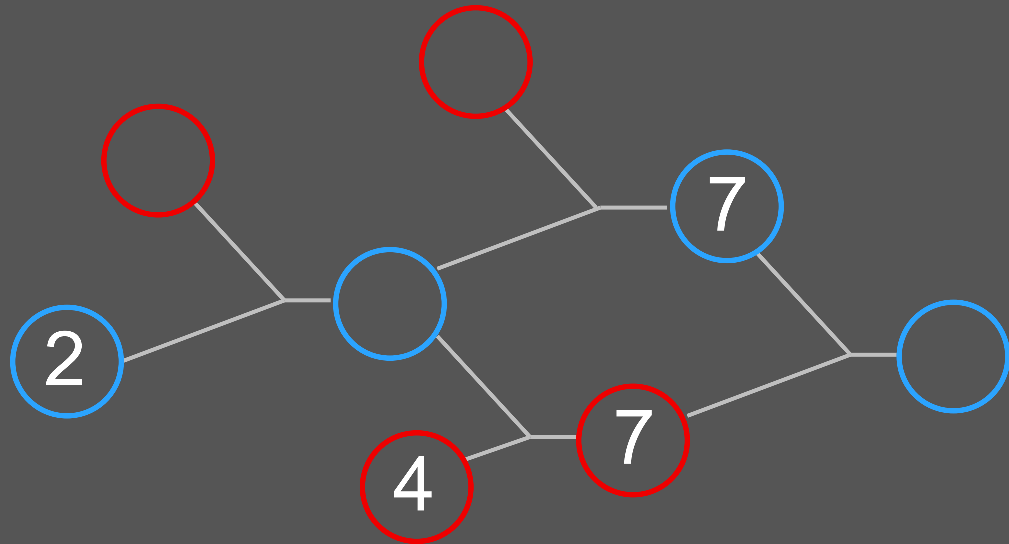
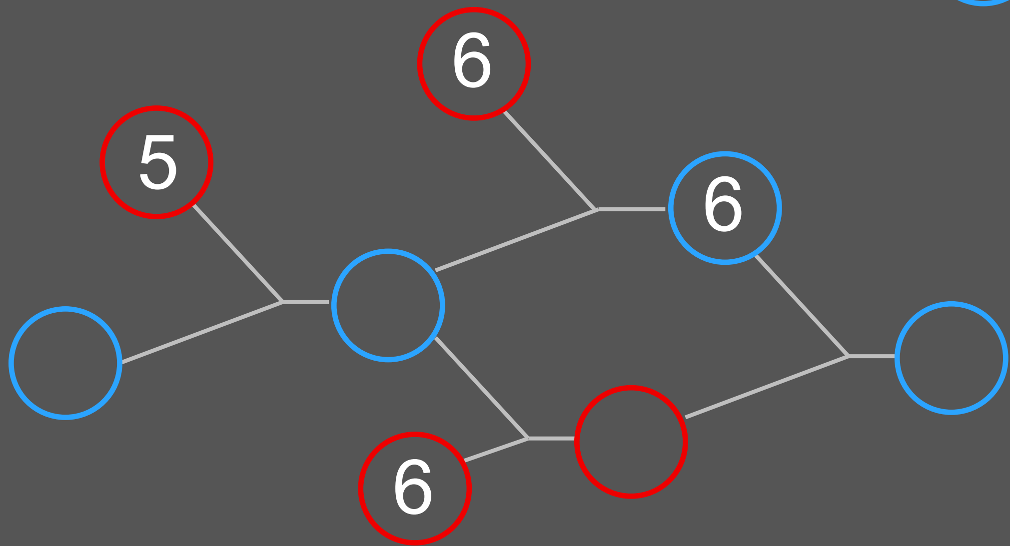
41 mapping webs 2

You've seen **mapping webs** before but here's how this one works : wherever mapping lines come from a blue circle and a red circle, you just square the number in the red circle and then subtract three times the number in the blue circle; the lines lead to another circle on the right and this is where the answer goes.

Here's an example :







Your problem: make a quick copy of these two mapping webs and try to fill in the blanks.

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42 cube calendar

On the shelf behind his desk, Mr Pascal (the maths master) has a daily calendar made of wooden cubes. The numerals on the first two cubes tell you which day of the month it is; and the letters on the remaining three cubes tell you the month. Mr Pascal made the calendar himself, using five plain wooden cubes (taken from his son's toybox!) and some white stick-on pvc letters.

Let's just think about the first two cubes : you have to be able to show all the dates in the month from the first to the thirty-first (no months have more than thirty-one days). To do this, you'll have to think carefully about which numerals you're going to stick onto which cubes. See if you can find a way of doing this so that all dates in the month are possible. Remember, each cube has six faces.

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43 10-pint target

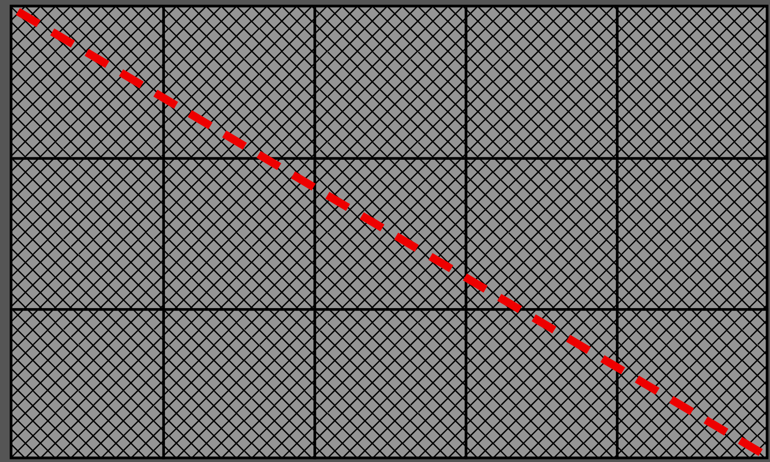
Nowadays we usually measure capacities of containers or volumes of liquid using litres as our basic unit . . . but in the not-so-distant past these things were measured in pints and gallons . . . and so to our problem :

Imagine you have two jars, a 6-pint jar (short and fat) and an 11-pint jar (tall and thin). Using nothing but these two jars, how could you measure out exactly 10 pints of water simply by pouring backwards and forwards? By the way : you are allowed as much water as you like from the tap and you may pour liquid away if you need to.

note *This is a very well-known problem. The numbers may be different but the wording is always something like : given an x-pint jar and a y-pint jar, how could you measure out exactly z pints? Of course, the units used might be litres instead of pints.*

44 ant and rectangle

Boris is an ant who loves maths. One day in the maths room he finds some drawings of rectangles, each one made up of squares. He thinks he'll investigate. He goes over to the first rectangle; then, starting at one corner, he walks straight across to the opposite corner, as in the drawing here :



How many squares does Boris cross? You can see that the answer is 7. All the rectangles the children have drawn are *prime rectangles* – this means they are rectangles where the two sides don't have any factors in common (except 1 of course). So, we're looking at rectangles like 7 x 2 or 5 x 3 or 4 x 5 – but not like 6 x 4 (because 2 goes into both 6 and 4) or 6 x 9 (because 3 goes into both 6 and 9).

Draw a few prime rectangles and see how many squares you cross going from one corner to the opposite corner. Can you find a simple rule which lets you work out, from the lengths of the sides, how many squares you'll cross?

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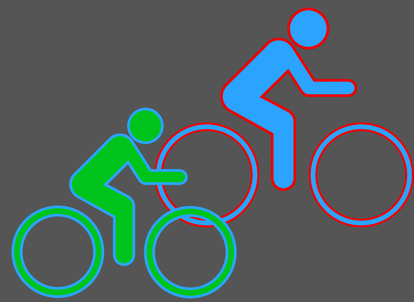
45 Alfred and Betty

Alfred and his wife Betty, keen to keep themselves fit and healthy, have recently bought a couple of rather handsome mountain-bikes. Alfred’s bike actually cost rather more than his wife’s but then, as he explained to her, he’s a more experienced cyclist and he needs the extra gears. Now, every Sunday morning the two of them go cycling in their local park. There they have a long, smooth, circular path which goes right round the park and which is perfect for cycling.



Alfred (either because he’s stronger or perhaps because his bike is lighter) is twice as fast as Betty. In fact, when they cycle round the park, the two of them always start off together and always finish together – except of course that in between Betty has completed one full circuit of the park and Alfred has completed two! It’s the same every week!

problem : Where on the circuit does Alfred overtake Betty?

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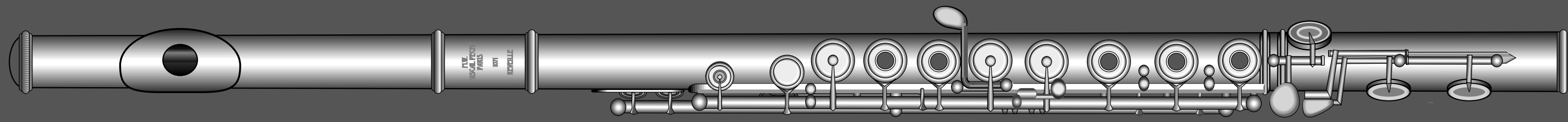
46 a tale of two flutes



Roland has a silver flute which he plays every day. He also has an old flute and one day, as he's hard-up, he decides to sell this old flute. It's quite tarnished and some of the keys don't work, so he knows he can't ask too much for it. In his mind he settles on a price which he thinks is fair.

So, Roland puts the flute up for sale on eBay – and waits for the bids to come in. By the end of 10 days, there are two bids. One of them is a bit low; in fact, it would have to be 25% higher to equal Roland's target. The other bid is quite high; in fact, you'd have to reduce it by 25% to equal Roland's target. The high bid is £32 higher than the low bid.

What exactly was the price Roland had in mind?

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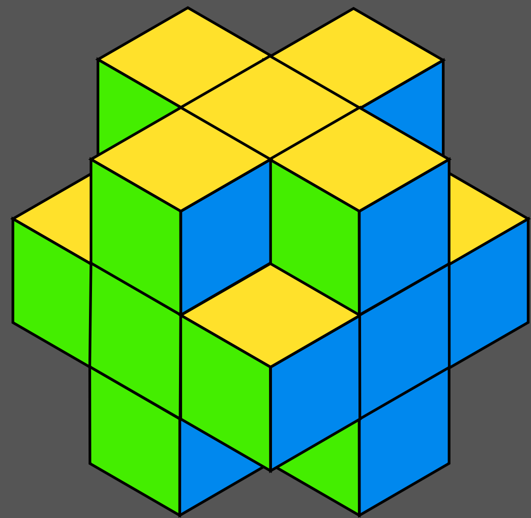
47 cutting corners

Sally had a box of 1cm cubes and she used some of them to make a 3 x 3 x 3 cube. To hold the cube together, she used some blu-tac from her desk. While Sally was out with her friends, her mischievous younger brother Jack went into her room and decided to pull all the corners off Sally's cube. On the right you can see a picture of the shape which remained. Jack thought it was quite an interesting shape.



- How many small cubes did Jack remove from the original large cube which Sally had made?
- How many small cubes are there now in this new (more interesting) shape?
- You know what surface area is. So, what's the total surface area of the shape on the right?

** to get the surface area of a shape, you just add together the areas of all the different faces of the shape.*

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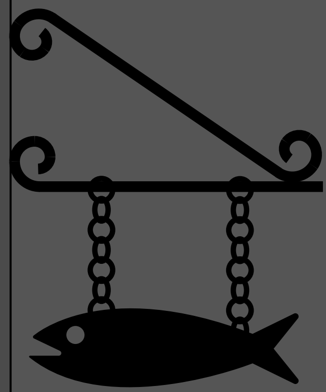
48 odd shops

At one end of York Road there's a row of five small shops. These shops are numbered 1, 3, 5, 7 and 9. One of the shops is a fish shop, one is a hat shop, one is a baker, one is a cycle shop and one is a shoe shop. Here are some useful facts about the positions of the shops :

- the fish shop is between the cycle shop and the shoe shop
- the cycle shop is not next to the hat shop
- the baker is not next to the hat shop
- the baker is at one end of the row, at number 1 in fact
- the hat shop and the shoe shop are next-door neighbours



Use this information to work out which shops go with which numbers.

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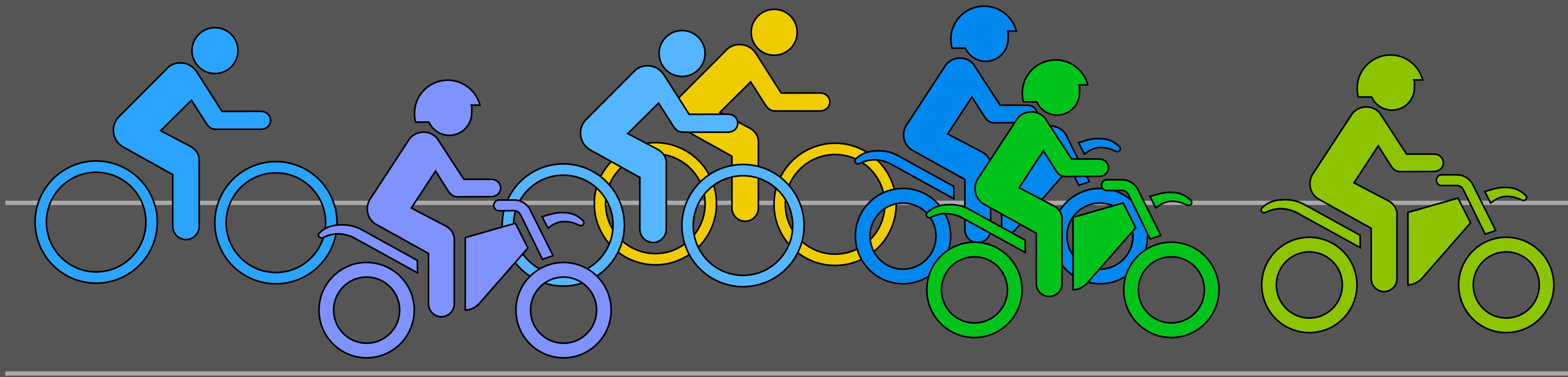
49 Wrekin Ride . . .

On the first day of March each year, a number of cyclists and motorcyclists set off on a hillside ride around the Wrekin, a hill in Shropshire (It's pronounced *ree–kin*). On the last Wrekin Ride, the cyclists and motorcyclists all set off together – but obviously the motorcyclists soon got ahead.



- On the last Wrekin Ride there were twice as many males as females.
- Of the motorcyclists, 14 were male and 13 were female, making 27 in all.
- Among the cyclists, there were three times as many males as females.
- There were more females on motorbikes than on bicycles.

Altogether, how many female riders took part? And how many of them were cyclists?

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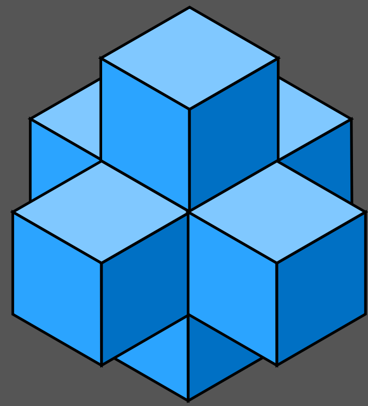
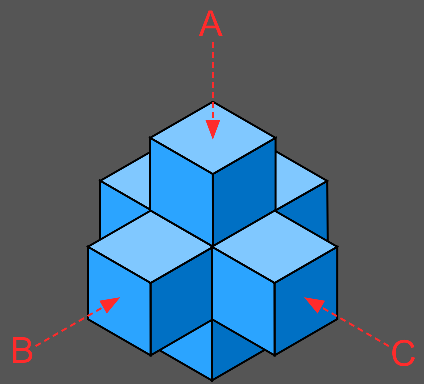
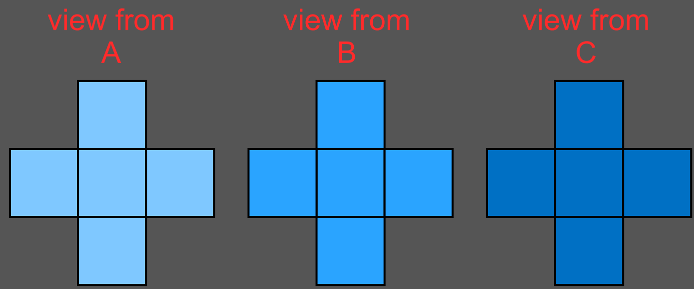
50 the blue cube

Look at this interesting shape from Jenny's collection. The shape started life as a blue 3 x 3 x 3 cube, made from individual 1cm cubes – but then Jenny decided to pull off every small cube except the middle one of each face. The shape Jenny finished up with is quite symmetrical, that's to say : you can look at it straight on from six different directions and it always looks the same.







. . . and of course for each of these you can look from exactly the opposite direction – which gives you a total of 6 different views, in three different colours but all the same in shape . . .

Here are three problems for you :

1 How many cubes did Jenny take from the original cube?

2 How many cubes are there in Jenny's new shape?

3 What's the surface area of this new shape?

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problem-solving
~ how to get started

Sometimes you look at a difficult problem and think, 'I really don't know where to start on this one!' Many people imagine that a real mathematician can look at any problem and know what he must do to solve it. But that's not how things are! Real mathematicians look at all sorts of problems and very often say to themselves, 'I don't know where I'm going to start on this one!' The interesting thing is searching for a way to unlock the problem and at times that can be very difficult. It can also be really interesting – as you try different things which might help you, you often discover new connections and new patterns. That's mathematics!

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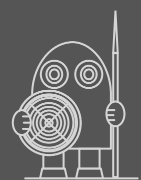
dog on the wing

Most people have never seen a dog playing football. But a few years ago in Battersea Park, South London, you could have seen such a dog. It's a large park, and daily there's lots going on: cycling, strolling, dog-walking, pram-pushing and games of all sorts.

But on Sunday mornings the big thing is football.There are serious games being played on the big pitches, with local teams and supporters, plus real referees to keep order and to ensure fair play. And there are also many smaller games going on in various locations around the park. It's in one of these smaller games that every week you'd have been able to see a dog, not a very big dog, playing for one of the sides. He didn't really have many football skills (he couldn't keep the ball in the air using his head) but he was very fast as he scampered down the wing, pushing the ball ahead of him with his nose and quickly finding a way around almost anyone who tried to take the ball from him. As you can guess, quite a crowd would gather to watch 'Dribbler', as he was called . . .

And 'dribbler' turns out to be a good way of remembering our useful tips for problem-solving (they're on the following pages). If you can just remember the word 'dribbler', then you've got the first letters of the important key words . . .



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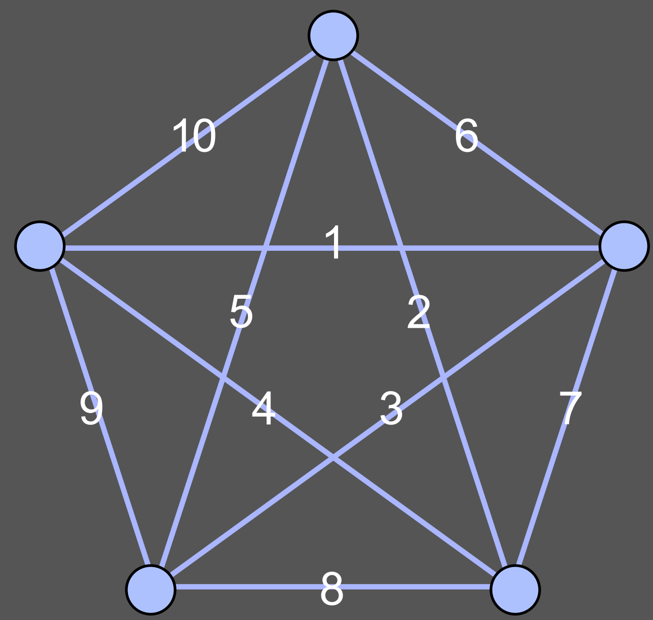
draw

Sometimes a problem you're given is based on a drawing or diagram. But there are other times when drawing something yourself suddenly makes the problem a lot more straightforward. What you draw doesn't have to be a work of art – it just has to make the problem clearer for you. You'd be surprised how many times a simple drawing or diagram makes it easier to see what's in a problem and perhaps makes it easier to solve.



no. of lines = 10

and so

no. of handshakes = 10

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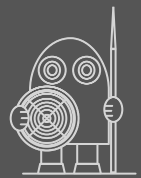
information

You've probably been told lots of times, 'read the question!' . . . There is some sense in this because the problem as you read it contains the information you'll need to solve it. Let's hope we're agreed on that but – as you start working on the problem, there are some questions you need to ask yourself :

- Are you using all the information you've been given? Sometimes you're stuck and it's just because there's a bit of information you do need in order to solve the problem and you've overlooked it.

- Have you been given more information than you need? If you're given lots of information, then it's tempting to dive in straight away and start doing some working-out but – it might just be that if you take a moment, you'll find that you really only need certain key facts.

- Can you perhaps look at the information in a completely different way? For example, suppose you've been told that a hot tap takes 6 minutes to fill a bath and a cold tap takes 12 minutes. You might not be sure how you can use this information as it stands but try looking at it this way : *the hot tap can fill 10 baths an hour and the cold tap can fill 5 baths an hour.* Suddenly, the answer is just around the corner!



bit-by-bit

Often you'll start working on a problem and find that you really can't think of any way of getting directly to the answer. But not all problems have to be solved by a clever method or by a smart way of going straight to the answer. At times a bit-by-bit approach is the way forward. Perhaps in arithmetic you've already come across the bit-by-bit way of working some things out. For example, if you had to work out 35% of £64, you might know that you could try to work out 35/100 of £64 but – it might be a lot easier to jot down 10%, then 20%, then 5% and then just total the results to get your final answer. Instead of trying to aim for an answer in one go, you take a bit-by-bit approach and you get there more easily. You can take the same approach with some difficult maths problems.

like

Sometimes you can look at a problem and you know straight away you've already solved another one just like it. If you can remember how you solved that problem, it might help you to solve this one.

