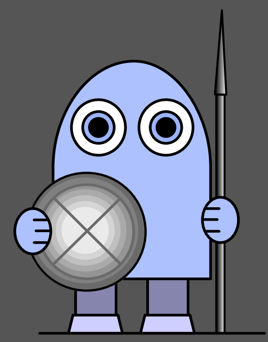
no problem !

book 1

answer book

four winds



Sports Day

We know where Kate and Anne came in the race, so here are the positions 1 to 5, with these two already listed :

1	
2	Kate
3	
4	
5	Anne

What else do we know? Well, we know that Mary was just behind Sanjit – which means that their positions must be next to each other. The only slots available for this are positions 3 and 4, so that's where we'll put Sanjit and Mary . . . and now of course there's only Jeremy left – so he must have come in at number 1 :

1	Jeremy
2	Kate
3	Sanjit
4	Mary
5	Anne

ans 2 time's up!

Here you can see the times with digital sums equal to 24 23, 22 and 21.

There's one obvious pattern there for you to see : the number of times in these answers begins with 1, then comes 3, then 6 and finally 10. Perhaps you recognise the sequence 1, 3, 6, 10 as the first four **triangle numbers**. The next triangle number after 10 is 15 and that's your answer to the last question.

There are also some patterns for you to find in the times themselves : to spot these it's best to look at the hours and minutes separately.

- *digital time total = 24*

19 59

- *digital time total = 23*

19 58

19 49 18 59

- *digital time total = 22*

19 57

19 48 18 58

19 39 18 49 17 59

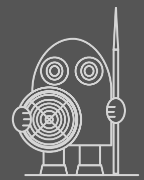
- *digital time total = 21*

19 56

19 47 18 57

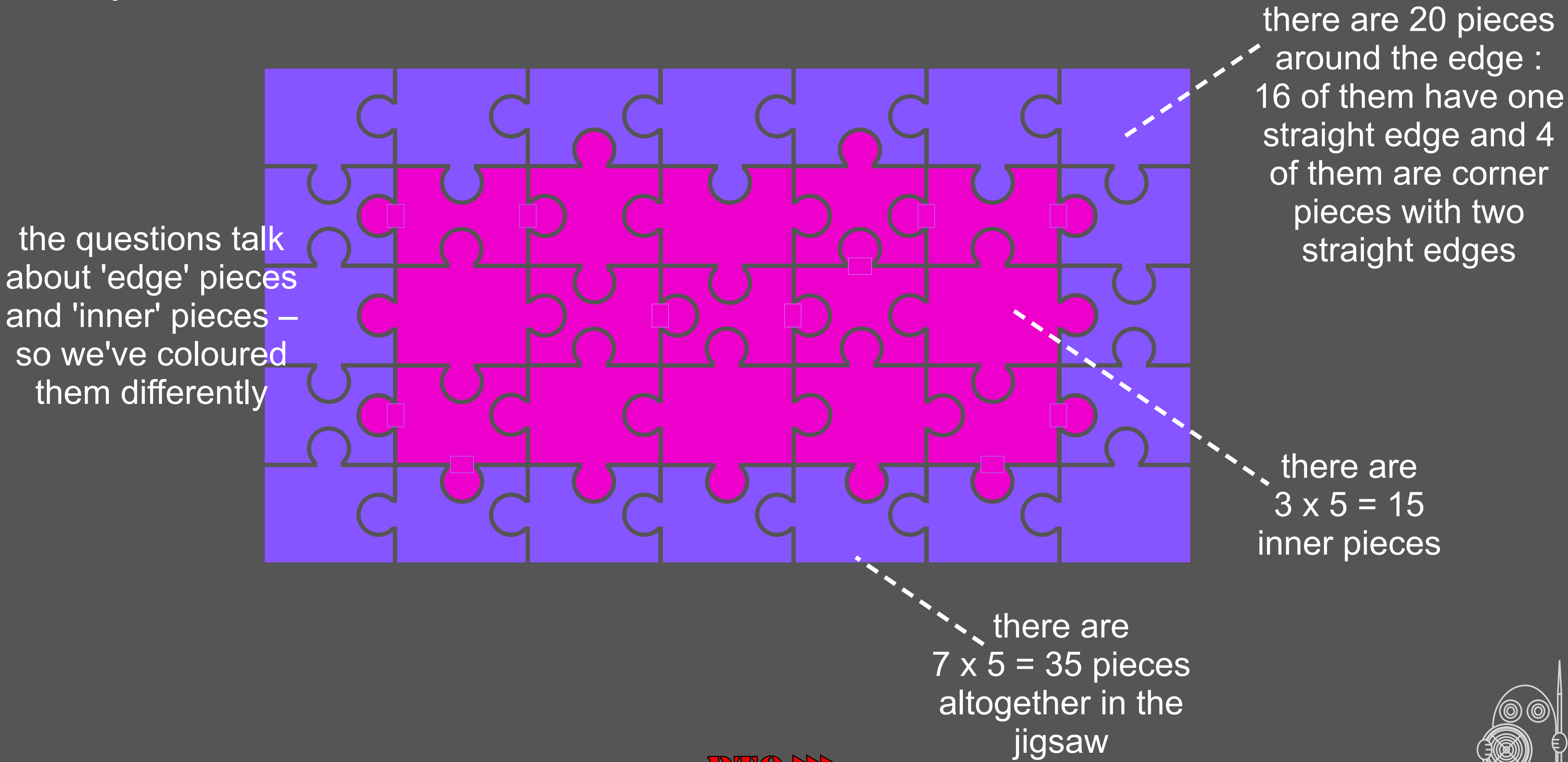
19 38 18 48 17 58

19 29 18 39 17 49 16 59



ans 3 an edgy problem

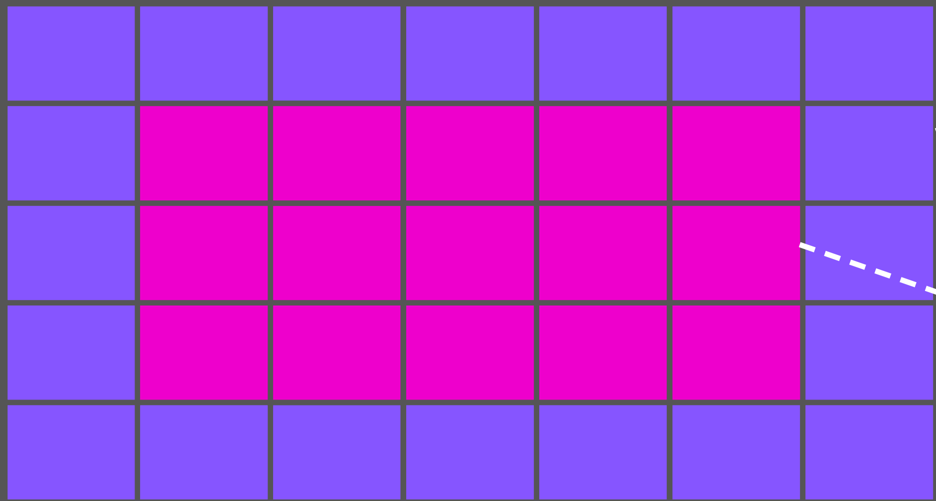
Syed's puzzle has quite a few pieces, that's for sure, and it's not clear at first what we're to do with the large numbers we're given. You might or might not have already met a problem like this one but – one thing we can do is to make up a simpler problem that's just like this one. For example, if we start with a 7 x 5 jigsaw, we can see more easily how the various numbers work out :



PTO

ans 3 an edgy problem

There's no real need to show the curly bits on the jigsaw pieces – we can just picture them as small rectangles. So now we can think of what we've got as a 7 x 5 jigsaw with a 5 x 3 jigsaw inside it :



In the larger jigsaw,
number of pieces =
7 x 5 = 35

In the smaller jigsaw,
number of pieces =
5 x 3 = 15

So for this jigsaw, our answers are :

total number of pieces = 35

number of inner pieces = 15

PTO

ans 3 an edgy problem

Coming back to Syed's jigsaw, we can think of it as a 54 x 27 jigsaw with *(make sure you understand this!)* a 52 x 25 jigsaw inside it :

In the larger jigsaw,
number of pieces =
54 x 27 = 1458

In the smaller jigsaw,
number of pieces =
52 x 25 = 1300

So for Syed's jigsaw, our answers are :

total number of pieces = <u>1458</u>

number of inner pieces = <u>1300</u>

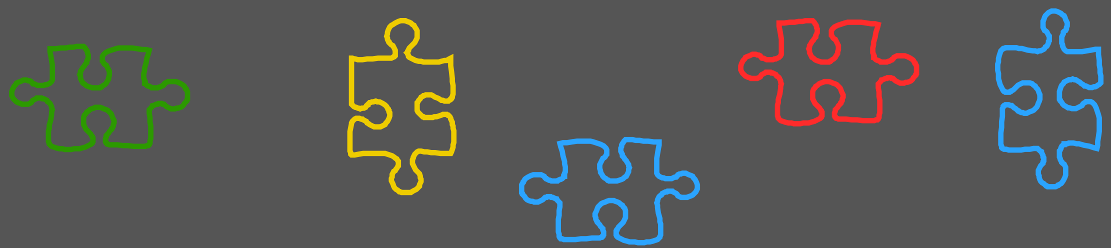
PTO

ans 3 an edgy problem

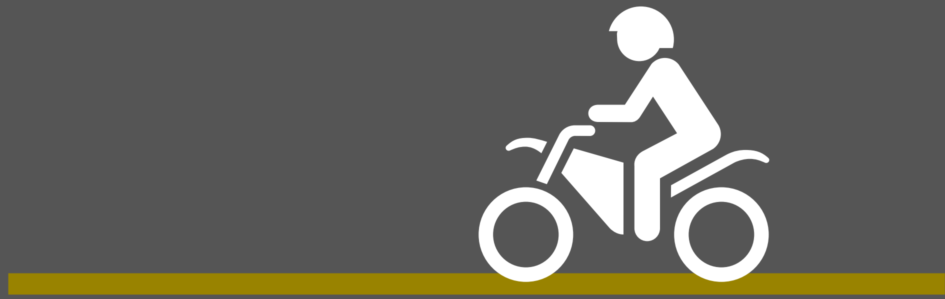
. . . and here's a more direct way of getting the answers :

- The total number of pieces is 54 x 27, which equals 1458

- There are 4 corner pieces, each with two straight edges. As for pieces with just one straight edge, there are 52 along each longer side of the puzzle and 25 along each shorter side of the puzzle, making a total of 154 pieces. So, altogether, there are 158 pieces with either one or two straight edges. Subtract this from the total number of pieces in the puzzle and you get 1458 – 158 = 1300. So altogether there are 1300 'inner pieces' (that's to say, pieces with no straight edge).



ans 4 easy rider



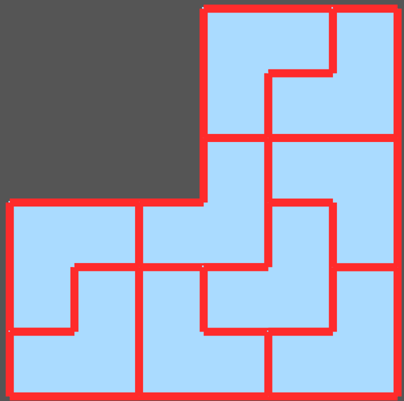
If Jamie is going to finish the season with an average position of 3rd, then we know that at the end of the season, his eight finishing positions must add up to 8 x 3 = 24. We know this because 24 ÷ 8 is how we get an average of 3.

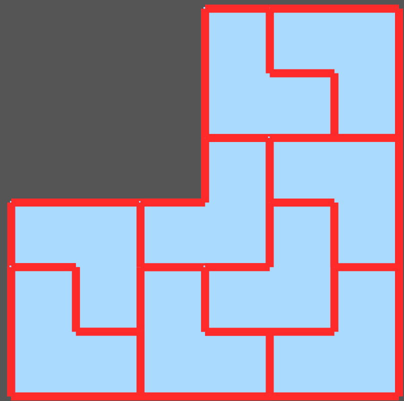
So far, Jamie's total from seven finishes is 21. But we've just worked out that the final total must be 24. That leaves a gap of just 3! So, to win a cash prize, Jamie must finish 3rd (or better) in his last race.

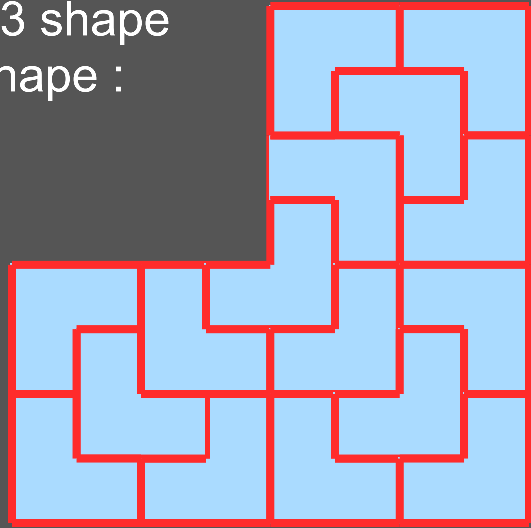
ANS 5

L is for learner

Here are two ways of tiling the L-3 shape and three ways of tiling the L-4 shape :