But even if you haven't solved something like this before, you might be able to make up a problem that's just like this one but much simpler. For example, looking at a problem about inner and outer pieces in a large jigsaw, you might feel unsure about how to handle the big numbers involved. So, make a quick drawing of, say, a simple 7 x 5 jigsaw (rectangles will do for pieces, no need to put in the curly bits). Now you can easily see how the numbers work out when you want to find things about inner and outer pieces or whatever. Next, you can apply what you've discovered to the dimensions of the larger jigsaw and perhaps feel more confident about the answers you get.

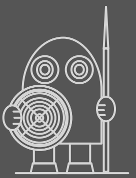
experiment

Remember, there's nothing to stop you trying out an answer to see whether it works . . . and if it isn't quite right, then think how best to change it – and try again! Look at this problem, for example :

Peter and Sue are brother and sister; Peter is 5 years older than Sue. Their two ages add up to 27. How old is Sue?

Let's experiment! If Peter and Sue were twins, then they would each be $13\frac{1}{2}$. But they're not twins, are they? In fact, Sue is younger. So let's start with a try of, say, Sue = 9. This would make Peter 14 and their total ages 23. That's too low, so let's try Sue = 10. With this try, Peter (5 years older) = 15 and the total of ages = 25. Too low again! Putting Sue = 11 makes Peter 16 and the age total = 27. Success! So our problem is solved : Sue is 11 yrs old.

You can use this method in many maths problems. Ignore anyone who says this is just guessing – because it's much more than that. Remember, when we tried an answer which didn't work, we didn't just wildly guess at another one, we stopped and thought – and then we made an intelligent correction (alteration) and tried it out. You could call this method the 'Estimate & Adjust' method if you like. (Doesn't this sound a whole lot better than 'guessing'?)



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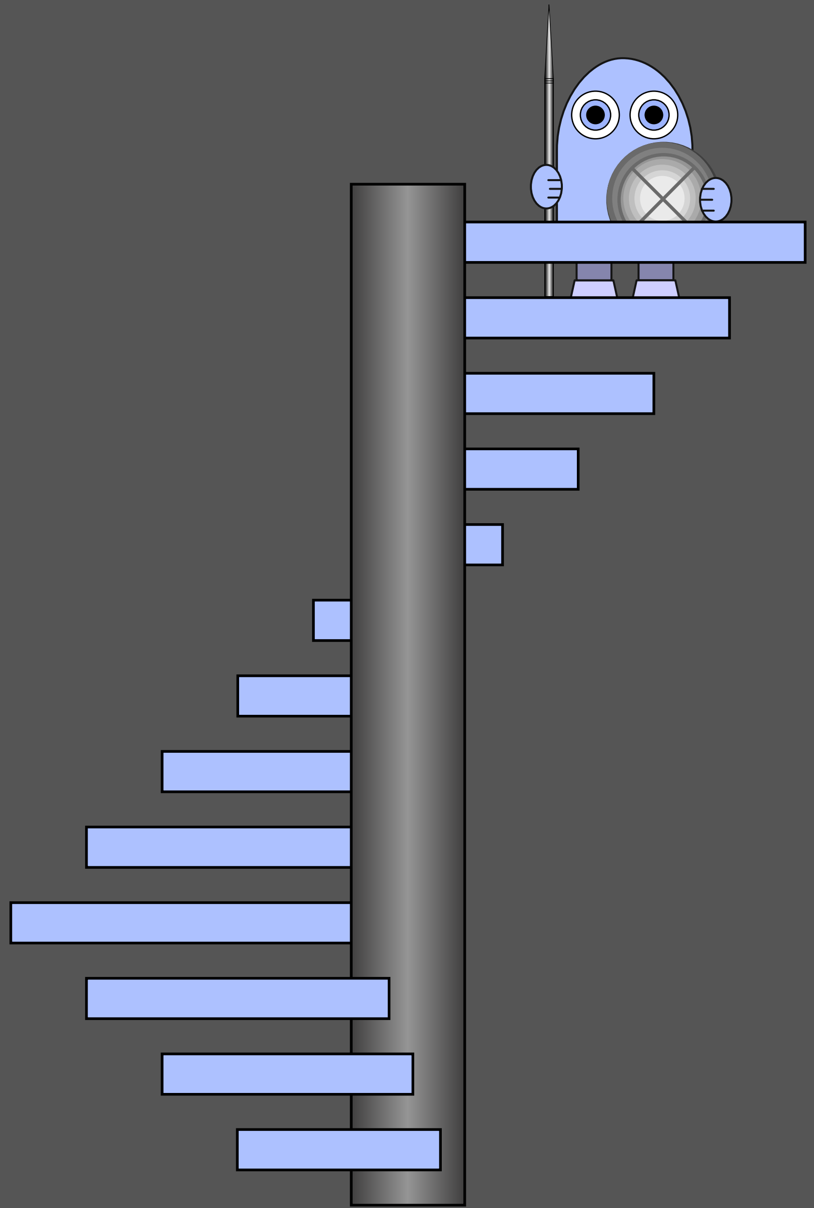

realistic

Well, when you've worked on a problem and finally got an answer, is it time to sit back and feel pleased with yourself? Not quite! There's actually one more important thing to do. Take a hard look at your answer and ask yourself: Is my answer reasonable . . . is it realistic? Say it was a question about the ages of different people in a family and you worked out that grandfather must be 15 years old. Is that a reasonable answer? Well, not really . . . most grandfathers are a good deal older than that. And, as you probably know, no chicken weighs 300kg, no girl can cycle at 85km/hr and no 10 year old boy is likely to have a height of 14cm – and yet these are all answers which have been given by real pupils.

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. . . and this gives you quite an easy way to remember how to get started on a problem if you're really stuck. Just remember the name 'dribbler' and you've got the first letters of the important key words :

** the one-stop chart is on the next page -->*

draw

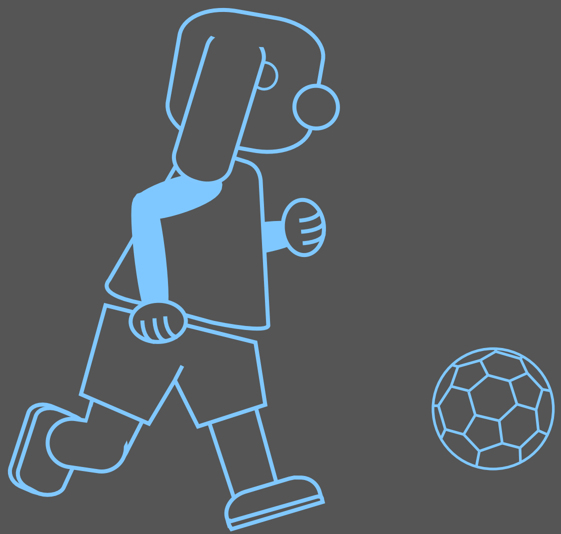
information

bit-by-bit

like

experiment

reasonable



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information

are you using all the information you've been given? can you look at the information in a different way?

experiment

can you try out an answer and then improve it?

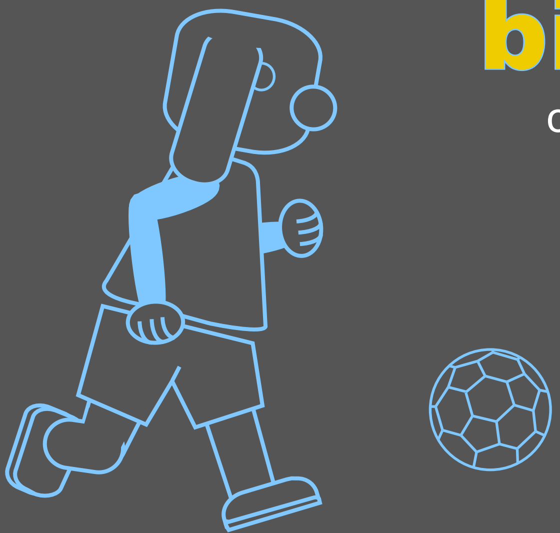
draw

can you draw something which will make the problem clearer?

dribbler

reasonable

and when you've found an answer : is your answer reasonable? is it realistic?



bit-by-bit

can you work out the answer in easy stages?

like

is the problem like something you've seen before – or can you make up a simpler one just like it?

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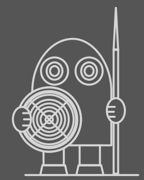
Sports Day

We know where Kate and Anne came in the race, so here are the positions 1 to 5, with these two already listed :

1	
2	Kate
3	
4	
5	Anne

What else do we know? Well, we know that Mary was just behind Sanjit – which means that their positions must be next to each other. The only slots available for this are positions 3 and 4, so that's where we'll put Sanjit and Mary . . . and now of course there's only Jeremy left – so he must have come in at number 1 :

1	Jeremy
2	Kate
3	Sanjit
4	Mary
5	Anne



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ans 2 time's up!

Here you can see the times with digital sums equal to 24 23, 22 and 21.

There's one obvious pattern there for you to see : the number of times in these answers begins with 1, then comes 3, then 6 and finally 10. Perhaps you recognise the sequence 1, 3, 6, 10 as the first four **triangle numbers**. The next triangle number after 10 is 15 and that's your answer to the last question.

There are also some patterns for you to find in the times themselves : to spot these it's best to look at the hours and minutes separately.

- *digital time total = 24*

19 59

- *digital time total = 23*

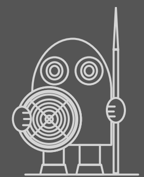
19 58
19 49   18 59

- *digital time total = 22*

19 57
19 48   18 58
19 39   18 49   17 59

- *digital time total = 21*

19 56
19 47   18 57
19 38   18 48   17 58
19 29   18 39   17 49   16 59



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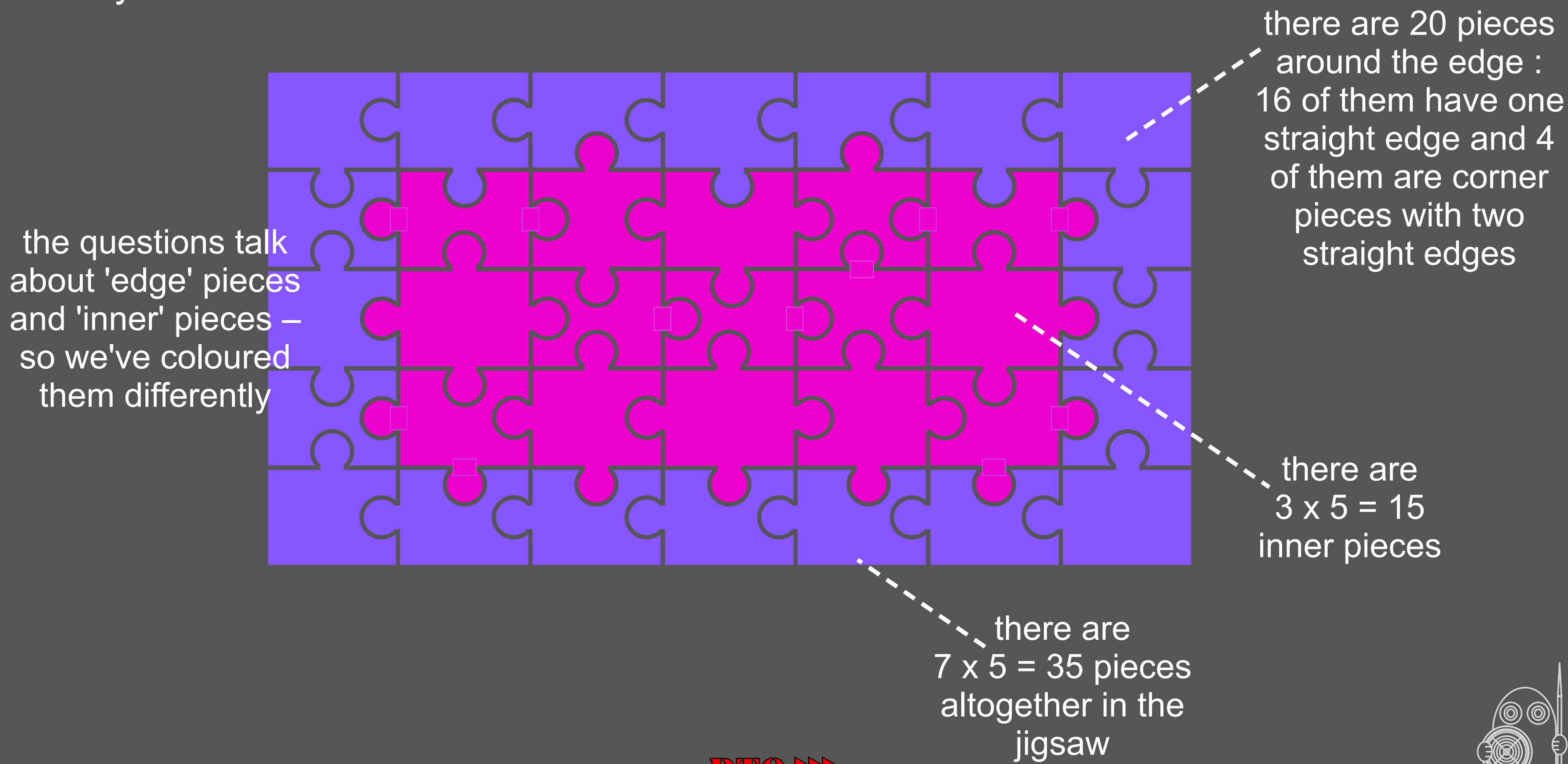
ANS 3 an edgy problem

Syed's puzzle has quite a few pieces, that's for sure, and it's not clear at first what we're to do with the large numbers we're given. You might or might not have already met a problem like this one but – one thing we can do is to make up a simpler problem that's just like this one. For example, if we start with a 7 x 5 jigsaw, we can see more easily how the various numbers work out :



PTO

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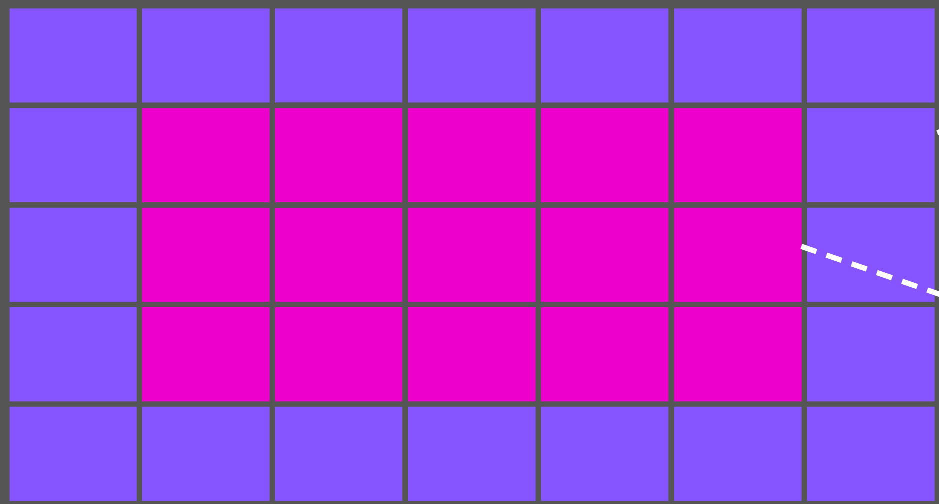
ans 3 an edgy problem

There's no real need to show the curly bits on the jigsaw pieces – we can just picture them as small rectangles. So now we can think of what we've got as a 7 x 5 jigsaw with a 5 x 3 jigsaw inside it :



In the larger jigsaw,
number of pieces =
7 x 5 = 35

In the smaller jigsaw,
number of pieces =
5 x 3 = 15

So for this jigsaw, our answers are :

total number of pieces = 35

number of inner pieces = 15

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PTO

ans 3 an edgy problem

Coming back to Syed's jigsaw, we can think of it as a 54 x 27 jigsaw with *(make sure you understand this!)* a 52 x 25 jigsaw inside it :

In the larger jigsaw,
number of pieces =
54 x 27 = 1458

In the smaller jigsaw,
number of pieces =
52 x 25 = 1300

So for Syed's jigsaw, our answers are :

total number of pieces = 1458

number of inner pieces = 1300

PTO

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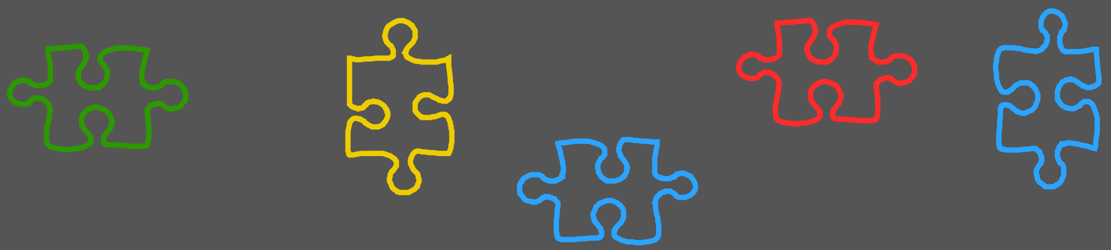
ans 3 an edgy problem

. . . and here's a more direct way of getting the answers :

- The total number of pieces is 54 x 27, which equals 1458

- There are 4 corner pieces, each with two straight edges. As for pieces with just one straight edge, there are 52 along each longer side of the puzzle and 25 along each shorter side of the puzzle, making a total of 154 pieces. So, altogether, there are 158 pieces with either one or two straight edges. Subtract this from the total number of pieces in the puzzle and you get 1458 – 158 = 1300. So altogether there are 1300 'inner pieces' (that's to say, pieces with no straight edge).



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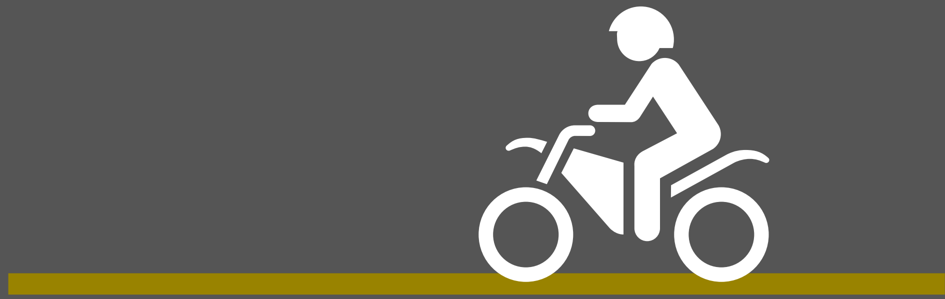
ans 4 easy rider



If Jamie is going to finish the season with an average position of 3rd, then we know that at the end of the season, his eight finishing positions must add up to 8 x 3 = 24. We know this because 24 ÷ 8 is how we get an average of 3.

So far, Jamie's total from seven finishes is 21. But we've just worked out that the final total must be 24. That leaves a gap of just 3! So, to win a cash prize, Jamie must finish 3rd (or better) in his last race.

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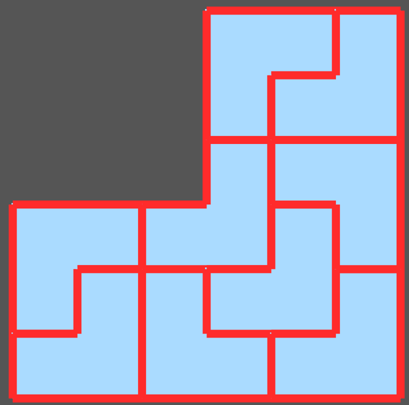
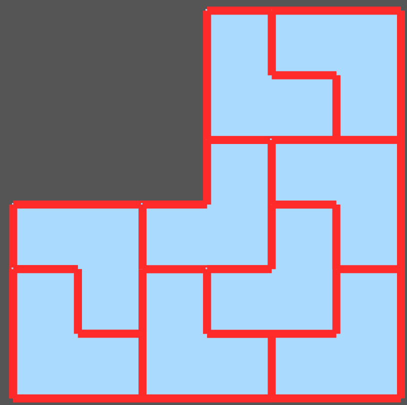
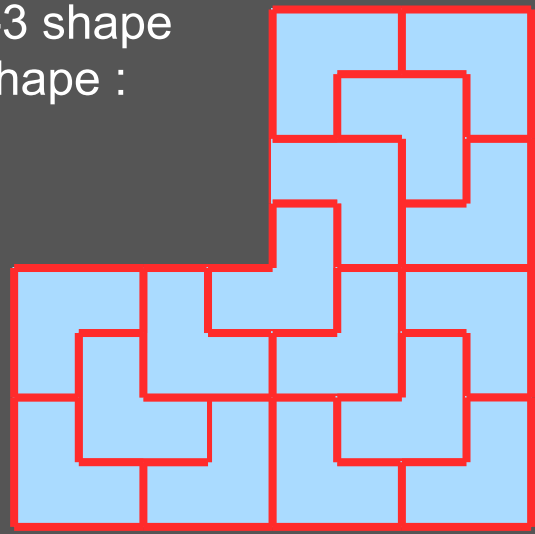
ANS 5

L is for learner

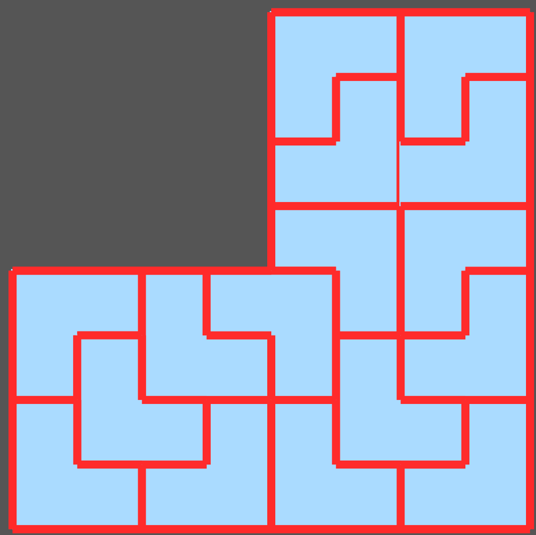
Here are two ways of tiling the L-3 shape and three ways of tiling the L-4 shape :

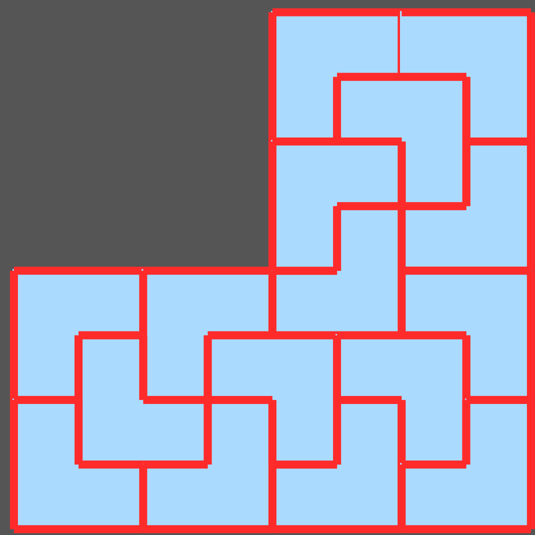






You use 1 tile for shape L-1 and 4 tiles for shape L-2, then 9 tiles for shape L-3 and 16 tiles for shape L-4.





1, 4, 9, 16 . . . I'm sure you'll recognise the **square numbers** here !

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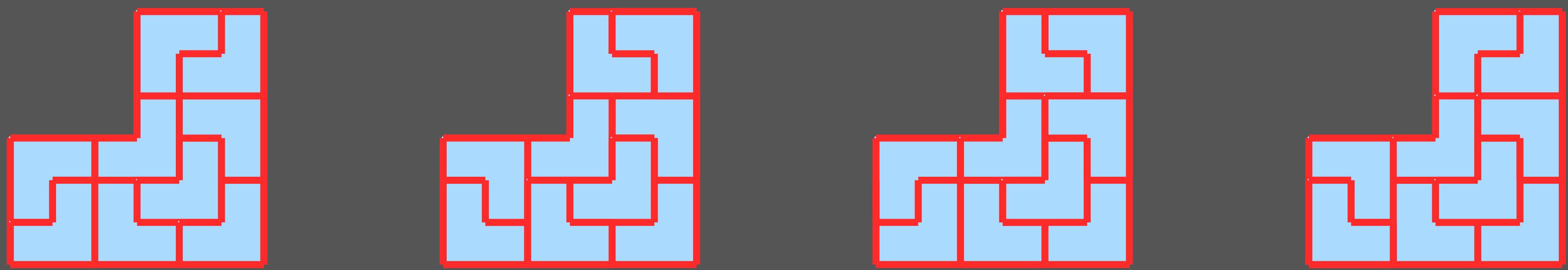
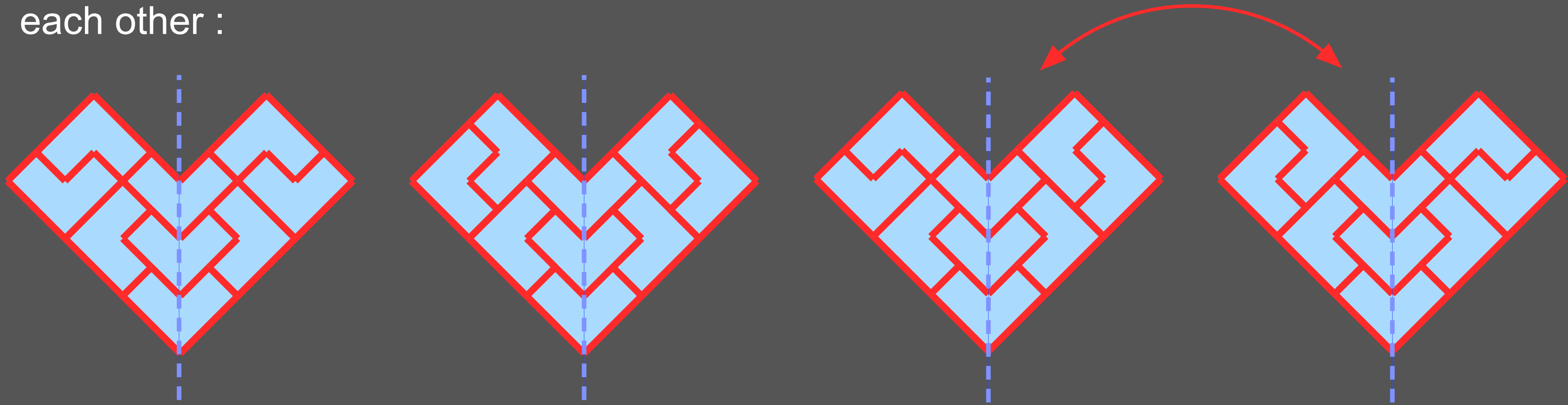
PTO ▶▶

ans 5 L is for learner

Actually, we found four ways of tiling the L-3 shape and here they are :

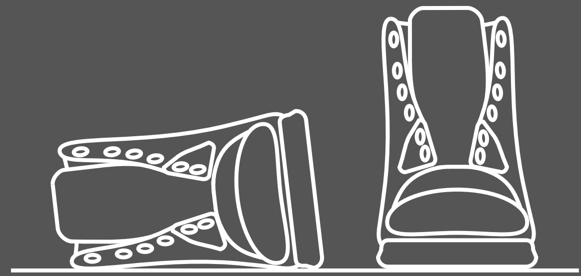


But when we tilted them by 45° we could see that the last two were mirror-images of each other :



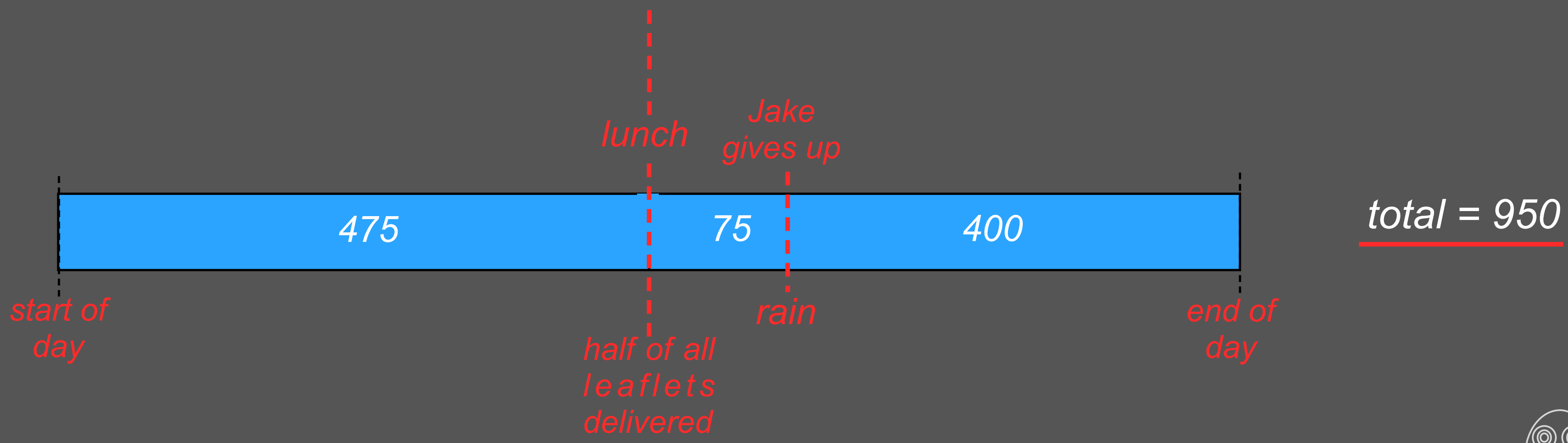
Finally, when it comes to the L-4 shape, we've shown you just two ways of tiling it. But how many ways could you find? And are they all really different?

ans 6 Jake's leaflets



We know that by lunch-time Jake had delivered half of the leaflets. This means that he had half the leaflets still in his sack. And we know that when Jake gave up, he'd delivered 75 more of these and he still had 400. So half the leaflets must add up to 475; which means that **all** the leaflets must be 475 x 2 = 950.

answer : Jake began his day with 950 leaflets, as this diagram shows :



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ans 7 getting warmer . . .

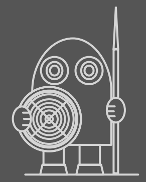
With just the four number cards 8, 2, 7, 4, our 'closest numbers' are :

- 7248
- 2874
- 4872
- 7248
- 4287
- 8742
- 8724
- 8427

Most of these are fairly easy – but with the last one there are two answers almost as close as each other, so here you might have had to do a bit of arithmetic to find what you were after . . .

$$\begin{array}{r} 8427 \\ -8351 \\ \hline 76 \\ \hline \end{array} \qquad \begin{array}{r} 8351 \\ -8274 \\ \hline 77 \\ \hline \end{array}$$

– so 8427 wins !



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in the hot seat . . .

probability

I'm sure you know that probability is about things happening, and it measures how likely different things are to happen. But how does it work? Well, to find the probabillity of some particular result, you just compare how many times you get this result with how many possible results there are. Here are two simple examples, which should give you something of the idea . . .

eg1 'If I toss two coins in the air, what's the probability of them both landing heads?'

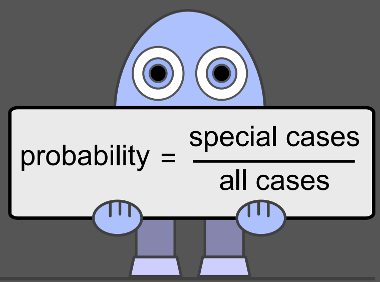
Well, the two coins could land HH, HT, TH or TT – and that's four possible results. Only one of them is HH, so we can write : probability (two heads) = 1/4

eg2 'If I throw down a normal dice, what's the probability that it will land with a prime number on top?'

There are six faces on the dice and only three of them are prime numbers (2, 3, 5), so we can write : probability (prime number on top) = 3/6 = 1/2

there are just two important things to remember :

1



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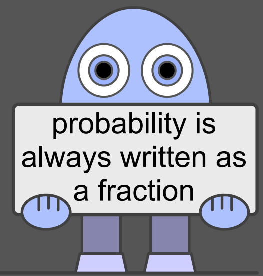

PTO

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ans 8 in the hot seat . . .

This is an easy question and here are two ways of getting to the answer:

1 Start by writing down all the possible ways in which the three children could be seated. It's easier to do this if you have a method (working alphabetically, for example) :

R S T
R T S
S R T
S T R
T R S
T S R

As you can see, there are 6 different ways of seating the children - but just two of these have Rosie in place number 1. So,

prob (Rosie in seat 1) = 2/6 = 1/3

2 Or, you could argue that by the sheer symmetry of the situation, the children have absolutely the same chance of ending up in any particular seat. So, for each child (including Rosie), we can write : prob (getting seat 1) = 1/3

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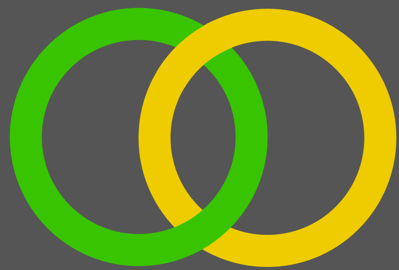
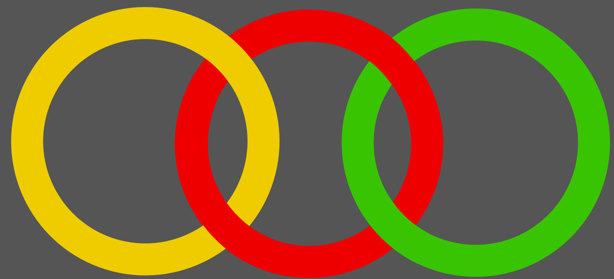
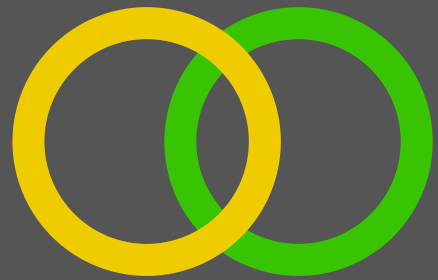
ans 9 overlapping rings



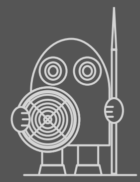
- Look hard at the original diagram – perhaps you can see that the green ring and the yellow ring are linked. And they will still be linked even if someone takes the blue ring away!



- This time, if someone does cut the blue ring and remove it, the red ring will still be linked to the yellow ring and to the green ring – but the yellow and green rings will not be linked to each other.



- Finally, if someone cuts both the blue ring and the red ring and then takes them away, the remaining two rings (green and yellow) will not be linked.



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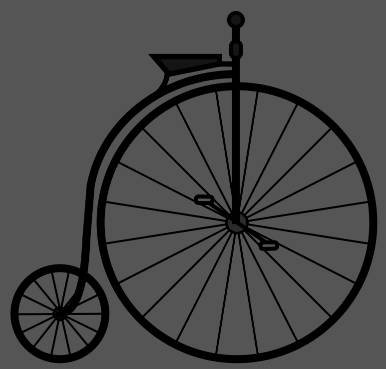
ans 10 The Pedalman

The number we're after is not hard to find . . .

On the right is a list of all nine possible numbers. As you can see, eight of the numbers have been crossed out – and next to each is the reason why that particular number has been disqualified.

Only one number remains : 87 (and yes, that is a multiple of 3). So there we have it – the address which the Maths Detectives need is

87, Old Montague Street



~~23~~	it's a prime number
~~49~~	it's a square number
~~62~~	it's not a multiple of 3
87	it ticks all the boxes !
~~105~~	it's a multiple of 7
~~121~~	it's a square number
~~210~~	it's a multiple of 7
~~169~~	it's a square number
~~188~~	it's not a multiple of 3

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ans 11

happy birthday, Ben !

There are different ways of going about this problem; one easy way is to try some different possible answers until we find what we're after. But – before we jump in and begin trying out lots of numbers, let's stop and think : If Ben's age is four times Annabelle's age, then Ben's age must be a multiple of 4. So let's try some multiples of 4 for Ben and next to them we'll put the corresponding ages for Annabelle . . . and on the line beneath we could put their ages next year. Let's begin by trying Ben = 4, then Ben = 8 and so on . . .

	Ben	Annabelle
this year	4	1
next year	5	2

	Ben	Annabelle
this year	8	2
next year	9	3

It didn't take long, did it? Our second try was just right. If Ben is 8 this year and Annabelle is 2, then everything works!

So, that's our answer : Ben is 8 and Annabelle is 2.

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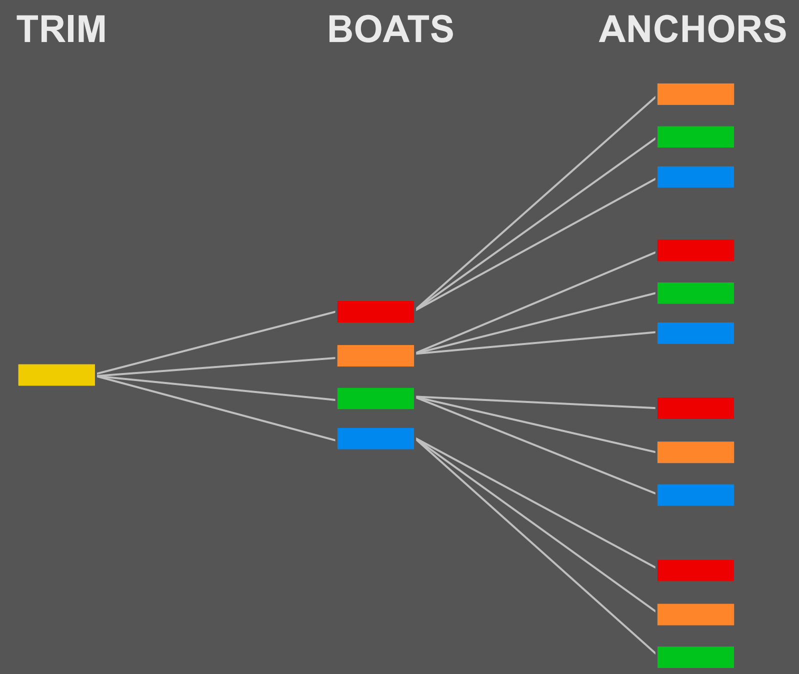
ans 12 **sailing by . . .**

The 'tree diagram' on the right shows you all the possibilities.

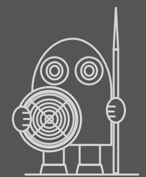
Starting on the left, the 'TRIM' column shows the only possible colour, yellow. With this there are 4 colours remaining which can be used for the BOATS part of the badge. So that's 4 possible combinations. Moving to the ANCHORS, each of the 4 combinations we've mentioned can be joined with any of 3 different colours.

giving us : 4 X 3 possibilities overall

or, in other words, given what we're told, 12 colour arrangements are possible . . .



note : remember, we're counting eg boats blue / anchors red as a quite different arrangement from boats red / anchors blue . . .



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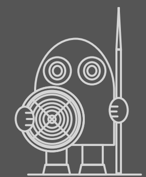
PTO

ans 12 sailing by . . .

. . . well, that's one way of solving the problem. But some people prefer to make a list of all the possibilities. On the right you can see such a list. If you choose to do this kind of problem by making a list of all combinations, it helps if you can find a definite system for making sure you've got all of them. Counting up, the final answer here is :

total number of possible arrangements = 12

TRIM	BOATS	ANCHORS
YELLOW	*BLUE*	*GREEN*
YELLOW	*BLUE*	*ORANGE*
YELLOW	*BLUE*	*RED*
YELLOW	*GREEN*	*BLUE*
YELLOW	*GREEN*	*ORANGE*
YELLOW	*GREEN*	*RED*
YELLOW	*ORANGE*	*BLUE*
YELLOW	*ORANGE*	*GREEN*
YELLOW	*ORANGE*	*RED*
YELLOW	*RED*	*BLUE*
YELLOW	*RED*	*GREEN*
YELLOW	*RED*	*ORANGE*



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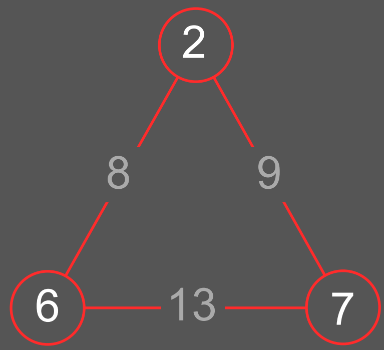
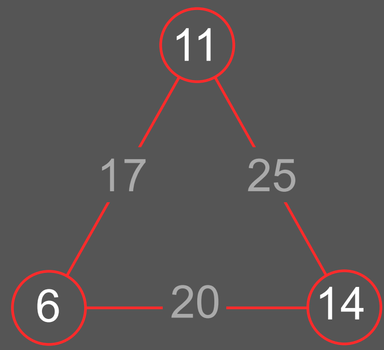
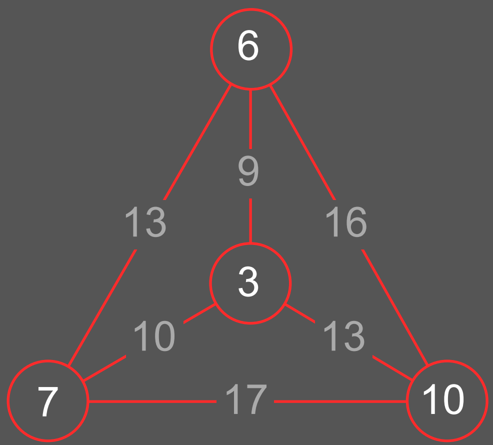
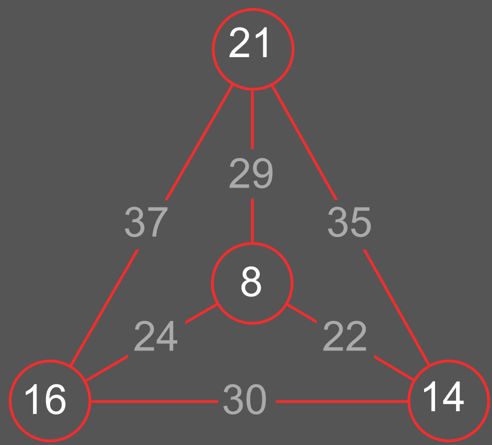
ans 13 number triangles

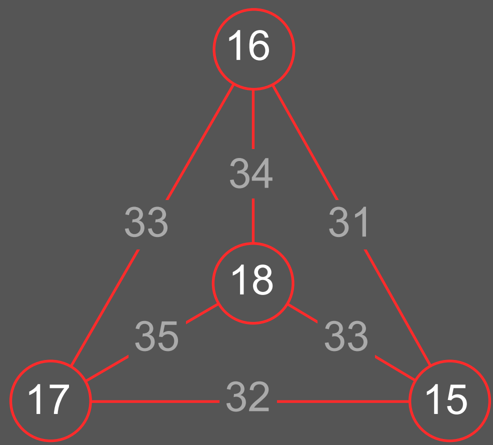
and here are the answers . . .