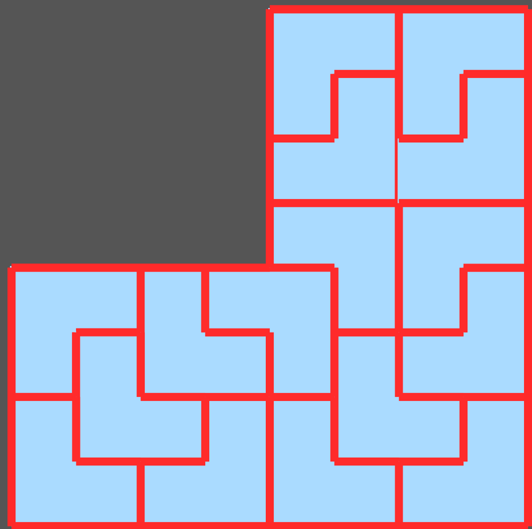


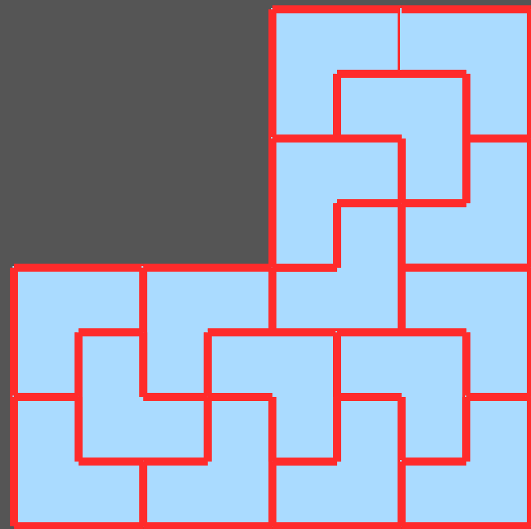




You use 1 tile for shape L-1 and 4 tiles for shape L-2, then 9 tiles for shape L-3 and 16 tiles for shape L-4.

1, 4, 9, 16 . . . I'm sure you'll recognise the **square numbers** here !



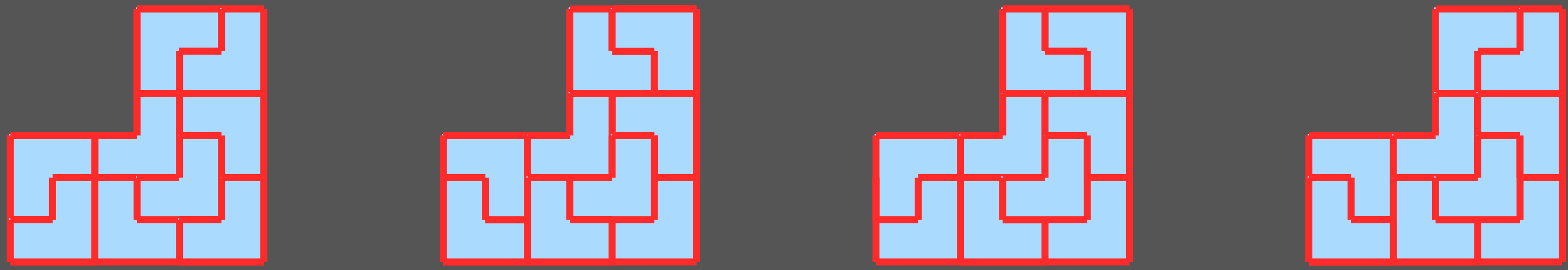


PTO ▶▶▶

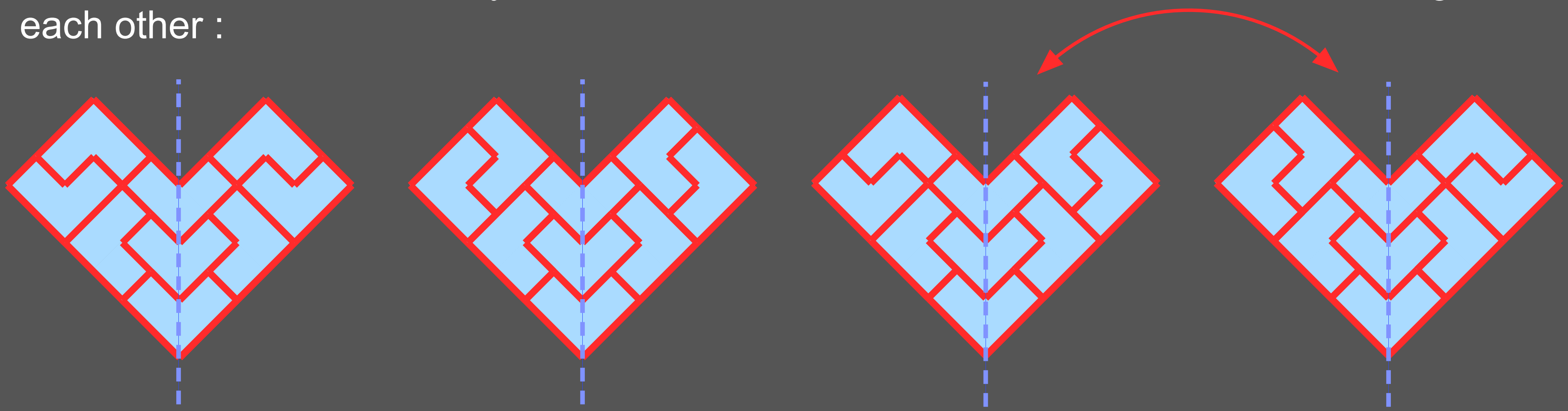
ans 5

L is for learner

Actually, we found four ways of tiling the L-3 shape and here they are :

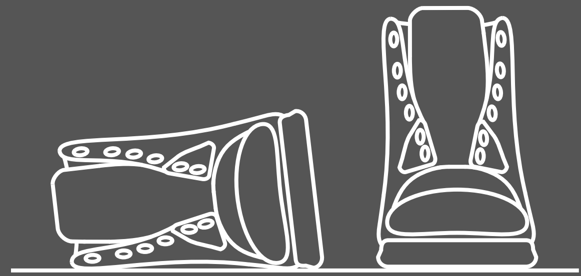


But when we tilted them by 45° we could see that the last two were mirror-images of each other :