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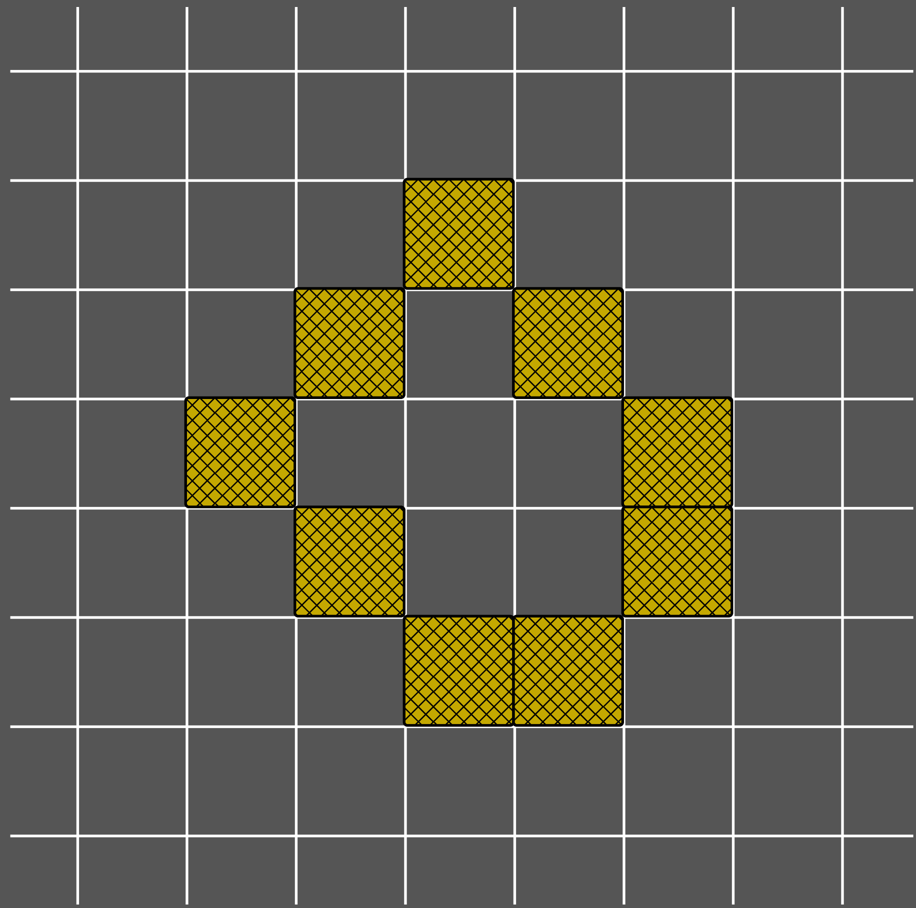
ans 14 penning sheep

6 squares seems to be the most you can enclose with 9 hay-bales – and here's one way of doing it :



**extra problem : Is this the only way of arranging 9 bales to enclose 6 squares?*
Special note: before you investigate this problem you'll need to decide what counts as 'different' arrangements eg if one arrangement is a mirror-image of another arrangement, do you want to count them as different arrangements ?

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ans 15 parcels-to-go

- Obviously for small parcels you're better off going with Parcels-to-Go, because you don't have to pay a basic charge.

- Perhaps the easiest way to get at the answer is just to work out the cost of sending parcels of different sizes with the two delivery firms. The chart on the right shows you how their charges work out :

	ParcelDrop	Parcels-to-Go
1kg	2.50	0.75
2kg	3.00	1.50
3kg	3.50	2.25
4kg	4.00	3.00
5kg	4.50	3.75
6kg	5.00	4.50
7kg	5.50	5.25
8kg	6.00	6.00

– You can see that at 8kg the rates charged by the two delivery companies are exactly the same. So that's the answer : 8kg

* *Now turn to next page . . .*

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PTO

ans 15 parcels-to-go

A different way of getting to this answer is to notice that Parcels-to-Go charge 25p more per kilogram than ParcelDrop. Of course you don't have to pay them the £2 basic charge – but how many kilograms would your parcel need to be in order for the extra 25p per kg to equal a £2 basic charge? Well, that's easy enough to work out . . . How many times does 25p go into £2? Answer : 8 times. Or in other words, when your parcel is as heavy as 8kg, the extra 25p per kilogram charged by Parcels-to-Go is exactly wiped out by the £2 basic charge which ParcelDrop makes you pay.

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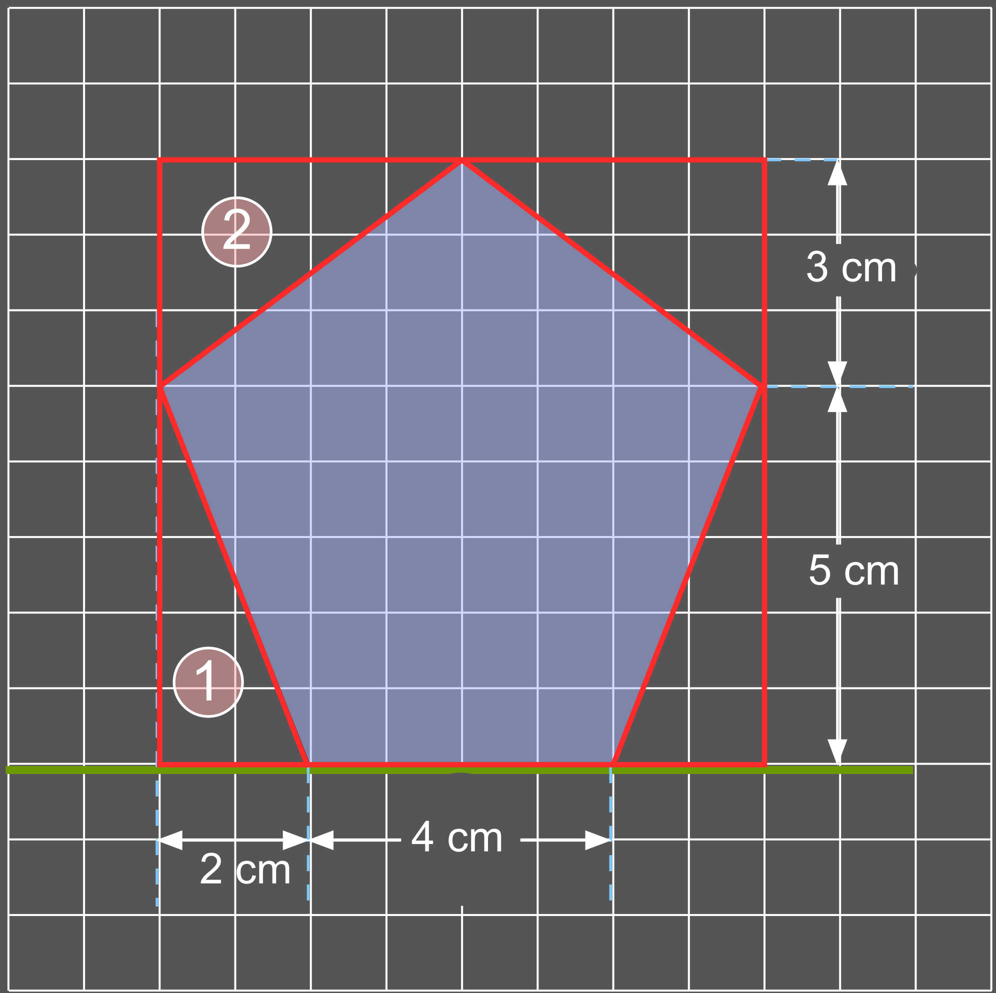
ANS 16 **five-a-side**



One way of finding the area of a pentagon like this is by subtraction : just find the area of the surrounding rectangle (it's a square in fact) and subtract the combined area of the four corner triangles . . .

area of red square = 8 x 8 = 64

area of triangle 1 = 5

area of triangle 2 = 6

as there are two of each triangle,

total area of triangles = 22

so, area of pentagon = 64 – 22

= 42 cm^2

PTO

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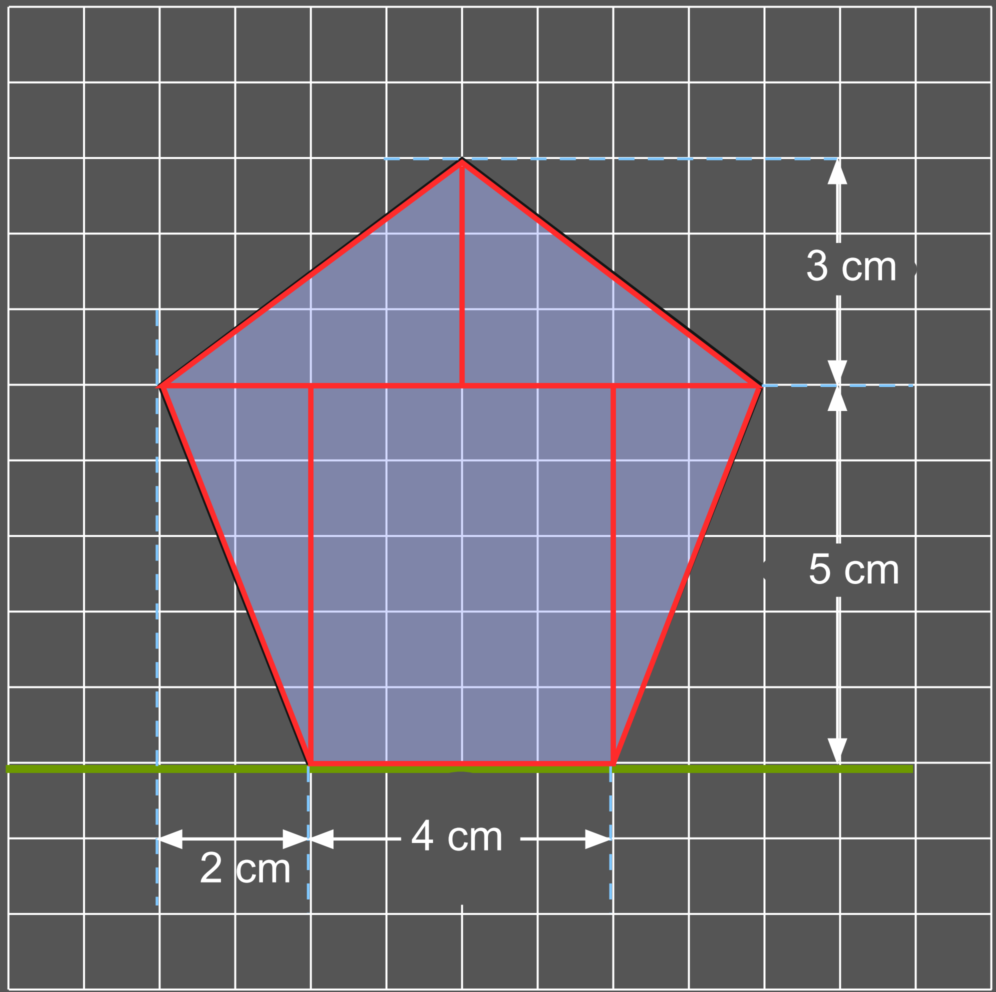
ans 16 **five-a-side**



On the other hand, many people find it easier to carve up the pentagon into a rectangle and four triangles, as in the diagram, and then simply add together the five areas . . .

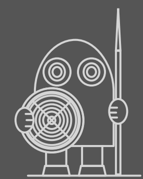
area of two top triangles = 12

area of two side triangles = 10

area of rectangle = 20

total area of rectangle plus four triangles = 12 + 10 + 20

= 42 cm²



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ans 17 Alice's party

You might have been able to solve this one by trial-and-error or in some other way but – if you have a problem like this one and you don't know where to begin, you can always set up a 'truth table'. What you do here is make a list of all possible arrangements – and then cross out the ones which you know you can't have. You should be left with just one possible arrangement and this of course is your answer.

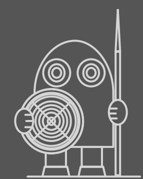
This is how the truth table method works with the 'Alice's party' problem :

CONDITION 1

This tells us that Debbie must have seat number 4. This means that we really only have the problem of how to fill seats 1, 2 and 3.

To make our lives easier, let's use the letters A, B, C and D to stand for Alice, Ben, Charlie and Debbie. Then we make a list of all the possible ways there are of filling these three places. You can see our list here on the right :

1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A



PTO

ans 17 **Alice's party**

NOT POSSIBLE	POSSIBLE
🔴	🟢

CONDITION 2

This tells us that we can't have any arrangement with A next to C or with C next to A, so straight away we can put a 🔴 next to two of the rows :

1	2	3	
A	B	C	
A	C	B	🔴
B	A	C	
B	C	A	🔴
C	A	B	
C	B	A	

CONDITION 4

This tells us that we can't have C in seat number 3, so that's a 🔴 next to two more rows :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	
C	B	A	

CONDITION 3

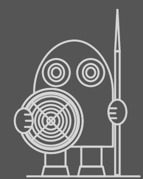
This tells us that we can't have any arrangement with A and C in seats 1 and 3. This happens in the first and last rows; the first row is already cancelled, so we need to put a 🔴 next to the last row :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	
C	B	A	🔴

And at last we have the answer to our problem; having Charlie, Alice, Ben and Debbie in seats 1, 2, 3 and 4 satisfies all the conditions we were given :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	🟢
C	B	A	🔴

our answer is // seat 1 : Charlie // seat 2 : Alice // seat 3 : Ben // seat 4 : Debbie //



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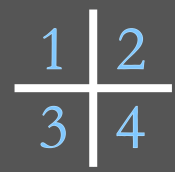
PTO

ans 17 Alice's party

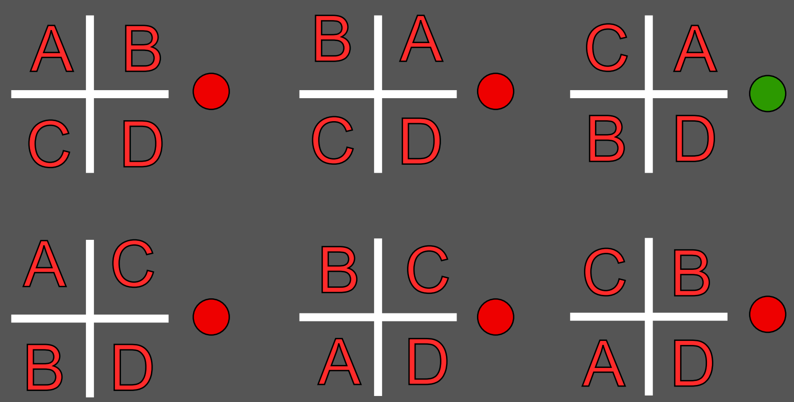
1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Of course, setting out the possibiblities in a table like the one on the left is the usual way to go about problems of this sort – but we could use a way which matches the seating plan, as in the diagram on the right :



Using this sort of 'truth table' means we can show all the possible seating arrangements just as they are . . . and with Debbie having to sit in place number 4, there are six possible seating arrangements.

Going through the four conditions we were given in the original question, we can mark each one as possible or not possible. When we do this, we have just one arrangement left, as you can see.



and once more we have // seat 1 : Charlie // seat 2 : Alice // seat 3 : Ben // seat 4 : Debbie //

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ans 18 Ben and Terry . . .

Every problem starts by giving you some information. This problem is a perfect example of how looking at this information in a different way can lead you straight to an answer :

Ben sells 1 ice-cream every 10 mins and Terry sells 1 ice-cream every 5 mins. It's hard to see how we can easily combine these two facts to get an overall figure for how many ice-creams an hour they are selling between the two of them. But we can re-write our information in this way :

Ben sells 6 ice-creams per hour

Terry sells 12 ice-creams per hour

So together Ben and Terry sell 18 ice-creams per hour !

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ans 19 . . . and Sally

As you will have realised, this problem is another example of how you need to take a different look at the given information in order to get to the answer :

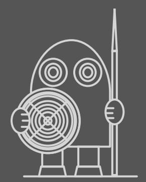
We know that we have one tap which takes 12 minutes to fill a bath and another tap which takes 6 minutes to fill the same bath. But there's no easy way of combining these figures. However, we can look at the information in a different way, like this :

hot tap : takes 12 mins to fill bath = can fill 5 baths an hour

cold tap : takes 6 mins to fill bath = can fill 10 baths an hour

so, hot and cold taps together can fill . . . 15 baths an hour

And our problem is solved! Saying that the hot and cold taps working together can fill 15 baths an hour comes to the same as saying that taken together they need just 4 minutes to fill one bath – and that's our answer!



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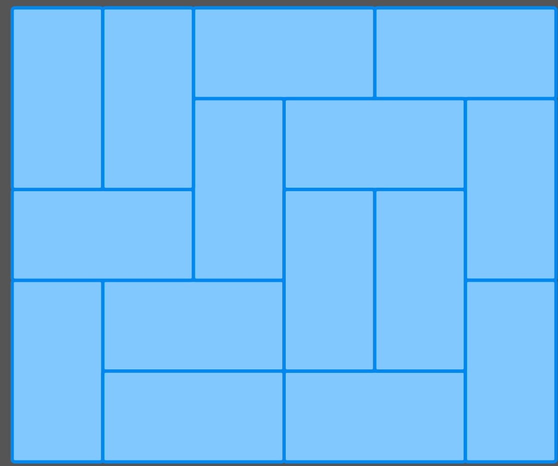
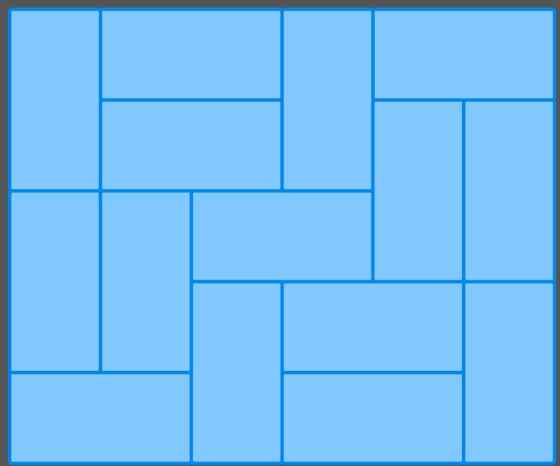
ans 20 whose fault is it ?



Here are two ways of solving the problem :



Perhaps you found a different way ?

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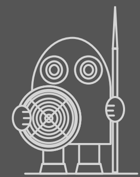
ans 21 **mr owl ate my metal worm**

palindromes with digit sum equal to 8

				.
101	202	303	404	.
111	212	313	.	1771
121	222	323	.	1881
131	232	333	.	1991
141	242	343	.	2002
151	252	353	.	2112
161	262	363	1001	2222
171	272	373	1111	2332
181	282	383	1221	.
191	292	393	1331	.

last palindrome year before 2002 1991

next palindrome year after 2002 2112



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ans 22 Ravi's mean question

How should we set about this problem? Well, we're looking for a set of three numbers and one thing we know is that one of them is double another of them. So, let's jot down some pairs of (a number + its double) and for each pair let's write down what the third number must be. (Remember, the mean of the three numbers is 10, so we know that they must add up to 30.)

20, 10	18, 9	16, 8	14, 7	12, 6	10, 5
0	3	6	9	12	15

The red dotted line joins the smallest and the largest of each set of three numbers. As you can quickly see, there's only one group of three numbers where the smallest is 10 less than the largest. So now we have our answer:

Ravi's three numbers were 6, 8 and 16

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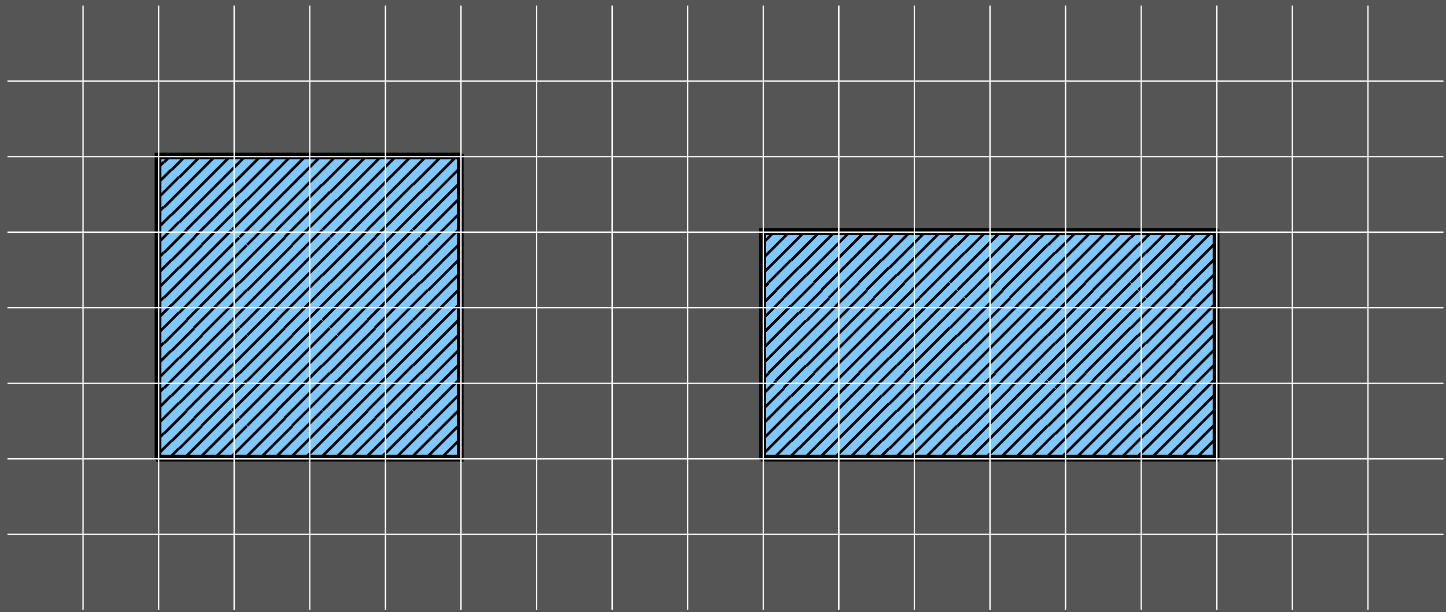
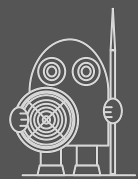
Here are a couple of suggestions :

A rectangle 4 cm x 4 cm has area 16 cm^2 and perimeter 16 cm



A rectangle 3 cm x 6 cm has area 18 cm^2 and perimeter 18 cm



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six-a-side !

- The one-hole shape uses 6 tiles, the two-hole shape uses 10 tiles and the three-hole shape uses 14 tiles.

- As you can see, Mark has just been adding 4 tiles each time, so the four-hole shape would use 18 tiles.

- We can think of the pattern this way : We begin with 2 tiles, then add 1 lot of 4 tiles to complete a 1-hole shape, add 2 lots of 4 tiles to complete a 2-hole shape, add 3 lots of 4 tiles to complete a 3-hole shape and so on. So, we have a pattern which is just :

  times the number of holes by 4 and add 2.

  So, if Mark used 150 tiles, he must have used 37 lots of 4 tiles (because 37 x 4 = 148) to begin with and then added 2 tiles to complete the pattern. Which means that it's a 37-hole pattern!