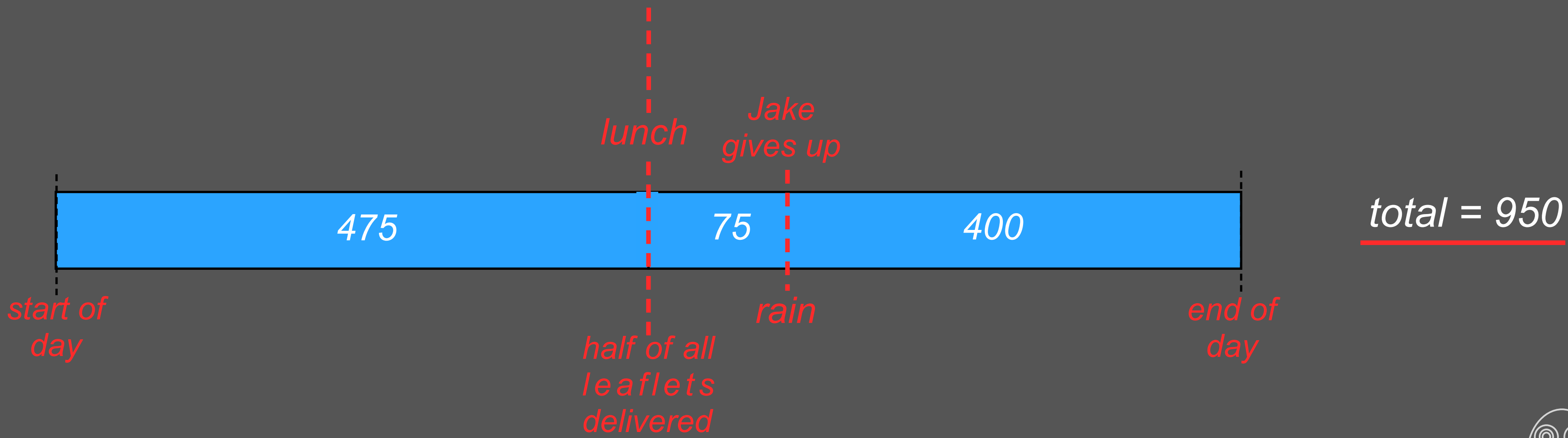
Finally, when it comes to the L-4 shape, we've shown you just two ways of tiling it. But how many ways could you find? And are they all really different?

ans 6 Jake's leaflets



We know that by lunch-time Jake had delivered half of the leaflets. This means that he had half the leaflets still in his sack. And we know that when Jake gave up, he'd delivered 75 more of these and he still had 400. So half the leaflets must add up to 475; which means that **all** the leaflets must be 475 x 2 = 950.

answer : Jake began his day with 950 leaflets, as this diagram shows :



ANS 7 getting warmer . . .

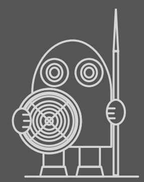
With just the four number cards 8, 2, 7, 4, our 'closest numbers' are :

- 7248
- 2874
- 4872
- 7248
- 4287
- 8742
- 8724
- 8427

Most of these are fairly easy – but with the last one there are two answers almost as close as each other, so here you might have had to do a bit of arithmetic to find what you were after . . .

$$\begin{array}{r} 8427 \\ -8351 \\ \hline 76 \\ \hline \end{array} \qquad \begin{array}{r} 8351 \\ -8274 \\ \hline 77 \\ \hline \end{array}$$

– so 8427 wins !



8 in the hot seat . . .

probability

I'm sure you know that probability is about things happening, and it measures how likely different things are to happen. But how does it work? Well, to find the probabillity of some particular result, you just compare how many times you get this result with how many possible results there are. Here are two simple examples, which should give you something of the idea . . .

eg1 *'If I toss two coins in the air, what's the probability of them both landing heads?'*

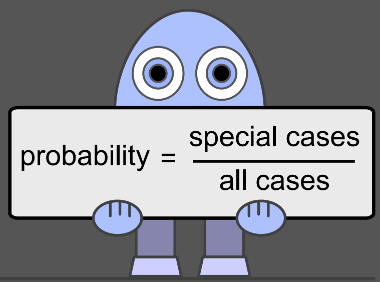
Well, the two coins could land HH, HT, TH or TT – and that's four possible results. Only one of them is HH, so we can write : probability (two heads) = 1/4

eg2 *'If I throw down a normal dice, what's the probability that it will land with a prime number on top?'*

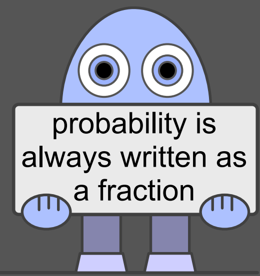
There are six faces on the dice and only three of them are prime numbers (2, 3, 5), so we can write : probability (prime number on top) = 3/6 = 1/2

there are just two important things to remember :

1



2



PTO

ans 8 in the hot seat . . .

This is an easy question and here are two ways of getting to the answer:

1 Start by writing down all the possible ways in which the three children could be seated. It's easier to do this if you have a method (working alphabetically, for example) :

R S T
R T S
S R T
S T R
T R S
T S R

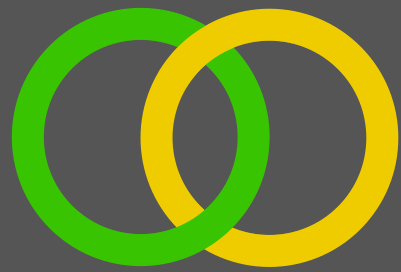
As you can see, there are 6 different ways of seating the children - but just two of these have Rosie in place number 1. So,

prob (Rosie in seat 1) = 2/6 = 1/3

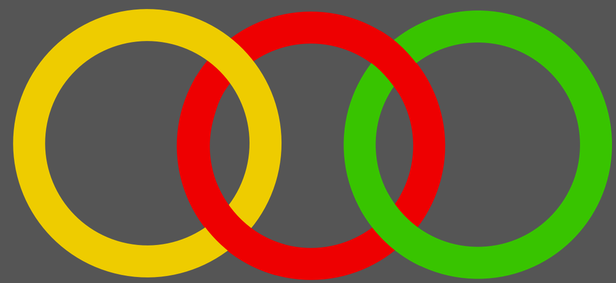
2 Or, you could argue that by the sheer symmetry of the situation, the children have absolutely the same chance of ending up in any particular seat. So, for each child (including Rosie), we can write : prob (getting seat 1) = 1/3

ans 9 overlapping rings

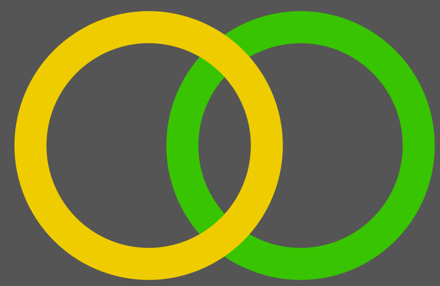
- Look hard at the original diagram – perhaps you can see that the green ring and the yellow ring are linked. And they will still be linked even if someone takes the blue ring away!



- This time, if someone does cut the blue ring and remove it, the red ring will still be linked to the yellow ring and to the green ring – but the yellow and green rings will not be linked to each other.



- Finally, if someone cuts both the blue ring and the red ring and then takes them away, the remaining two rings (green and yellow) will not be linked.



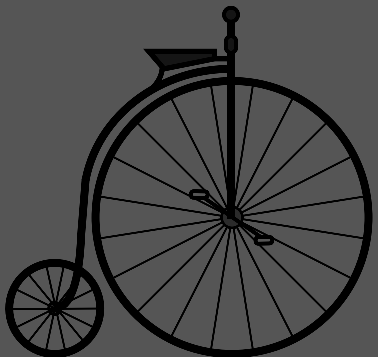
ans 10 The Pedalman

The number we're after is not hard to find . . .

On the right is a list of all nine possible numbers. As you can see, eight of the numbers have been crossed out – and next to each is the reason why that particular number has been disqualified.