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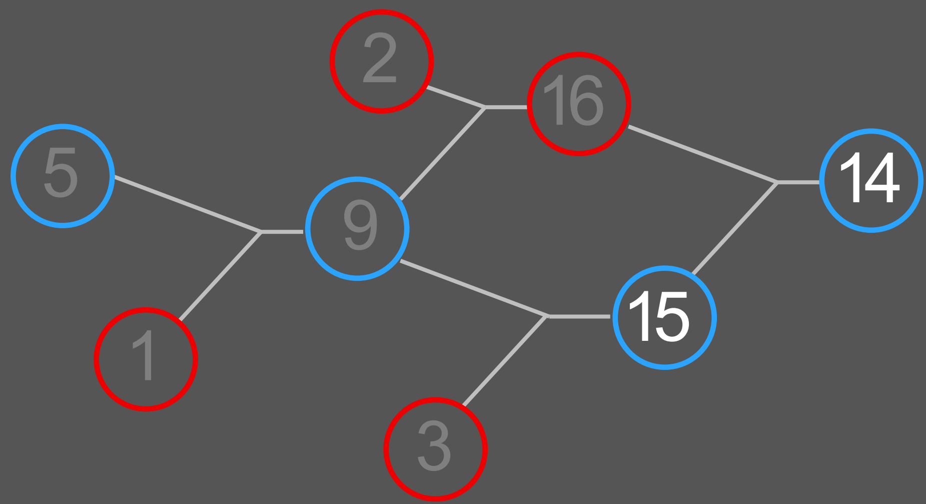
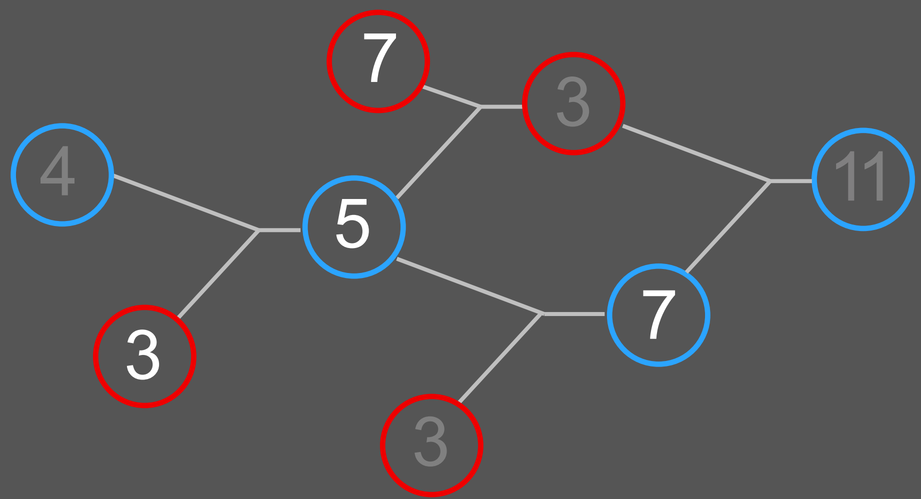
ans 25 mapping webs 1

Remember, the rule for this mapping web is : to combine two numbers, just double the 'blue' number and then subtract the 'red' number. For the web on the right, you simply have to work forwards; to complete the one below, you'll need to do some working backwards. Here are the answers :





* answers for missing numbers shown in white

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ans 26 three brothers

One way of handling this questions is :

- make a list or chart which shows all the possible combinations, that's to say teacher / seaside, teacher / country etc
- put S, J and P in each
- go through the seven pieces of information given in the question and cross out any initials ruled out eg 'John is not a teacher' lets us cross out all the Js in that row
- finally, just three initials remain (one on each row) so there are your answers!

	seaside	country	town
teacher	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ ~~P~~	S ~~J~~ ~~P~~
fireman	~~S~~ J ~~P~~	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ ~~P~~
builder	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ P	~~S~~ ~~J~~ ~~P~~

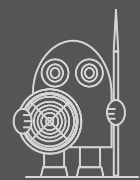
conclusions :

1 John is the fireman

2 the builder lives in the country

3 Simon is a teacher

4 John lives by the sea



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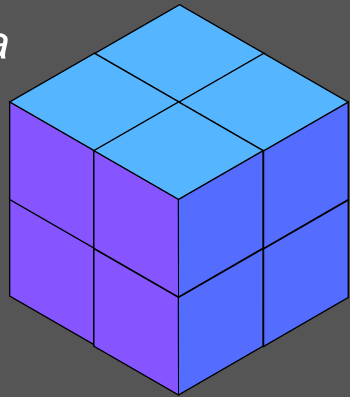
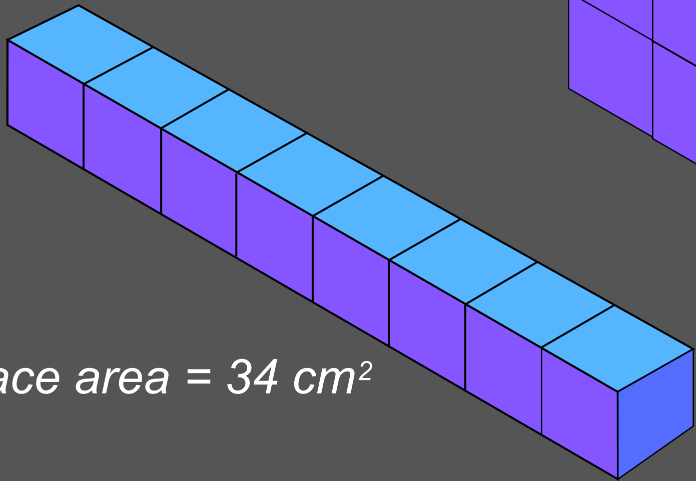
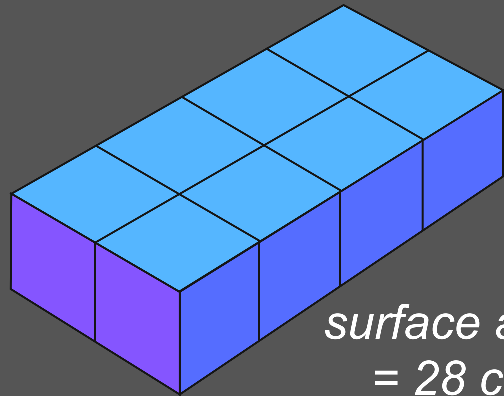
ans 27 cube surfaces

surface area
= 24 cm^2



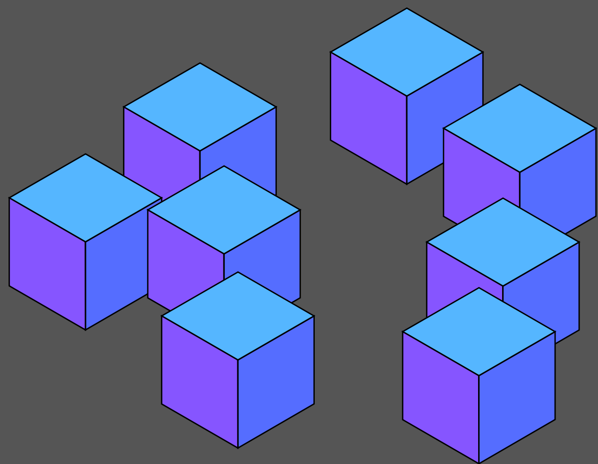


surface area = 34 cm^2



surface area
= 28 cm^2

total surface area
= 8 x 6 = 48 cm^2



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ans 28 a grandson called Ben

- The second piece of information gives us a good place to start. Because, if the digits of the number add up to 8, then it must be one of these :

  08, 17, 26, 35, 44, 53, 62, 71, 80

- Since John's parents are each 45 yrs old, the first five of these numbers can be ruled out straight away. This leaves us with just 53, 62, 71 and 80.

- The first bit of information we have is that our number is a prime number. Obviously 62 is an even number, so it isn't prime . . . and 80 has lots of factors, so it isn't prime . . . so now we're left with 53 and 71 (both prime numbers).

- 53 is ruled out for another reason : If Grandfather really is 53, he must have become a father at the age of 8 - most unlikely!

- final confident answer : Grandfather is 71 !

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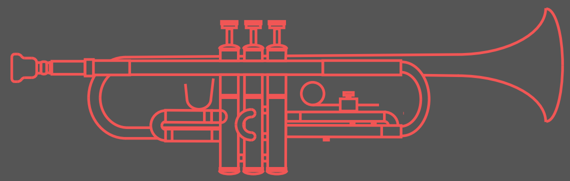


Using U for up and D for down, you could hold the three valves in these different positions :

U	U	U
U	U	D
U	D	U
U	D	D
D	U	U
D	U	D
D	D	U
D	D	D

– and that's 8 different combinations altogether !

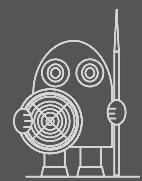
** Of course, a real trumpet can play more than 8 different notes, which tells you that there's more to playing a trumpet than just learning these 8 combinations.*

special note :

In fact, trumpeters usually call the valves 1,2 and 3 and they list the combinations like this (ps. 0, as you might guess, just means 'valve left open') :

0	0	0
0	0	3
0	2	0
0	2	3
1	0	0
1	0	3
1	2	0
1	2	3

– that's 8 different combinations altogether!



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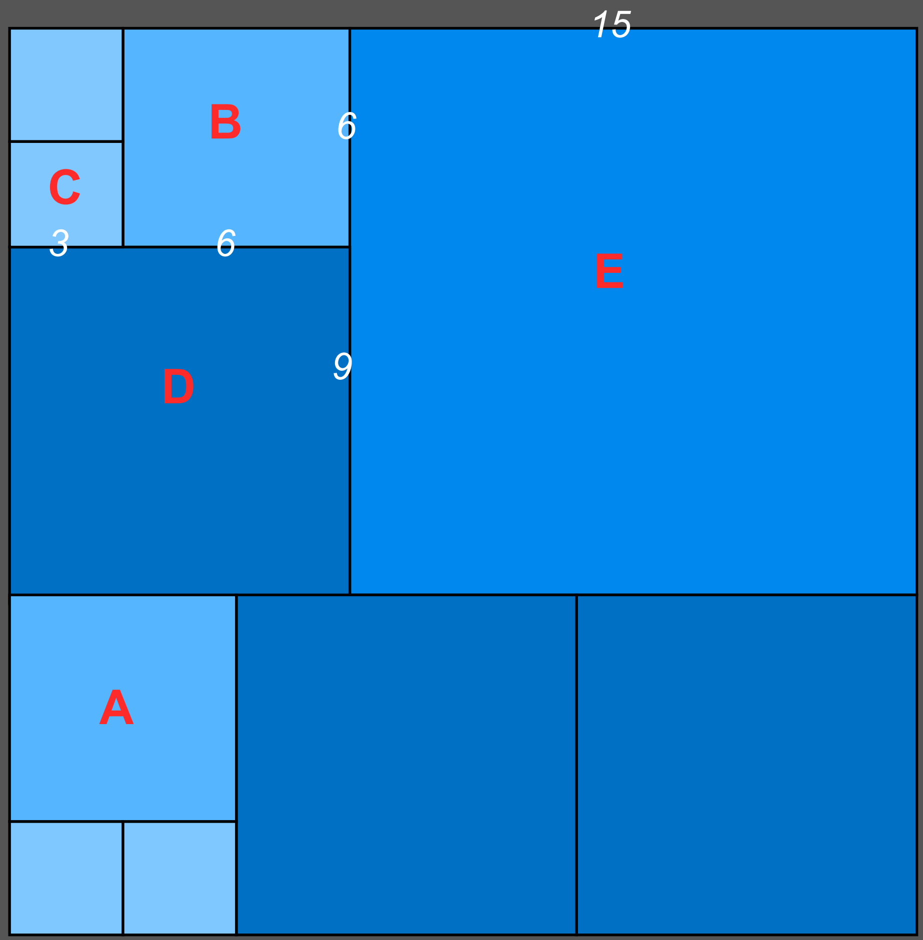
ans 30 all square



If square A has an area of 36, then square B, which is the same, also has an area of 36, which means it has sides of length 6.

Square C obviously has sides exactly half as long as square B's sides, so square C's sides must be just 3.

By simple addition, this gives us 9 for the side length of square D.

Then once again by simple addition, square E must have side length 6 + 9 = 15.

Squaring the 15 gives us the area of E :

$$15 \times 15 = \underline{225}$$

and that's your answer !

PTO

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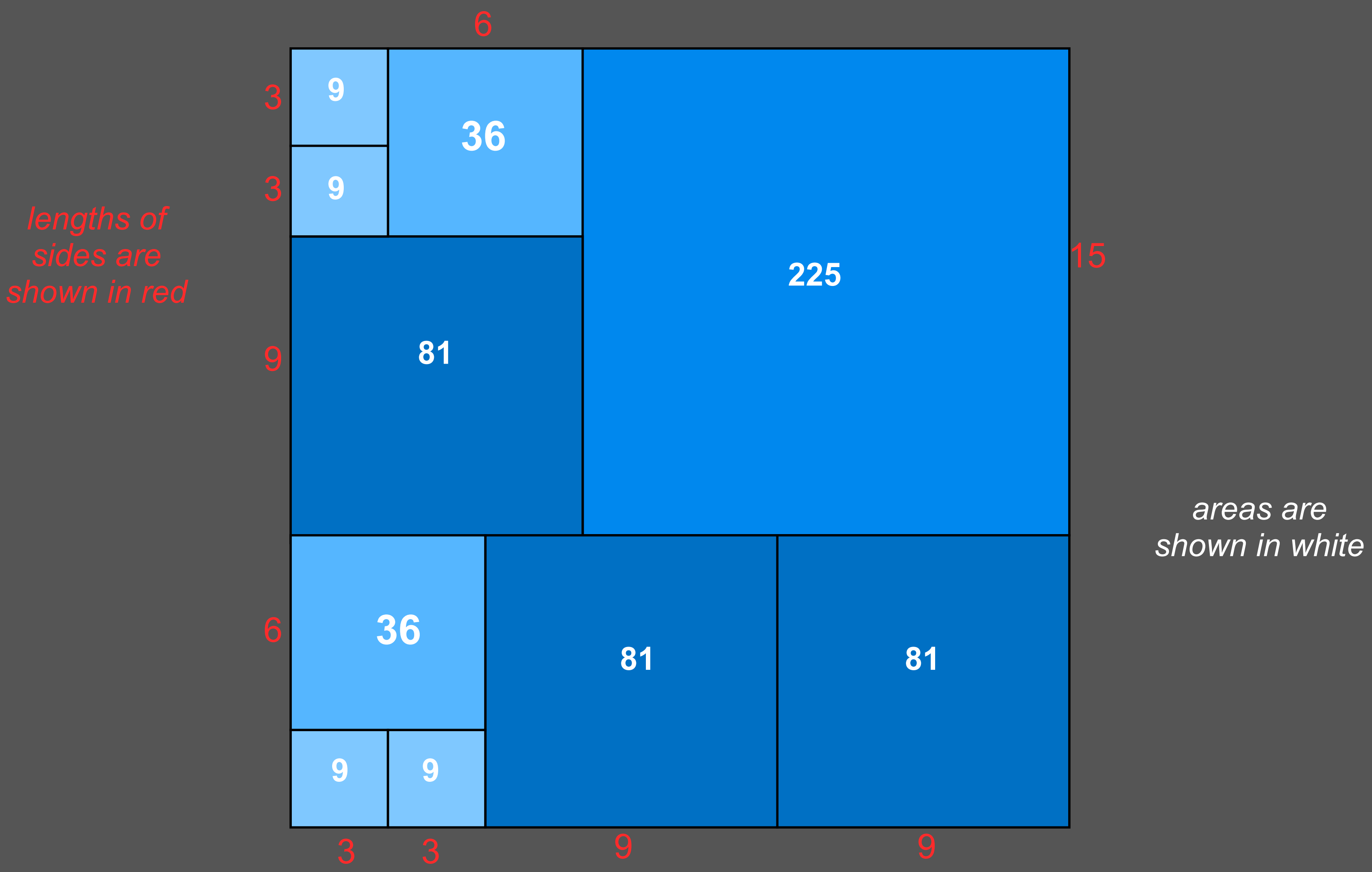
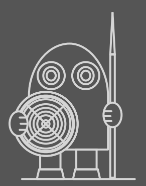
ans 30 all square

For your information, here is the same diagram with all lengths and areas shown :





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ans 31 Mr Average

Where to begin? Well, straight away we know that B weighs 40kg and that this is 80% of D's weight; perhaps you can see (or easily work out) that D must weigh 50kg. What else do we know? Let's use $\square$ to stand for the average (mean) of the group. Now we know three more things : We know that A's weight is $\square$, we know that C's weight is $\square\square$ and we know that the total weight of the whole group is exactly $\begin{matrix}\square\square\\\square\square\end{matrix}$ (that's to say four times the average).

Of course, the total weight of the whole group is made up by adding together the four weights of A, B, C and D. We can show this as follows :

Mr A's weight $= \square$

Mr B's weight = 40kg

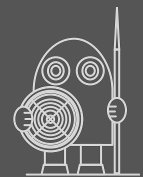
Mr C's weight $= \square\square$

Mr D's weight = 50kg

$$\square + 40 + \begin{matrix}\square\\\square\end{matrix} + 50 = \begin{matrix}\square\square\\\square\square\end{matrix}$$

So, what does this mean? You might recognise it as an equation but all that matters is that everything on the left comes to the same as what's on the right). And what this amounts to is just this :

$$\begin{matrix}\square\\\square\square\end{matrix} + 90 = \begin{matrix}\square\square\\\square\square\end{matrix}$$



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ans 31 **Mr Average**

So, we're left with : 3 lots of $\square$ plus 90 must be the same as four lots of $\square$

and this tells us that $\square$ must be worth 90 : $\square = 90$

And now it's not very hard to work out the weights of all four athletes :

Mr A's weight is 90kg

Mr B's weight is 40kg

Mr C's weight is 180kg

Mr D's weight is 50kg

At last we have answers to our two problems! First of all, Mr D weighs 50kg and secondly, the four men together weigh 360kg – exactly at the safety limit !

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ans 32 the long bench

One pretty standard way of handling problems like this is to begin by making a list of all the equally likely possible seating arrangements. You can see such a list here on the right :

Next, we identify those arrangements which have Tom sitting in place number 1. As you can see straight away, out of the 12 possible arrangements, there are just two which have Tom in place number 1. So, we can write

probability (Tom sits in place 1) = 2/12 = 1/6

1	2	3	4
X	R	S	T
X	R	T	S
X	S	R	T
X	S	T	R
X	T	R	S
X	T	S	R
R	S	T	X
R	T	S	X
S	R	T	X
S	T	R	X
T	R	S	X
T	S	R	X

– and that's our answer! Probability of Tom sitting in place 1 = 1/6

PTO

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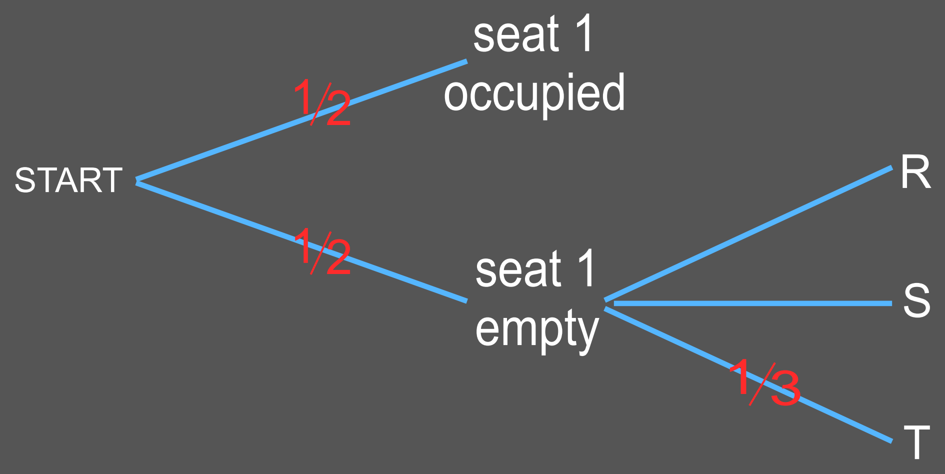
ANS 32 the long bench

Some people prefer a 'tree diagram' method of looking at problems like this one. Such a diagram for this problem is shown on the right :



You can see how the diagram works : To begin with there's an equally likely probability of either seat 1 or seat 4 being occupied when our three arrive. We show this on the diagram with two branches, each marked '1/2'. Then when seat 1 is empty, there's a 1/3 probability that Tom will sit in it. We need to work out 1/3 of 1/2, which of course is the same as 1/3 x 1/2, or 1/6. And so we have our answer :

probability of Tom sitting in seat number 1 = 1/6

PTO

the long bench

Here's a slightly different way of looking at the problem (you might see it as a more helpful way of explaining the tree diagram on the previous page) :

As the friends arrive, the probability that seat 1 is already occupied is 1/2 . . . and so the probability that this seat is empty is also 1/2.

So now we can say that there's only a 1/2 chance of any of the friends taking seat 1. But if seat 1 is empty and it's there to be taken, there's only a 1/3 chance that it's Tom who gets it.

Which means that Tom has a 1/3 of a 1/2 probability of sitting down in place number 1. And as you probably know, 1/3 of 1/2 just means 1/3 x 1/2, which is of course 1/6.

answer : probability of Tom sitting in place 1 = 1/6

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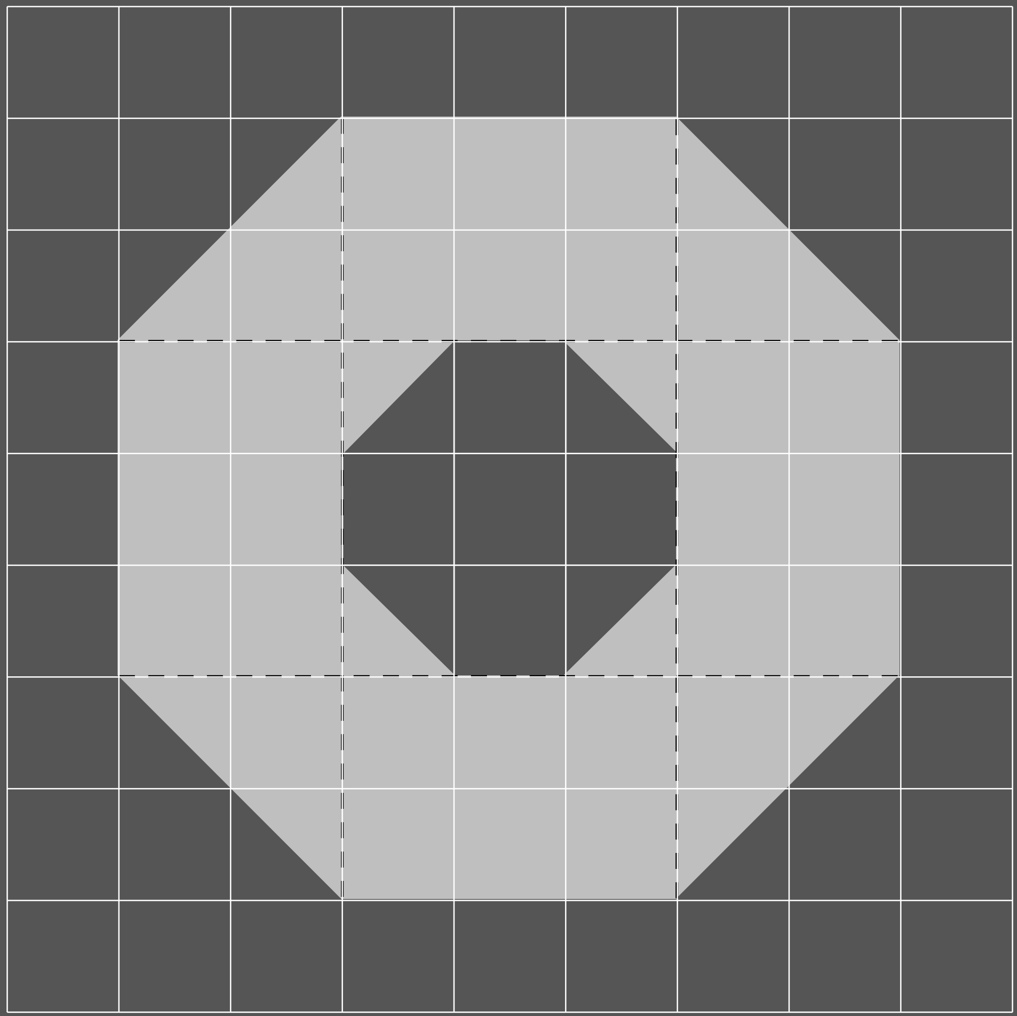
ANS 33 an octagon ring



This is the same diagram as in the question but here we've drawn the gridlines over the ring. Now it's easier perhaps to see that the ring is made up of two different kinds of shape, that's to say, triangles and rectangles. Altogether we have :

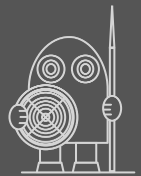
4 triangles, each of area 2 squares

4 triangles, each of area 1/2 square

plus

4 rectangles, each of area 6 squares

– which means that the total area of the ring must be exactly 34 squares.



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PTO

ans 33 an octagon ring

. . . or of course you might have done this a different way, that's to say by starting with a square and subtracting :

first of all, the large square around our shape has an area of 7 x 7 = 49 squares

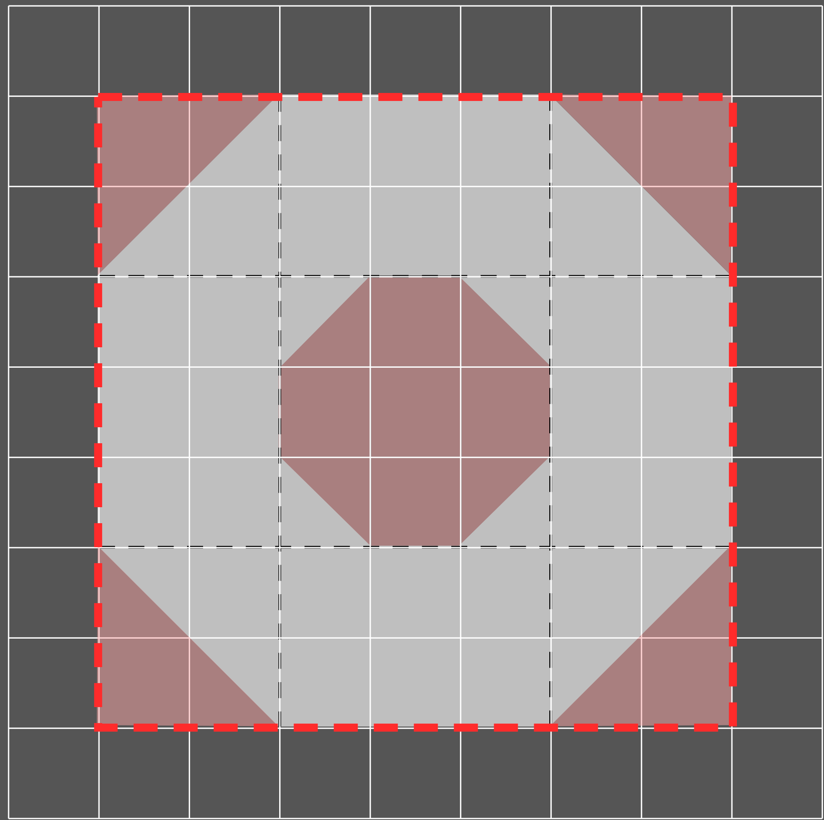
Now take away from this

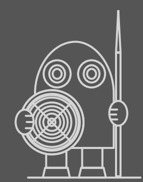
4 corner triangles, each of area 2 squares

and an octagon in the middle, area 7 squares

– which gives us a final result of

49 – 15 = 34 squares





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ans 34 you're my number wall !

No great challenge with this one! It takes only a moment's thought to see that the missing number on the bottom row must be 3 – and then it's just a matter of adding pairs of numbers to complete the wall . . .

			40			
		19		21		
	10		9		12	
4		6		3		9

			48			
			10			
3		6		4		15

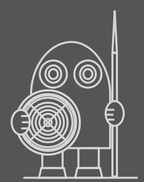
. . . but this one's harder! Obviously, the two missing numbers on the bottom row must add up to 10. We have no idea what they could be, so let's just try 6 and 4 – and see what happens.

We now have a bottom row which reads :

3, 6, 4, 15

Adding upwards, we can easily fill in the blanks (we've used red numerals for these). And straight away we can see that this works! But is this the only possible answer?

			48			
		19		29		
	9		10		19	
3		6		4		15



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PTO

ans 34 you're my number wall !

Let's try a different pair of numbers : 2 and 8. These numbers also add up to 10, so let's see what happens. Adding upwards, the blanks are soon filled in, as shown on the right – and, as you can see, once again everything works! So now we have another solution to the problem.

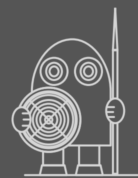
48
15 33
5 10 23
3 2 8 15

1 and 9 are two more numbers adding up to 10 and once again, starting with these on the bottom line and filling in the blanks, reveals another arrangement which works well.

48
14 34
4 10 24
3 1 9 15

In fact, it turns out that you can fill in the blanks on the bottom line with any pair of numbers you like, as long as they add up to 10.

By the way, this answer is another example of the value of experimenting when you're not sure how to begin; we just tried some numbers and we quickly solved the problem! We also found that this was yet another example of a problem having more than one solution . . .



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ans 35 remainders

The numbers which divide exactly by 2 are 2, 4, 6, 8 . . . So the numbers which divide by 2 and leave a remainder of 1 are all of these same numbers but with 1 added to each of them, in other words 3, 5, 7, 9 . . .

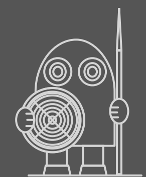
In the same way, the numbers which divide exactly by 3 are 3, 6, 9, 12 . . . and so those which divide by 3 and leave a remainder of 2 are just the same multiples of 3 with 2 added to each one, that's to say 5, 8, 11, 14 . . .

You can handle dividing by 4 and dividing by 5 in just the same way. On the right is a short version of the four lists you get from these calculations :

÷ 2 gives rem 1	÷ 3 gives rem 2	÷ 4 gives rem 3	÷ 5 gives rem 4
3	5	7	9
5	8	11	14
7	11	15	19
9	14	19	24
11	17	23	-
13	20	-	-
15	-	-	-
-	-	-	-
-	-	-	-
-	-	-	49
-	-	51	54
-	53	55	59
55	56	59	
57	59		
59			

59 is the first number to appear in all four columns – although you do have to work out a whole lot of rows to get there. To save space we've left out a lot of the middle rows and instead we've just shown the beginning and end numbers in the four sets.

So, 59 is the answer to our challenge !



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ans 36 parking mad . . .

Here are the three cars : a sports car, a saloon and an estate car . . .

The colours are easy : we're told that the sports car is blue and that the family saloon is red – so the estate car must be yellow . . .

And from what we're told, Dr Brown's car is obviously the red family saloon – and Miss Green's car is not blue, so it must be the yellow estate car – leaving the blue sports car for Mr Smith . . .

Mr S Dr B Miss G

answer : Mr Smith owns the sports car.

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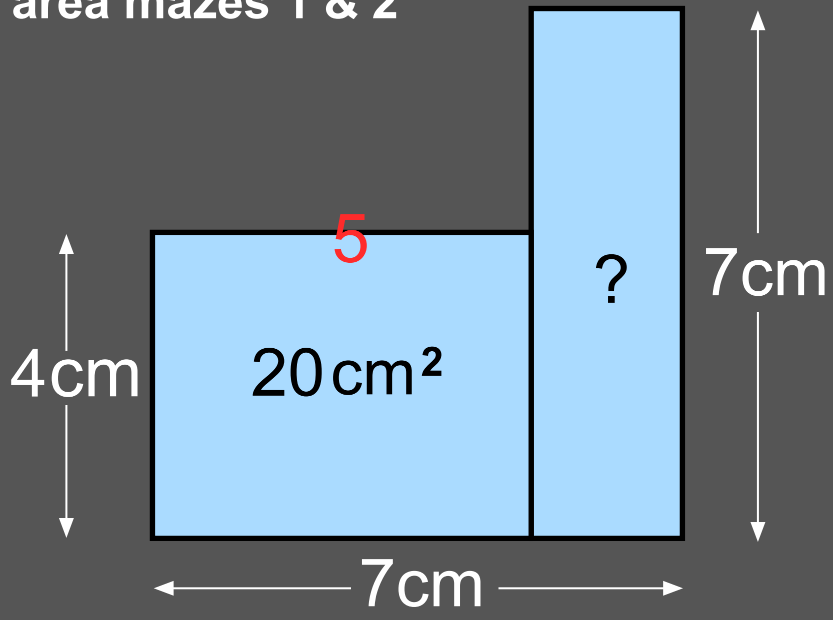
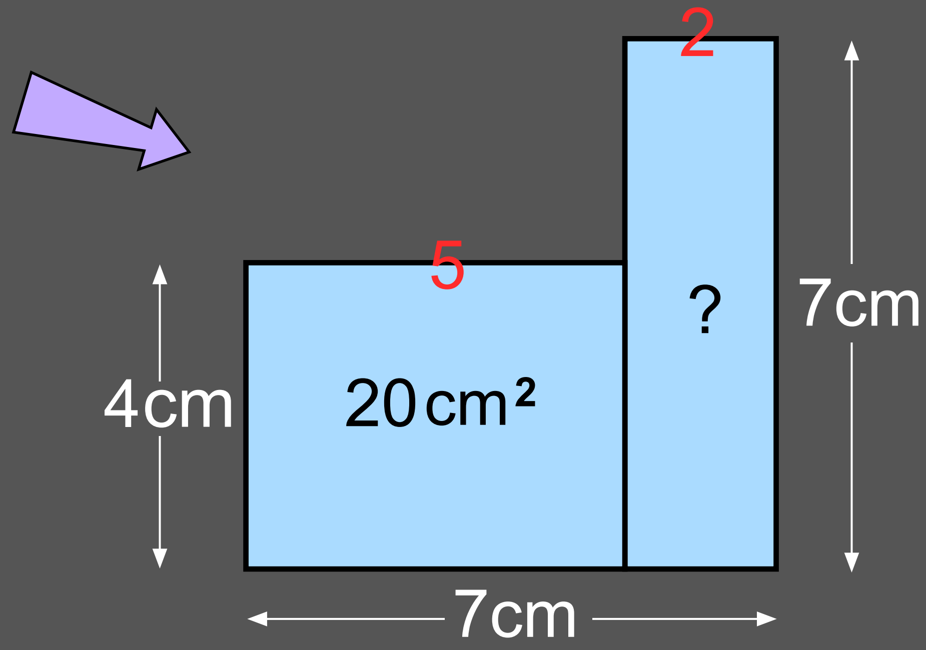
ans 37 **area mazes 1 & 2**

(a)





First of all, look at the rectangle on the left. Its area is 20cm^2 ; one side is 4cm and so the other side must be 5cm.

This means that the rectangle on the right must be 2cm wide . . .

. . . and so the area of this rectangle must be 14cm^2

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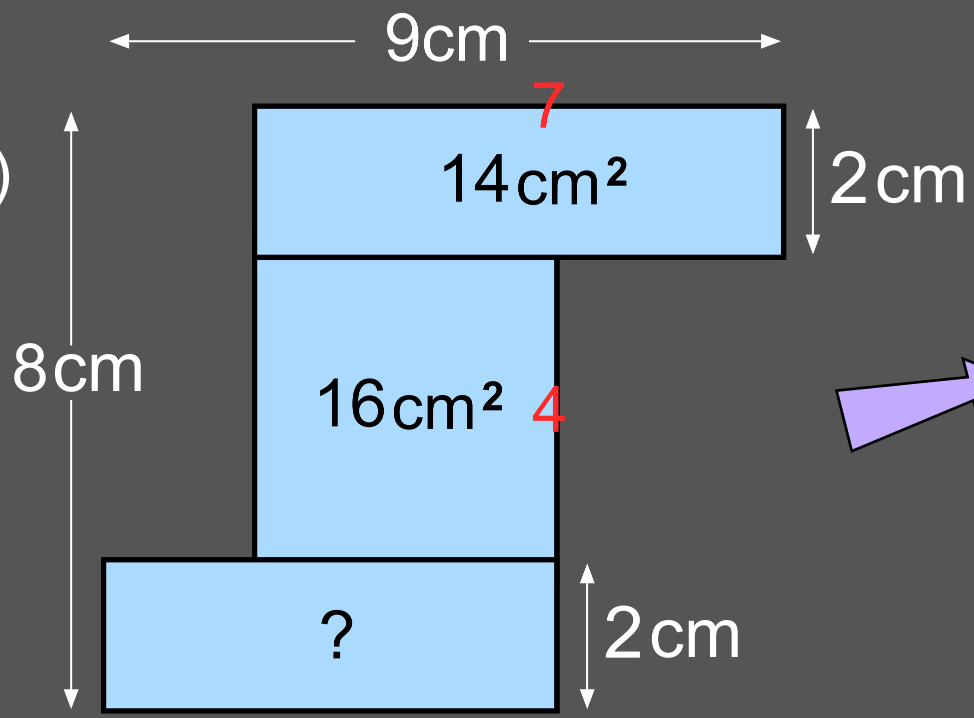
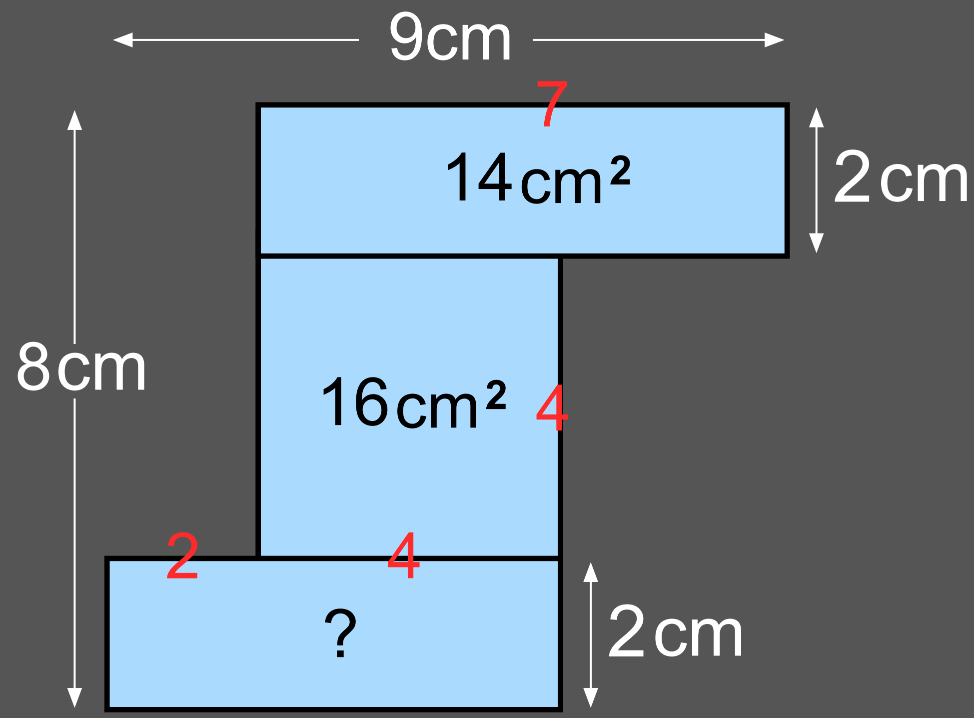
PTO

ans 37 area mazes 1 & 2

(b)





To begin with, notice that the top rectangle has an area of $14cm^2$ and one of its sides is 2cm; so the other side must be 7 cm. Also, the figure as a whole is 8cm deep and so by subtraction (taking away the two lots of 2cm), we get 4cm for one side of the middle rectangle.

With an area of $16cm^2$ and one side of 4cm, the other side of the middle rectangle must also be 4cm. And, by subtraction, the remaining length of the bottom rectangle must be 2cm. So its two sides must be 6cm and 2cm, giving an area of $\underline{12cm^2}$

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ans 37 area mazes 1 & 2

These two small problems are examples of an interesting type of puzzle called the 'Area Maze'.

With each area maze you are asked to find just one thing, which will be either a length or an area. To solve any area maze problem, you need only one bit of maths knowledge : how to find the area of a rectangle. Also (you may be pleased to know), all of these problems can be solved without using fractions or decimals, so there's just one rule you have to follow : stick to whole numbers! It's this rule which means that some area maze problems will send you quite a long way round to get to the answer – just like finding your way round a real maze . . .

日本

** Area Mazes first appeared in Japan (where they were called 'Menseki Meiro') and they quickly became very popular. They were invented by Naoki Inaba, a man who has thought up lots of different maths problems and puzzles . . . If you enjoy solving these, there are lots of Area Maze books available (but be careful – many of the books jump up rapidly from easy questions to really quite hard ones).*

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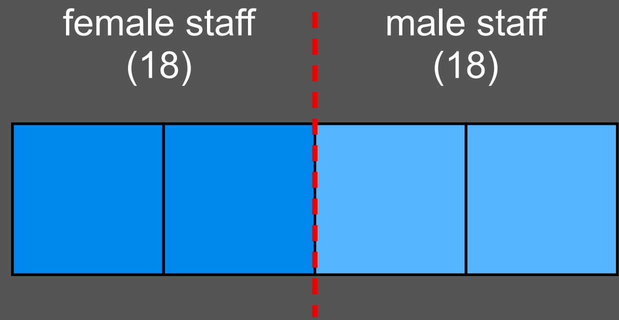
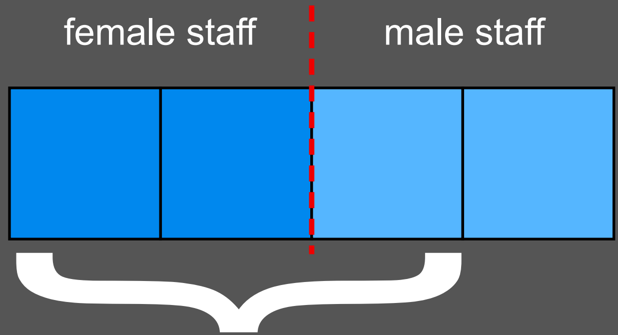
ans 38 teaching equality

There are different ways of going about this problem but one way is to use diagrams to make things clearer:

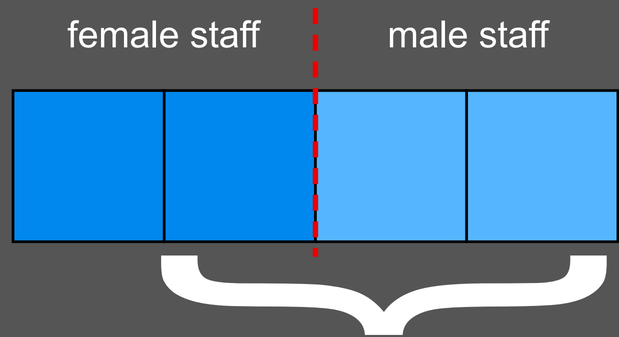
Suppose we use one square to stand for 9 members of staff, then the balance of male/ female staff at the School can be shown like this :



On any half-day, three-quarters of the teachers are present. Three-quarters of 36 is 27, or in other words, three full squares on our diagram. And so we can show the largest and the smallest numbers of females present at any one time :



largest no. of female teachers = 18



smallest no of female. teachers = 9

– and so question 1 is already answered : As there are always at least 9 female teachers present, the Inspector can indeed be sure of finding a female teacher there whenever he calls.

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ans 39 DIY magic square

PROBLEM 1 : The first thing to do with problems like this is to find the magic total. Adding up the numbers in the right-hand column gives us 24 and so this is our magic total. The middle row has two numbers in it, 15 and 1, which add up to 16. So we need an 8 in the centre to make our required 24. In the same way, we can work out that the two missing corners must be After this, it's easy to fill in the missing numbers in the centre column : they must be 11 and 5 . . . and that's it – problem solved!