Only one number remains : 87 (and yes, that is a multiple of 3). So there we have it – the address which the Maths Detectives need is

87, Old Montague Street



~~23~~	it's a prime number
~~49~~	it's a square number
~~62~~	it's not a multiple of 3
87	it ticks all the boxes !
~~105~~	it's a multiple of 7
~~121~~	it's a square number
~~210~~	it's a multiple of 7
~~169~~	it's a square number
~~188~~	it's not a multiple of 3

happy birthday, Ben !

There are different ways of going about this problem; one easy way is to try some different possible answers until we find what we're after. But – before we jump in and begin trying out lots of numbers, let's stop and think : If Ben's age is four times Annabelle's age, then Ben's age must be a multiple of 4. So let's try some multiples of 4 for Ben and next to them we'll put the corresponding ages for Annabelle . . . and on the line beneath we could put their ages next year. Let's begin by trying Ben = 4, then Ben = 8 and so on . . .

	Ben	Annabelle
this year	4	1
next year	5	2

	Ben	Annabelle
this year	8	2
next year	9	3

It didn't take long, did it? Our second try was just right. If Ben is 8 this year and Annabelle is 2, then everything works!

So, that's our answer : Ben is 8 and Annabelle is 2.

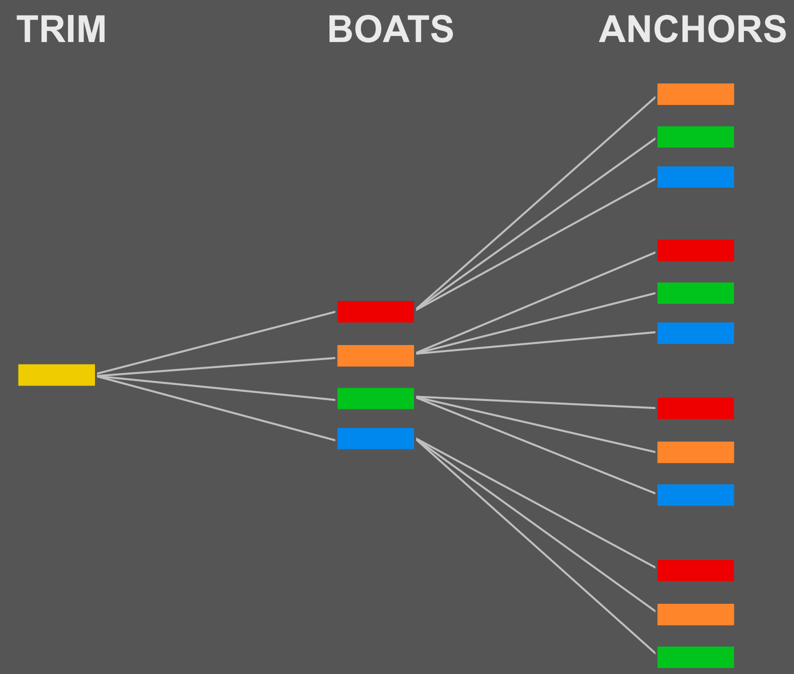
ans 12 sailing by . . .

The 'tree diagram' on the right shows you all the possibilities.

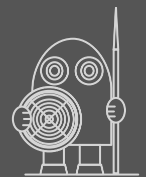
Starting on the left, the 'TRIM' column shows the only possible colour, yellow. With this there are 4 colours remaining which can be used for the BOATS part of the badge. So that's 4 possible combinations. Moving to the ANCHORS, each of the 4 combinations we've mentioned can be joined with any of 3 different colours.

giving us : 4 X 3 possibilities overall

or, in other words, given what we're told, 12 colour arrangements are possible . . .



note : remember, we're counting eg boats blue / anchors red as a quite different arrangement from boats red / anchors blue . . .



PTO

ans 12 sailing by . . .

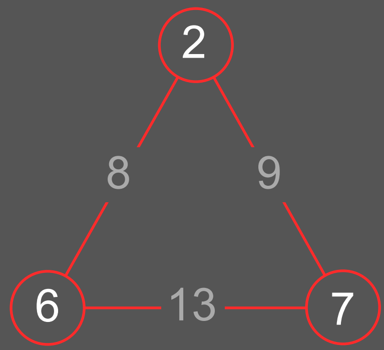
. . . well, that's one way of solving the problem. But some people prefer to make a list of all the possibilities. On the right you can see such a list. If you choose to do this kind of problem by making a list of all combinations, it helps if you can find a definite system for making sure you've got all of them. Counting up, the final answer here is :

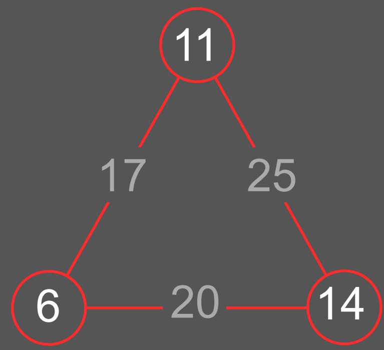
total number of possible arrangements = 12

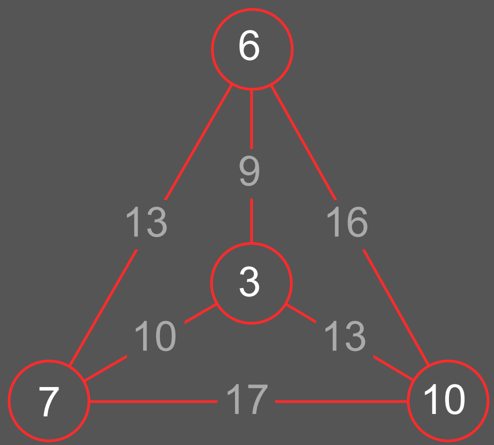
TRIM	BOATS	ANCHORS
YELLOW	*BLUE*	*GREEN*
YELLOW	*BLUE*	*ORANGE*
YELLOW	*BLUE*	*RED*
YELLOW	*GREEN*	*BLUE*
YELLOW	*GREEN*	*ORANGE*
YELLOW	*GREEN*	*RED*
YELLOW	*ORANGE*	*BLUE*
YELLOW	*ORANGE*	*GREEN*
YELLOW	*ORANGE*	*RED*
YELLOW	*RED*	*BLUE*
YELLOW	*RED*	*GREEN*
YELLOW	*RED*	*ORANGE*

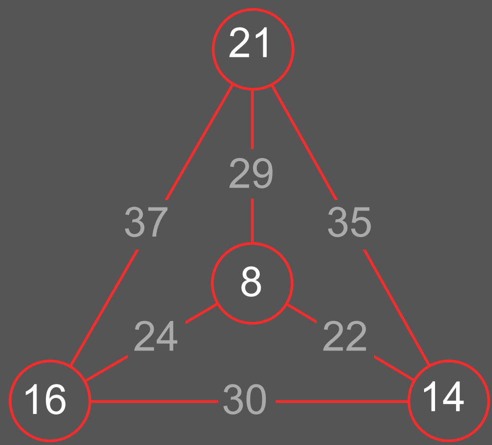
ans 13 number triangles

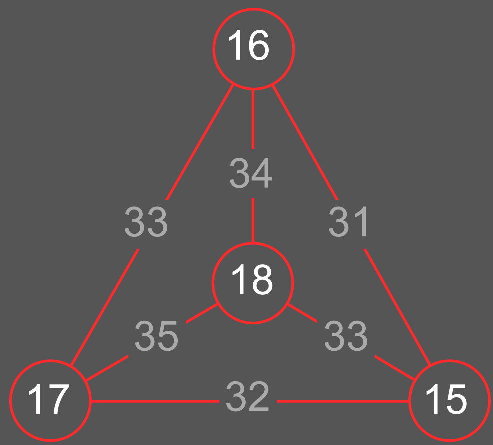
and here are the answers . . .