3	11	10
15	8	1
6	5	13

MT = 24

PROBLEM 2 : How can we get started on problem 2? Obviously we can just try our numbers in lots of different positions and hope that sooner or later we'll hit on an arrangement which works. But looking for an answer this way might take an awful long time! Maybe we can make things easier for ourselves if we can get straight to the magic total or perhaps to the centre number. On the next pages you'll find some ideas which might help . . .

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PTO ▶▶

ans 39 DIY magic square

finding the magic total . . .

Let's just think about the rows in a magic square. As you already know, each of the three rows in a 3 x 3 magic square must add up to the magic total. So if you add the three rows together, then of course you'll have three times the magic total. But stop and think : what are you adding together when you add up the three rows? Well, obviously you're adding up the three rows. But wait! Each row contains three of the nine numbers which make up the magic square – and so adding the three rows together is just the same as adding together all nine numbers. Thinking of our problem, we know that our nine numbers are 0, 1, 2, 3, 4, 5, 6, 7 and 8. These numbers add up to 36, which means that our three rows must add up to 36. So each row must add up to 12, or in other words,

magic total = 12

If you find this thinking a bit hard to follow, don't worry! It's not an easy idea, so you might need to go over it once or twice before you're sure you really understand it. It's an important idea though, so it's worth the effort!

With magic total = 12, perhaps you can already say what the centre number must be . . .

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PTO ▶▶▶

DIY magic square

the centre number . . .

You probably remember that in problem 1 the centre number was exactly one-third of the magic total. This connection holds true for all 3 x 3 magic squares. We know that the magic square we're going to construct has magic total = 12 and so the centre number must be 4. This leads us to some more useful facts . . .

these two numbers
must add up to 8

We know that 4 is our centre number. This means that whether we're looking at rows, columns or diagonals, the numbers on either side of the centre must add up to 8 (since 12 is the magic total). See the diagram on the left, which illustrates this.

So the pairs on either side of the centre 4 must be : 0,8 1,7 2,6 3,5

All we have to do now is to find how to arrange these pairs . . .

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PTO

ans 39 DIY magic square

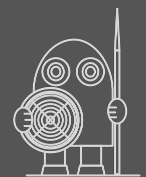
Well, first of all we found this one way of arranging the pairs. But then we looked again and we found some more ways of arranging the same pairs. In the end we found a total of eight different solutions. You can see them all on the next page. Obviously, they all have magic total = 12 and they all have 4 as the centre number.

3	2	7
8	4	0
1	6	5

MT = 12

Have a look at the next page and compare the eight different answers. Do you think they really should count as different solutions to the problem? Or do you think that we should see them as just one solution rearranged in different ways? This is the sort of thing mathematicians often have to decide.

And – can you describe what you have to do to solution 1 to get each of the others? (For example, you get solution 2 by simply rotating solution 1 clockwise through 90˚.)



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PTO ⏩

ans 39 **DIY magic square**

3	2	7
8	4	0
1	6	5

1	8	3
6	4	2
5	0	7

5	6	1
0	4	8
7	2	3

7	0	5
2	4	6
3	8	1

7	2	3
0	4	8
5	6	1

5	0	7
6	4	2
1	8	3

1	6	5
8	4	0
3	2	7

3	8	1
2	4	6
7	0	5

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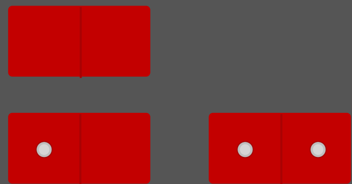
ans 40 **domino faces**

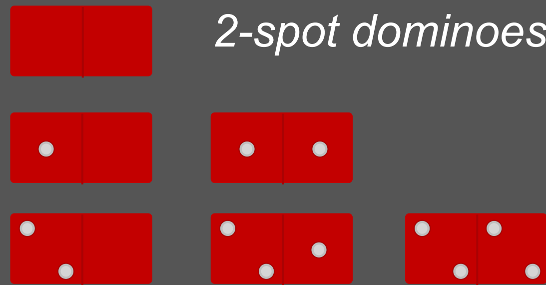
You'll find the answers to all four questions in the diagrams below and on the next few pages, which show domino sets from 0-spot to 6-spot, as well as a 9-spot set.

0-spot domino

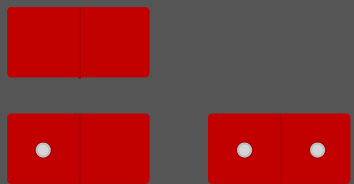
1-spot dominoes

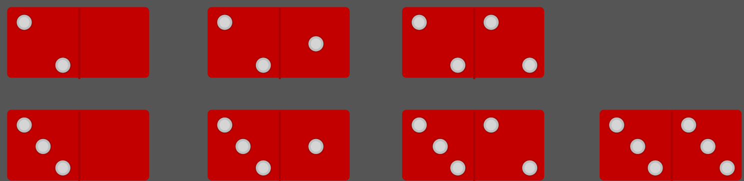


2-spot dominoes



3-spot dominoes





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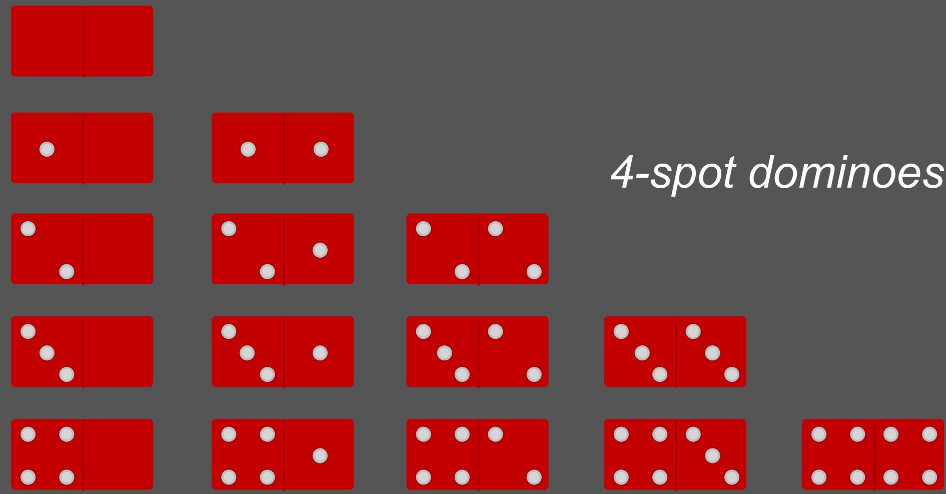
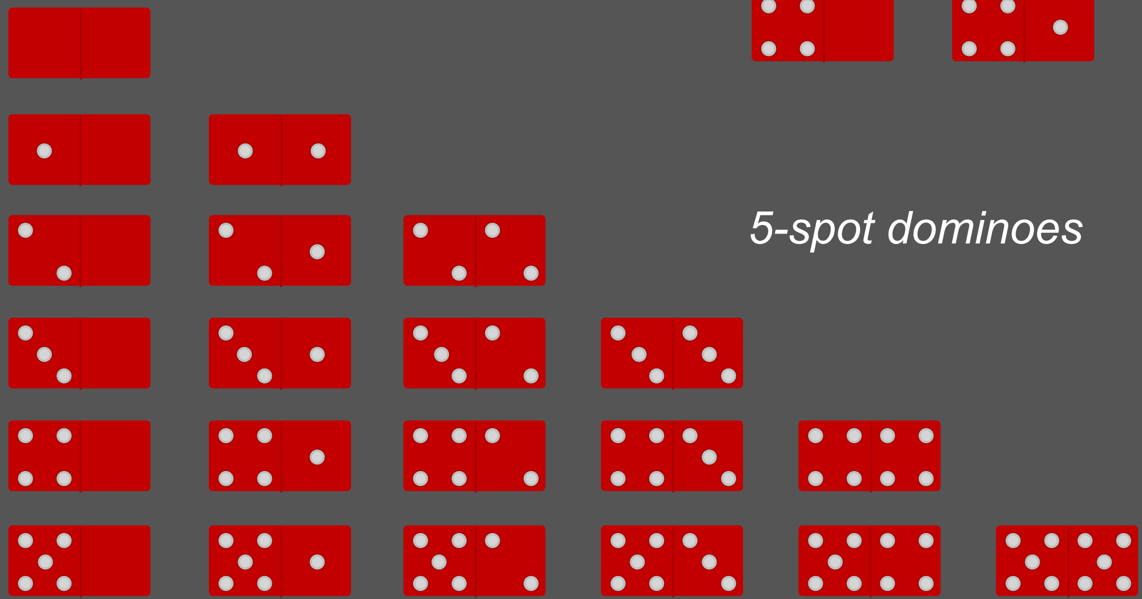
ans 40 **domino faces**



4-spot dominoes

5-spot dominoes



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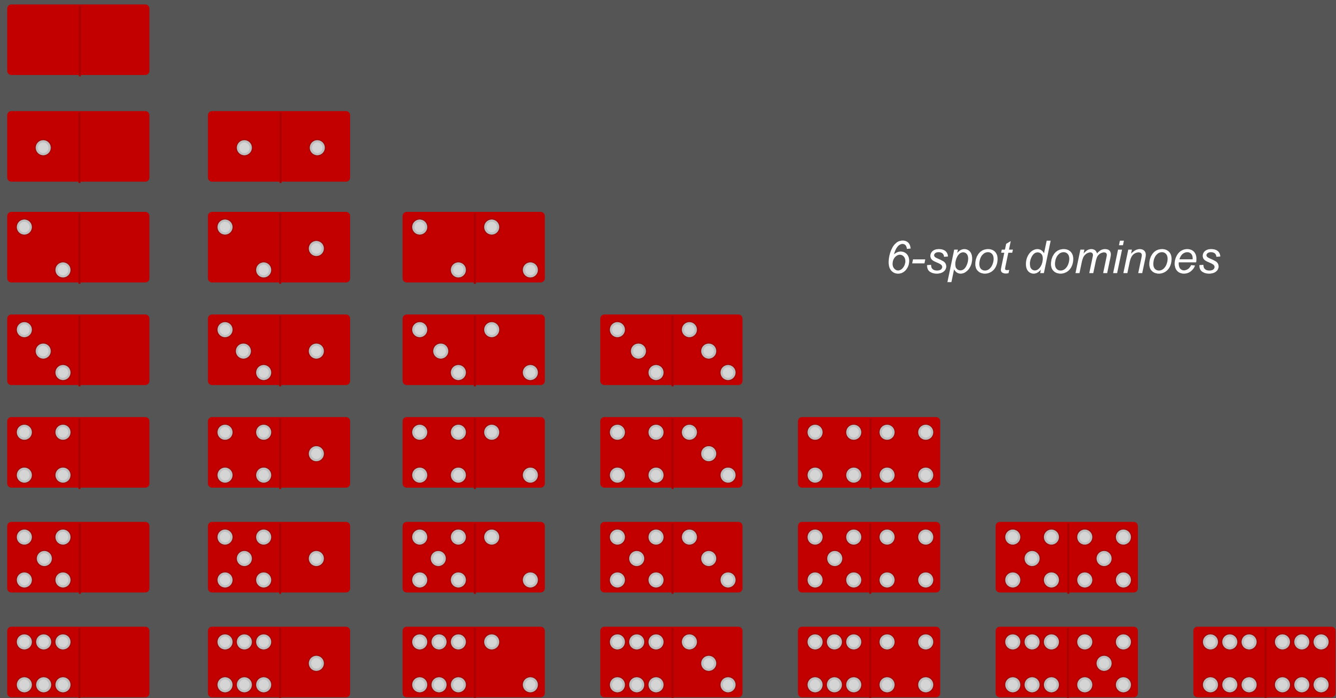
40 domino faces



6-spot dominoes

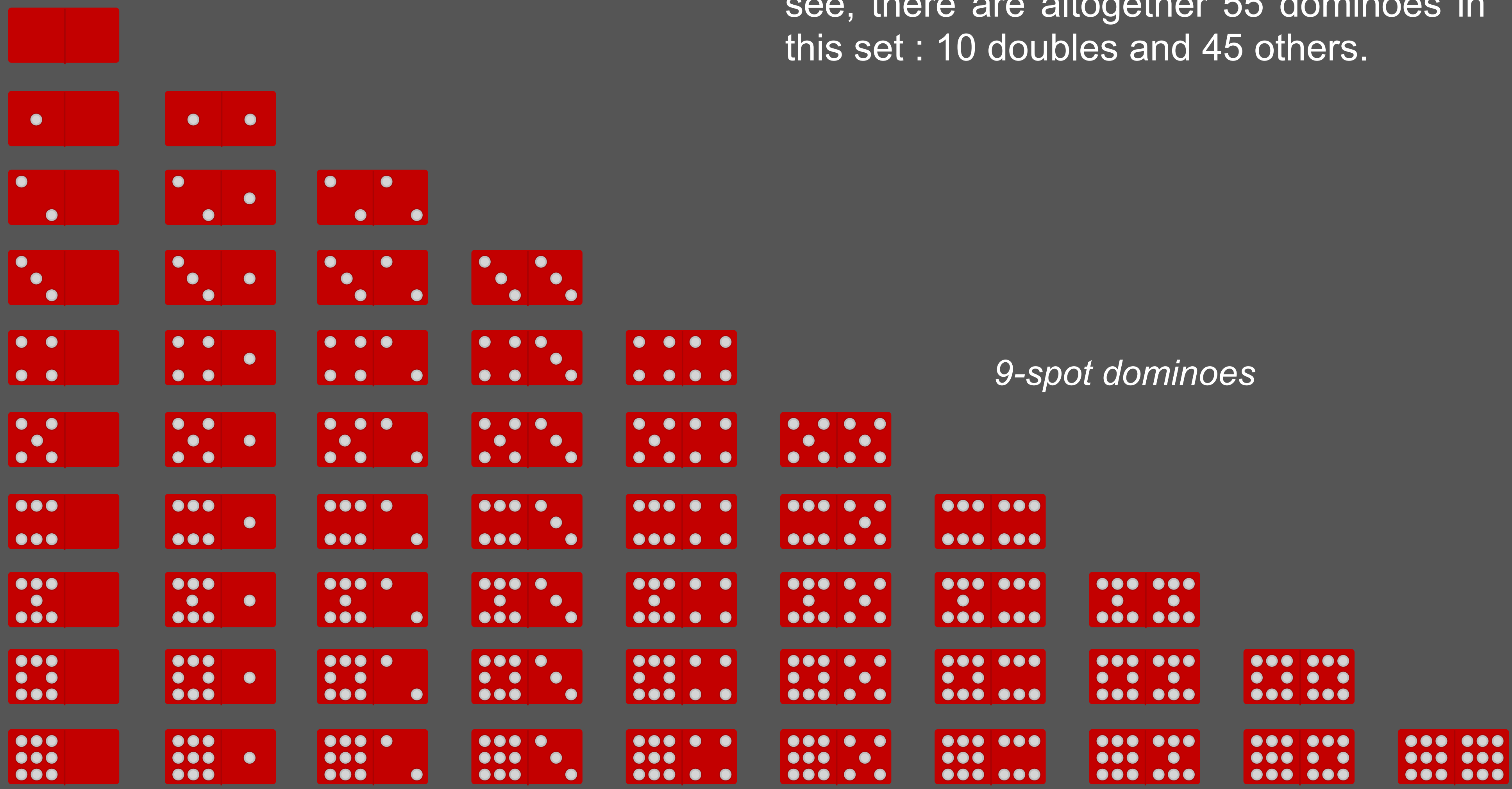
Notice that the number of dominoes in these sets goes 1, 3, 6, 10, 15, 21, 28 . . .

These numbers are of course the triangle numbers! As you probably know, the triangle numbers come up quite often in maths investigations. One simple way to get the triangle numbers is to start with 1, then add 2, then add 3, then add 4 and so on.

PTO

ans 40 domino faces

Finally, the 9-spot dominoes. As you can see, there are altogether 55 dominoes in this set : 10 doubles and 45 others.



9-spot dominoes

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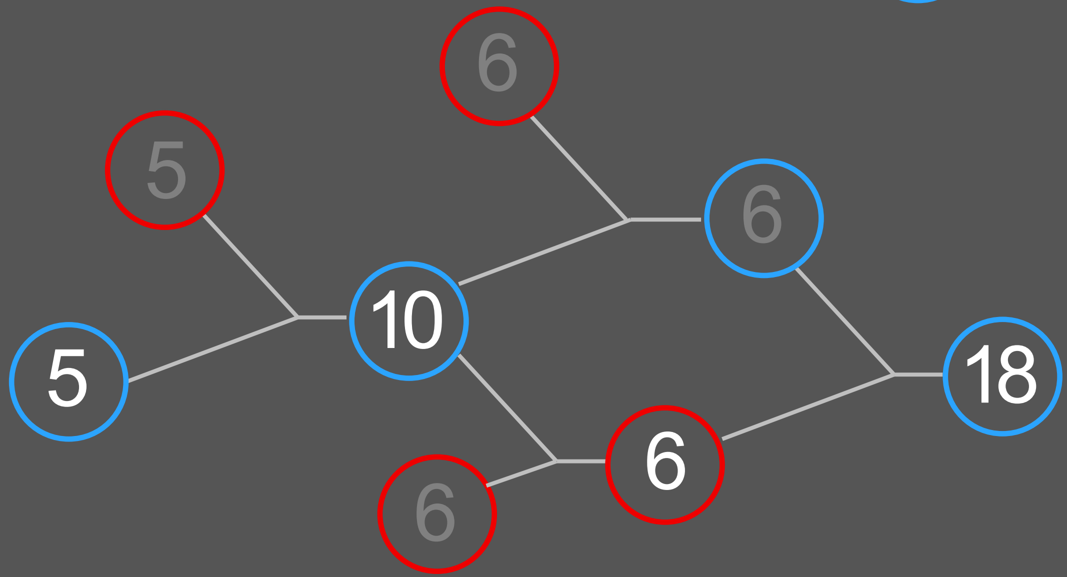
ans 41 **mapping webs 2**

Remember the rule for this mapping web : to combine two numbers, square the 'red' number and then subtract three times the 'blue' number. As you probably discovered, to complete these two webs, you need to do working backwards as well as forwards.

Here are the answers :



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ans 42 cube calendar

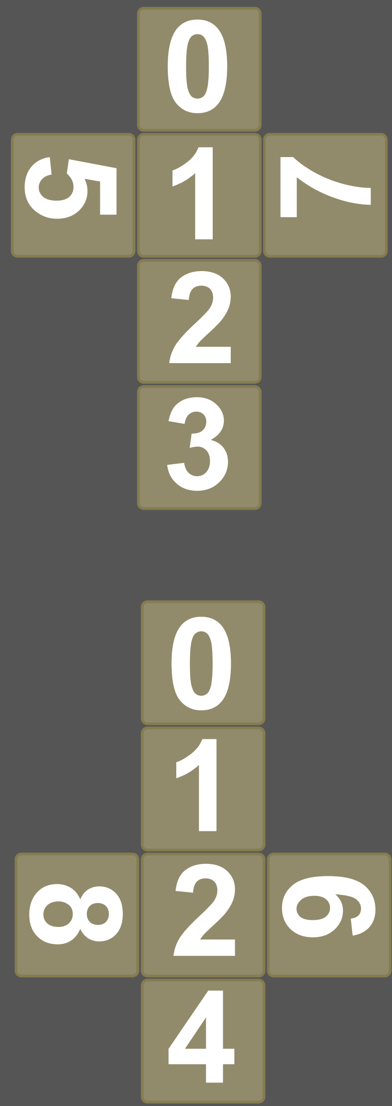
31 is the greatest number of days there are in any month, so we need our cubes to be able to display all numbers from 1 to 31. We can make our life easier by noticing one or two things :

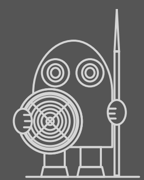
Because of 11 and 22, both 1 and 2 have to be on each cube. Next, because 0 has to be combined with every number from 1 to 9, there must be a 0 on each cube (since a cube has only 6 faces). And because 1 and 2 have to be combined with every number from 1 to 9, there must be a 1 and a 2 on each cube (which we've already proved).

Finally, which numbers don't we need? We can forget about 10 because that's covered by 01. We can forget about 20 and 30 in the same way. We can also forget about 21 and 31 because they're covered by 12 and 13. And as you never need both a '6' and a '9' on the same date, you can use just one of them to stand for both (by turning one upside-down when you need to).

	~~10~~	~~20~~	~~30~~
01	11	~~21~~	~~31~~
02	12	22	
03	13	23	
04	14	24	
05	15	25	
06	16	26	
07	17	27	
08	18	28	
~~09~~	19	29	

And here's one way of arranging the numerals on the two cubes :





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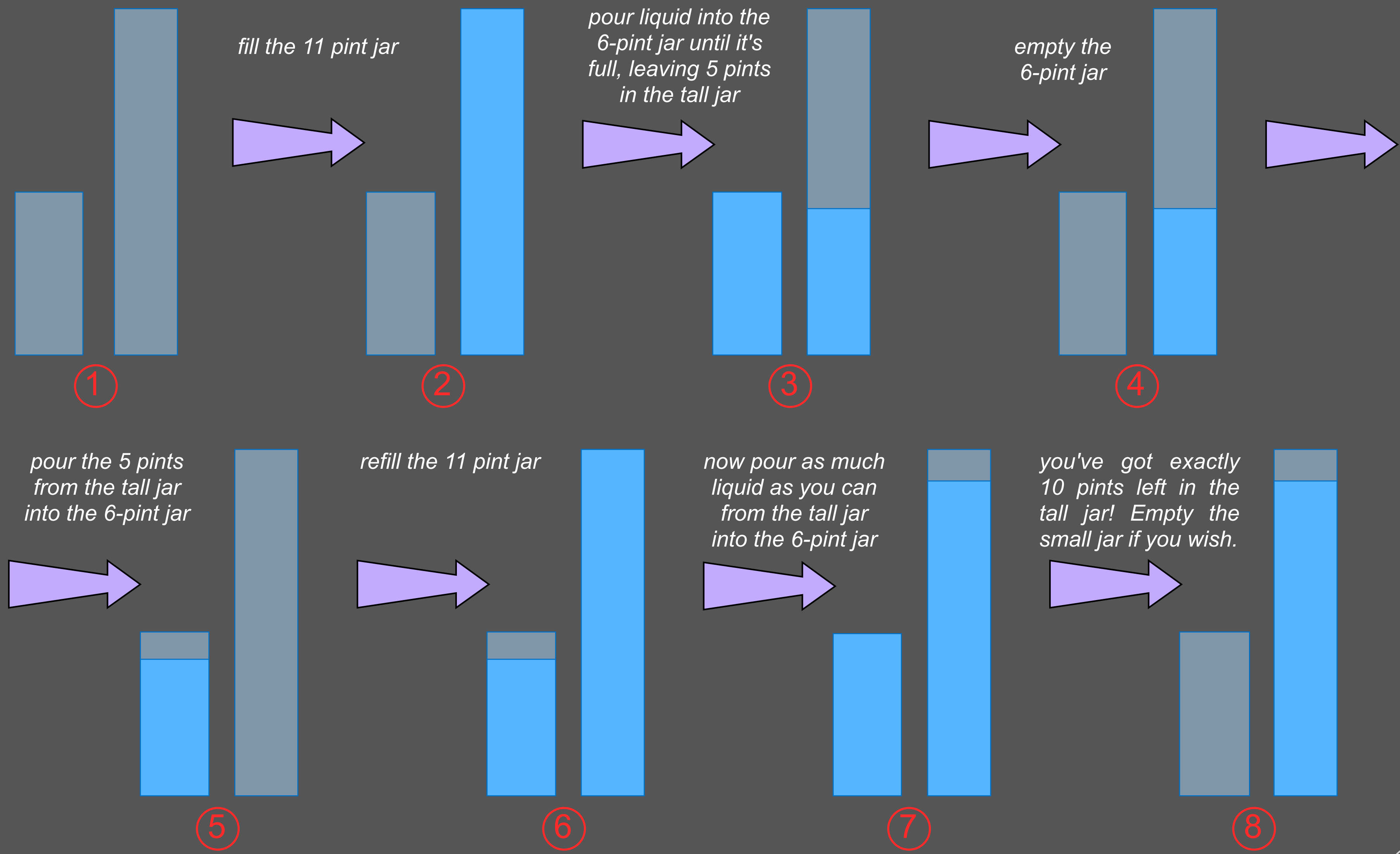
ans 43 10-pint target

Here's one way of solving the problem :



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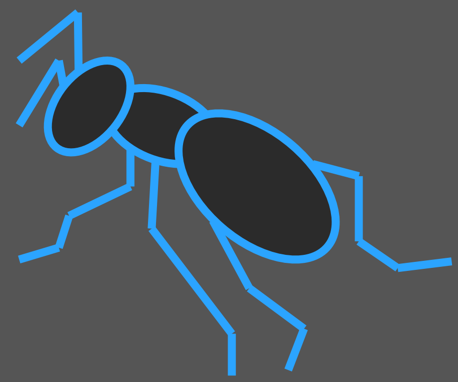
ans 44 ant and rectangle

With luck, you will have found the rule. It really is a simple one . . .

To find the number of squares your diagonal will cross, just add together the two sides and subtract 1.



For example, if you have a 7 x 9 rectangle, your diagonal will cross 15 squares (that's 7 + 9 = 16, and then 16 minus 1 gives you 15.)

Second example : If you have a 3 x 11 rectangle, your diagonal will cross 13 squares. (Because 3 + 11 = 14 and then 14 – 1 = 13.)

nb Remember, this simple rule applies to prime rectangles only.

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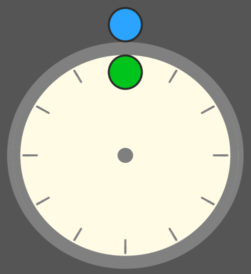
ans 45 Alfred and Betty

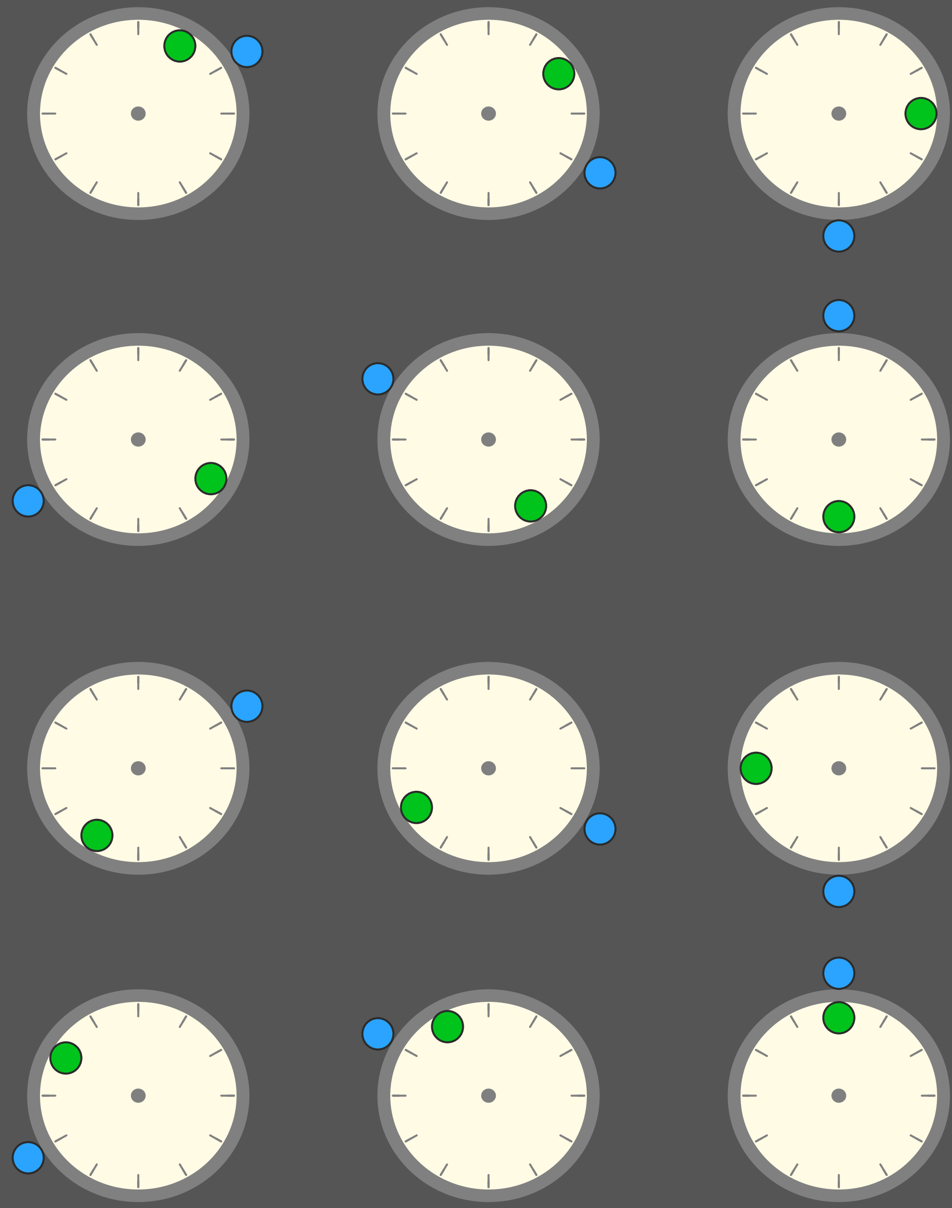
Instinctively you guess that Alfred will pass Betty somewhere along the way – until you think about it carefully! To make things easier, imagine the two of them cycling round a giant clock face :

Here's the starting position :



Alfred

Betty



Let's suppose they start at '12 o'clock' (as in the above diagram) – then the diagram on the right shows their positions as Betty reaches 1pm, 2pm, 3pm, 4pm . . . and as you'll see, although they come together again as Betty reaches 12 o'clock once more, Alfred never actually overtakes her!

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ans 46 a tale of two flutes

Before we start, it's really important to remember that when you see '25%', it always means 25% of something or other – so you have to be clear what it's 25% of you're after.

How does this affect our problem? Well, we need to remember that making the low bid 25% higher means increasing the low bid by 25% of itself . . . and in the same way, reducing the high bid by 25% means reducing the high bid by 25% of itself.

There's no obvious way of doing this, so perhaps we might use the 'trial and improvement' method here – or in other words, let's try some numbers and see what happens.

So, to begin . . . What should our initial guess be? It's useful if you know all about the price of second-hand flutes but most of us don't. Whatever you guess, you'll home in on the right answer in the end – but of course a good guess gets you there more quickly. If your first guess turns out to be wildly far out, then you can always start again with a new one. Let's start off with a guess of £90 for Roland's target sale price :

1st guess = £90

so, the high bid would have to be £120 (because reducing this by 25% gives you £90)

and the low bid would have to be £72 (because increasing this by 25% gives you £90)

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PTO

ans 46 a tale of two flutes

difference between low and high bids = £120 – £72 = £48

£48 is quite a bit too high (we know this difference has to come to £32), so let's make our second guess £75

2nd guess = £75

– and this means the high bid must be £100 and the low bid must be £60 . . . so now the important difference is £40, which is still too high but – we're getting nearer!

3rd guess = £60

Smiles all round! This guess gives you £80 for the high bid and £48 for the low bid - and a difference between the two bids of exactly £32 ! So, we can confidently say :

answer : Roland's target price was £60

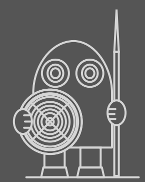
CHECK

If Roland's target = £60, then the high bid must = 4/3 of £60, which is £80.

And, with a target of £60, then the low bid must = 4/5 of £60, which is £48.

difference between these two bids is £80 – £48 = £32

check complete √



PTO

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a tale of two flutes

For those of you who like fractions, another way of looking at things would be this :

The high bid must be reduced by 25% to equal Roland's target – or to put it in fraction-speak, Roland's target is just 3/4 of the high bid. This is the same as saying that the high bid is 4/3 of Roland's target. To save writing things out all the time, let's call Roland's target R and the high bid H. Then what we have is :

$$H = 4/3 \text{ times } R$$

Now let's look at the low bid; we know we must add 25% to this to equal Roland's target – or to put things in fraction-speak, Roland's bid is 5/4 of the low bid. Let's call the low bid L. Then what we have is :

$$L = 4/5 \text{ times } R$$

PTO

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So we can write the difference between L and H like this :

$$H - L = 4/3\ R - 4/5\ R$$

We need to turn everything to fifteenths to do the subtraction :

$$H - L = 20/15\ R - 12/15R$$

So the difference between L and H is 8/15 of R. But we know this difference is exactly £32. We can carry on like this :

$$£32 = 8/15 \text{ of } R$$

$$\text{so,} \quad £4 = 1/15 \text{ of } R$$

And we're almost home! Because if £4 is 1/15 of R, the whole of R must be 15 lots of £4, or £60.

answer : Roland's target was £60

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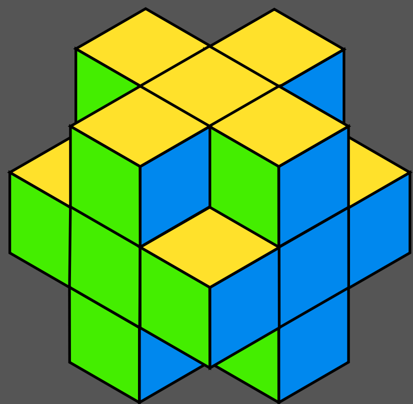
ans **47** **cutting corners**

- You know that a cube has 8 corners – and if you look carefully, you'll see that Jack has indeed removed 8 cubes from the corners of the original large cube.

- The original 3 x 3 x 3 cube was made from 27 individual 1cm cubes. With the 8 corner cubes taken away, there are now 19 cubes in the remaining shape.

- There are different ways of finding the total surface area of the shape which Jack made. Here are just a couple of ways :

1 There are 6 faces to the 'altered cube' and the middle cube in each face is showing just a $1cm^2$ area. So that's $6cm^2$ to begin with. Then each of the other cubes is joined on by 2 of its faces – this means each of these other cubes is showing 4 faces : there are 12 of these 'stuck-on' cubes and 12 x 4 = $48cm^2$.
So, total surface area = $6cm^2$ + $48cm^2$ = $54cm^2$.



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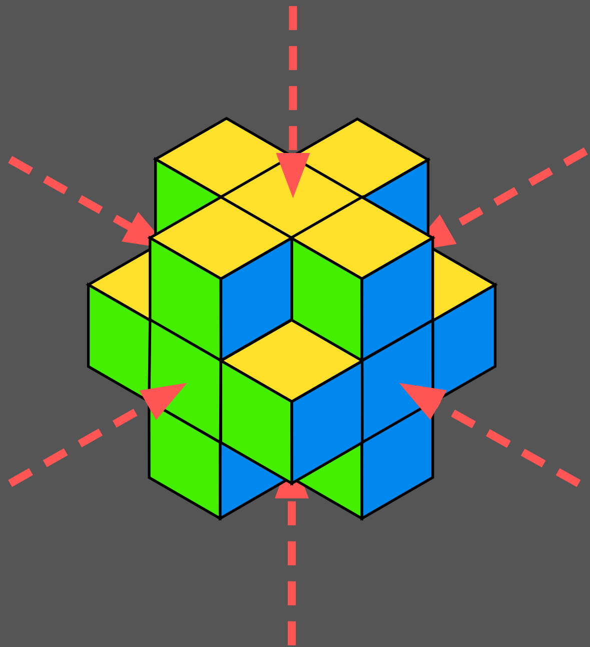
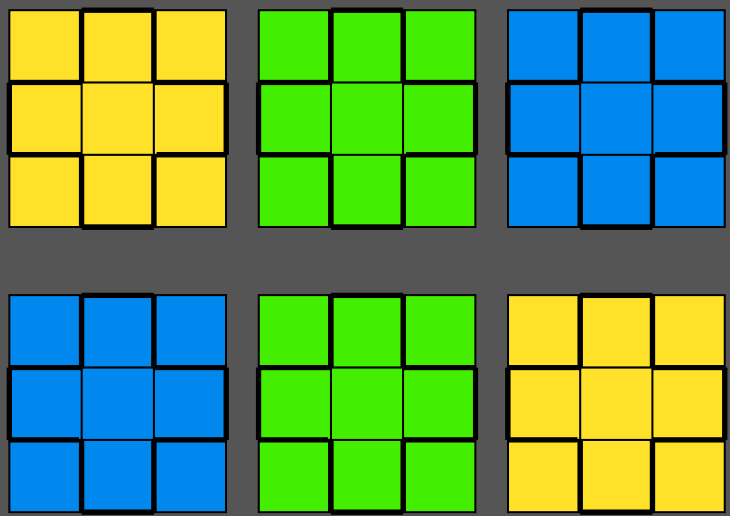
ans 47 cutting corners



2 Another way is by looking at the cube straight on from one direction eg looking along the line of one of the red arrows (see the diagram on the right). From each direction you'll see a total of 9 cubes facing you :



That's 6 straight-on views, each one showing 9 separate $1cm^2$ surfaces. So, total surface area = 6 x 9 = $54cm^2$.

ans 48 odd shops

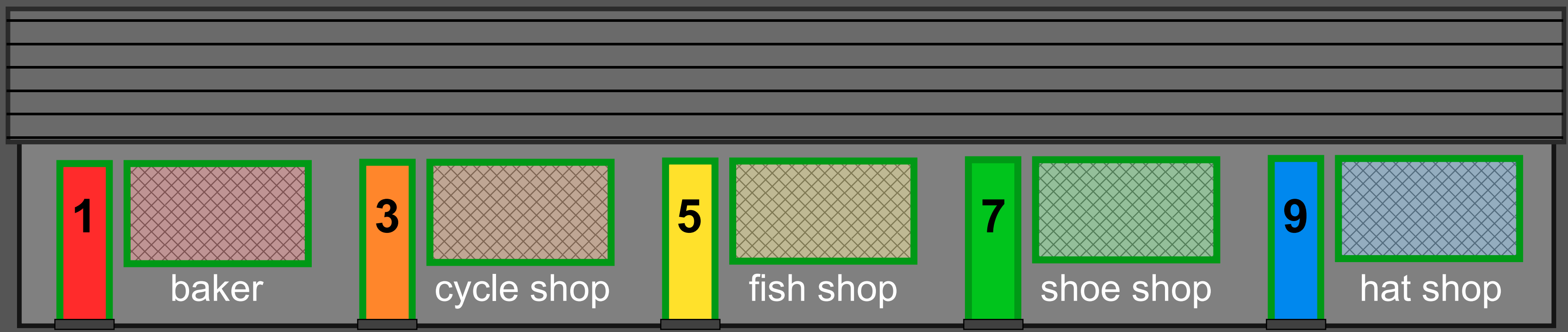
1 The baker is at no 1

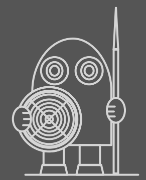
2 The fish shop is between the cycle shop and the shoe shop – so we know these three shops must be together, arranged either CFS or SFC

3 So, remembering that B is at no 1, we could have : BCFSH or BSFCH

4 But we're told that C is not next to H, so that leaves just : BCFSH

5 We're also told that H and S are next-door neighbours; we haven't needed this extra bit of information – but it confirms our answer !





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ans 49 Wrekin Ride

Looking at motorcyclists, we know that at the start there were 14 males and 13 females, as we show here :

			TOTALS
m	14		
f	13		

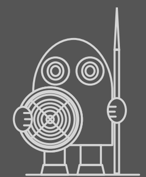
But of course we don't know how many cyclists there were – and we don't know how many were male and how many were female . . .

. . . except that the number of females was lower than 13. What to do next? Well, we could try some numbers and see the results . . .

Let's start with a low number. Suppose there were just 2 female cyclists; then there must have been 6 male cyclists, so now our table looks like this :

			TOTALS
m	14	6	20
f	13	2	15

Interesting – but as you can see when you look at the totals, not really what we want. We need the final total of males to be double the final total of females . . . So, the next thing is to try different numbers for female cyclists; let's try 2, 4, 6, 8 and so on.



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ans 49 Wrekin Ride

		TOTALS
14	6	20
13	2	15

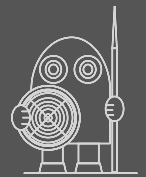
		TOTALS
14	12	26
13	4	17

		TOTALS
14	18	32
13	6	19

		TOTALS
14	24	38
13	8	21

		TOTALS
14	30	44
13	10	23

		TOTALS
14	36	50
13	12	25



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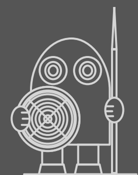
ans 49 Wrekin Ride

And there we have it ! Finally when we try 12 as the number of female cyclists, we get final male / female totals which work : overall number of females = 25 and overall number of males = 50. Here's the result from the last table shown again :

			TOTALS
m	14	36	50
f	13	12	25

– which means that in answer to the two problems we were set, altogether 25 females started the race and 12 of them were cyclists.

Of course, this isn't the only way to solve the proble, so you might have found a different way of getting to the answers . . .



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ans 49 Wrekin Ride

Looking for patterns :

Just looking at totals, we're told that the female total should be half of the male total. In other words, what we need is to have 2 x female total = male total. If you go back to the page where we tried out different numbers of female cyclists, you'll see that as we tried out 2, 4, 6 . . . for the female cyclists, we got: 15, 17, 19 . . . for the female totals. Doubling these totals gave you numbers which were: 10 too high, then 8 too high, then 6 too high . . . So there we have a pattern – and we would expect that carrying on in this way for three more steps, would give us totals which would be 4, 2 and finally 0 too high. The 0 result happens when we've hit just the right number for the female cyclists, which of course is 12 – meaning that 25 females started the race.

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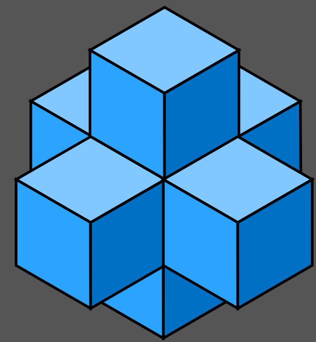
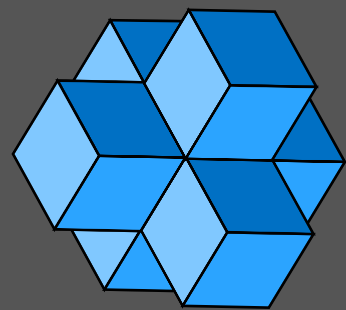
ans 50 the blue cube

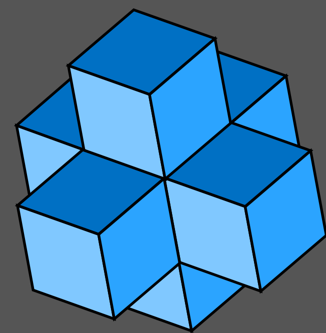
There are different ways of going about these problems.



1 With the new shape, you've got a 'very middle' cube with another cube stuck onto each of its 6 faces. So the new shape must have 7 cubes in it altogether. As a 3 x 3 x 3 large cube, the original shape must have had 27 1cm cubes to start off with. So Jenny must have removed 20 small cubes from that shape.



2 Jenny's new shape has 7 cubes in it. You can see this by looking at it – or you can reason it out like we've done above in answering question 1.



3 The new shape is completely symmetrical – it has 6 cubes sticking out from a 'very middle' cube. As you can see, each of these 'sticking-out' cubes is showing 5 faces to the world, and each of these faces has an area of $1cm^2$. So, the new shape must have a surface area of $30cm^2$. Another way of doing this is to notice that viewed directly from 6 different angles the shape always has 5 small squares facing you; so the total surface area must be 5 x 6 = $30cm^2$. (Notice that, compared with the original large cube, the new shape has lost about three-quarters of its cubes but it still has more than half the original surface area.)

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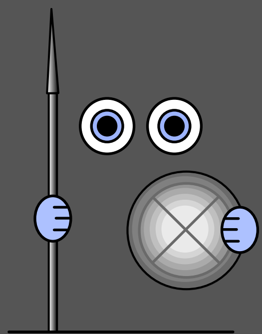
no problem! book

50 maths problems with answers



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