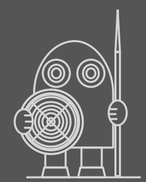






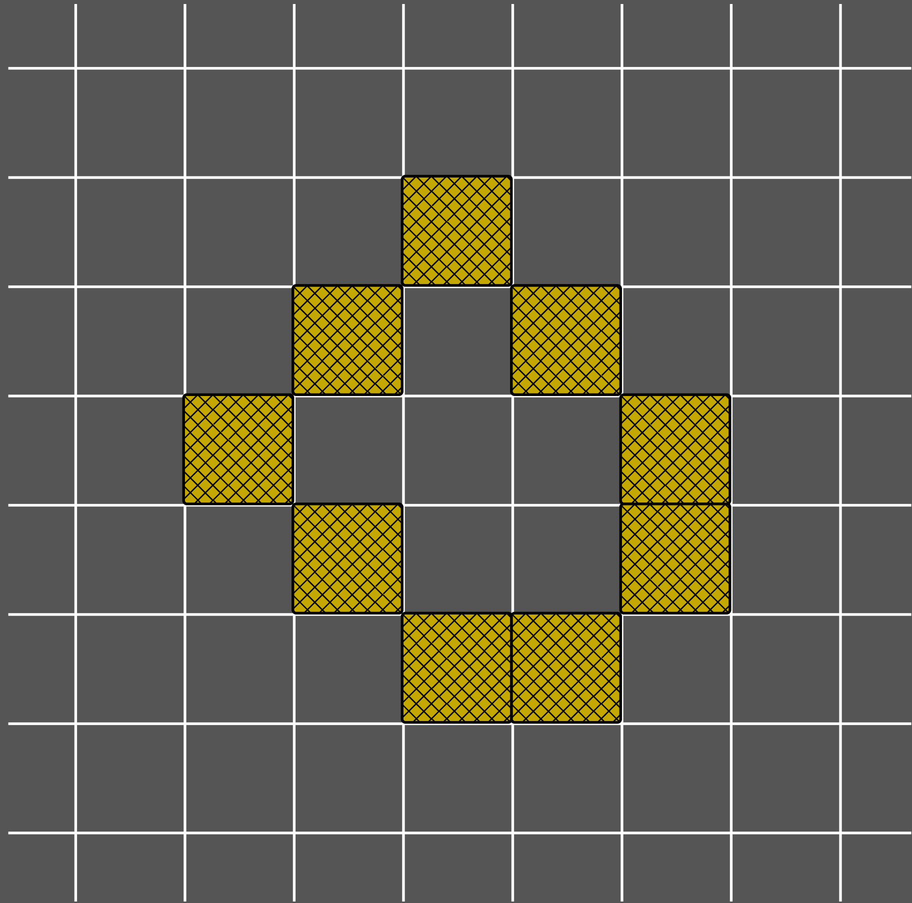






ans 14 penning sheep

6 squares seems to be the most you can enclose with 9 hay-bales – and here's one way of doing it :



**extra problem : Is this the only way of arranging 9 bales to enclose 6 squares?*
Special note: before you investigate this problem you'll need to decide what counts as 'different' arrangements eg if one arrangement is a mirror-image of another arrangement, do you want to count them as different arrangements ?

ans 15 parcels-to-go

- Obviously for small parcels you're better off going with Parcels-to-Go, because you don't have to pay a basic charge.

- Perhaps the easiest way to get at the answer is just to work out the cost of sending parcels of different sizes with the two delivery firms. The chart on the right shows you how their charges work out :

	ParcelDrop	Parcels-to-Go
1kg	2.50	0.75
2kg	3.00	1.50
3kg	3.50	2.25
4kg	4.00	3.00
5kg	4.50	3.75
6kg	5.00	4.50
7kg	5.50	5.25
8kg	6.00	6.00

– You can see that at 8kg the rates charged by the two delivery companies are exactly the same. So that's the answer : 8kg

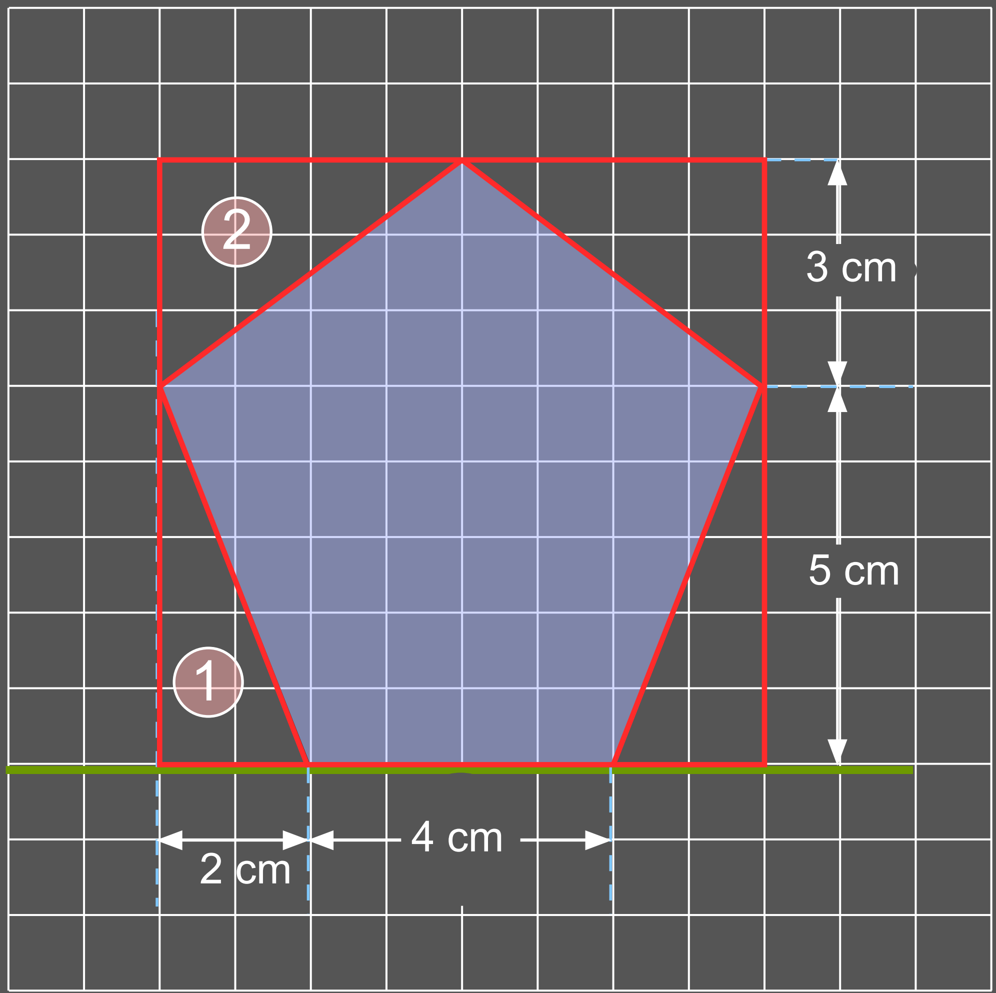
* *Now turn to next page . . .*

PTO ▶▶

ans 15 **parcels-to-go**

A different way of getting to this answer is to notice that Parcels-to-Go charge 25p more per kilogram than ParcelDrop. Of course you don't have to pay them the £2 basic charge – but how many kilograms would your parcel need to be in order for the extra 25p per kg to equal a £2 basic charge? Well, that's easy enough to work out . . . How many times does 25p go into £2? Answer : 8 times. Or in other words, when your parcel is as heavy as 8kg, the extra 25p per kilogram charged by Parcels-to-Go is exactly wiped out by the £2 basic charge which ParcelDrop makes you pay.

ans 16 five-a-side



One way of finding the area of a pentagon like this is by subtraction : just find the area of the surrounding rectangle (it's a square in fact) and subtract the combined area of the four corner triangles . . .

area of red square = 8 x 8 = 64

area of triangle 1 = 5

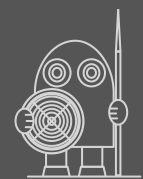
area of triangle 2 = 6

as there are two of each triangle,

total area of triangles = 22

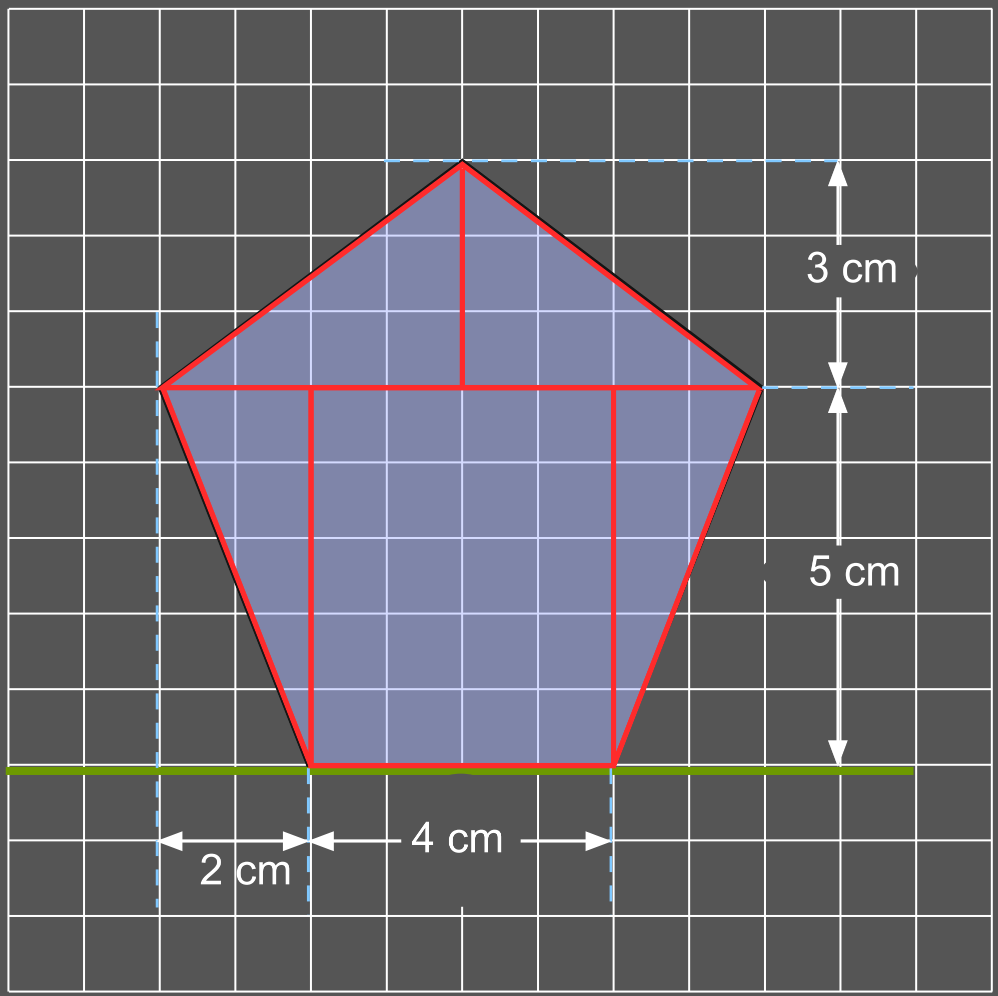
so, area of pentagon = 64 – 22

= 42 cm^2



PTO ►►

ans 16 **five-a-side**



On the other hand, many people find it easier to carve up the pentagon into a rectangle and four triangles, as in the diagram, and then simply add together the five areas . . .

area of two top triangles = 12

area of two side triangles = 10

area of rectangle = 20

total area of rectangle plus four triangles = 12 + 10 + 20

= 42 cm²

ans 17 Alice's party

You might have been able to solve this one by trial-and-error or in some other way but – if you have a problem like this one and you don't know where to begin, you can always set up a 'truth table'. What you do here is make a list of all possible arrangements – and then cross out the ones which you know you can't have. You should be left with just one possible arrangement and this of course is your answer.

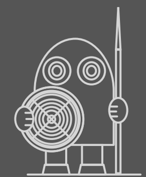
This is how the truth table method works with the 'Alice's party' problem :

CONDITION 1

This tells us that Debbie must have seat number 4. This means that we really only have the problem of how to fill seats 1, 2 and 3.

To make our lives easier, let's use the letters A, B, C and D to stand for Alice, Ben, Charlie and Debbie. Then we make a list of all the possible ways there are of filling these three places. You can see our list here on the right :

1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A



ans 17 Alice's party

NOT POSSIBLE	POSSIBLE
🔴	🟢

CONDITION 2

This tells us that we can't have any arrangement with A next to C or with C next to A, so straight away we can put a 🔴 next to two of the rows :

1	2	3	
A	B	C	
A	C	B	🔴
B	A	C	
B	C	A	🔴
C	A	B	
C	B	A	

CONDITION 4

This tells us that we can't have C in seat number 3, so that's a 🔴 next to two more rows :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	
C	B	A	

CONDITION 3

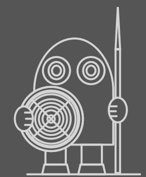
This tells us that we can't have any arrangement with A and C in seats 1 and 3. This happens in the first and last rows; the first row is already cancelled, so we need to put a 🔴 next to the last row :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	
C	B	A	🔴

And at last we have the answer to our problem; having Charlie, Alice, Ben and Debbie in seats 1, 2, 3 and 4 satisfies all the conditions we were given :

1	2	3	
A	B	C	🔴
A	C	B	🔴
B	A	C	🔴
B	C	A	🔴
C	A	B	🟢
C	B	A	🔴

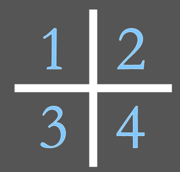
our answer is // seat 1 : Charlie // seat 2 : Alice // seat 3 : Ben // seat 4 : Debbie //



ans 17 Alice's party

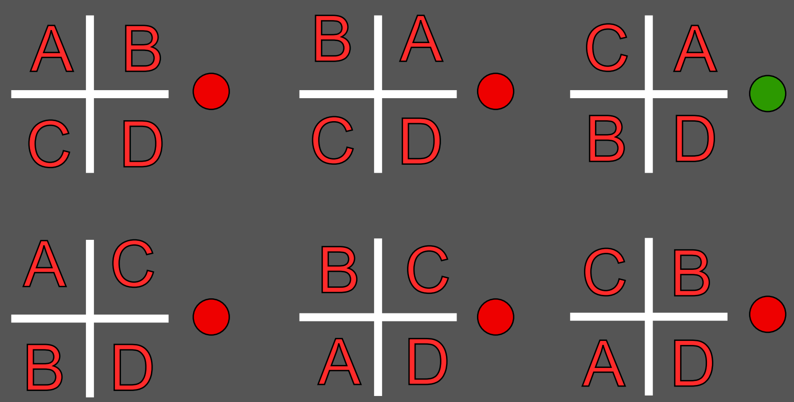
1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Of course, setting out the possibiblities in a table like the one on the left is the usual way to go about problems of this sort – but we could use a way which matches the seating plan, as in the diagram on the right :



Using this sort of 'truth table' means we can show all the possible seating arrangements just as they are . . . and with Debbie having to sit in place number 4, there are six possible seating arrangements.

Going through the four conditions we were given in the original question, we can mark each one as possible or not possible. When we do this, we have just one arrangement left, as you can see.



and once more we have // seat 1 : Charlie // seat 2 : Alice // seat 3 : Ben // seat 4 : Debbie //

ans 18 Ben and Terry . . .

Every problem starts by giving you some information. This problem is a perfect example of how looking at this information in a different way can lead you straight to an answer :

Ben sells 1 ice-cream every 10 mins and Terry sells 1 ice-cream every 5 mins. It's hard to see how we can easily combine these two facts to get an overall figure for how many ice-creams an hour they are selling between the two of them. But we can re-write our information in this way :

Ben sells 6 ice-creams per hour

Terry sells 12 ice-creams per hour

So together Ben and Terry sell 18 ice-creams per hour !

ans 19 . . . and Sally

As you will have realised, this problem is another example of how you need to take a different look at the given information in order to get to the answer :

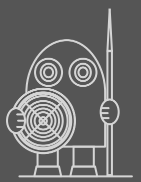
We know that we have one tap which takes 12 minutes to fill a bath and another tap which takes 6 minutes to fill the same bath. But there's no easy way of combining these figures. However, we can look at the information in a different way, like this :

hot tap : takes 12 mins to fill bath = can fill 5 baths an hour

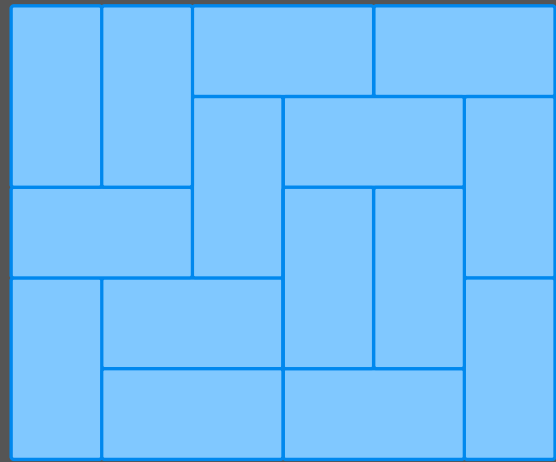
cold tap : takes 6 mins to fill bath = can fill 10 baths an hour

so, hot and cold taps together can fill . . . 15 baths an hour

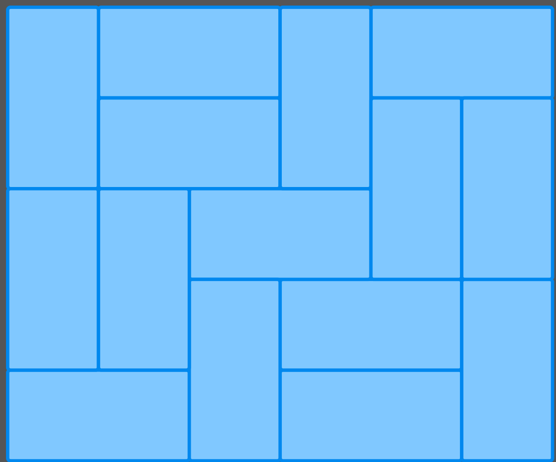
And our problem is solved! Saying that the hot and cold taps working together can fill 15 baths an hour comes to the same as saying that taken together they need just 4 minutes to fill one bath – and that's our answer!



ans 20 whose fault is it ?



Here are two ways of
solving the problem :



Perhaps you found a different way ?

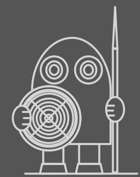
ans 21 **mr owl ate my metal worm**

palindromes with digit sum equal to 8

				.
101	202	303	404	.
111	212	313	.	1771
121	222	323	.	1881
131	232	333	.	1991
141	242	343	.	2002
151	252	353	.	2112
161	262	363	1001	2222
171	272	373	1111	2332
181	282	383	1221	.
191	292	393	1331	.

last palindrome year before 2002 1991

next palindrome year after 2002 2112



ans 22 Ravi's mean question

How should we set about this problem? Well, we're looking for a set of three numbers and one thing we know is that one of them is double another of them. So, let's jot down some pairs of (a number + its double) and for each pair let's write down what the third number must be. (Remember, the mean of the three numbers is 10, so we know that they must add up to 30.)

20, 10	18, 9	16, 8	14, 7	12, 6	10, 5
0	3	6	9	12	15

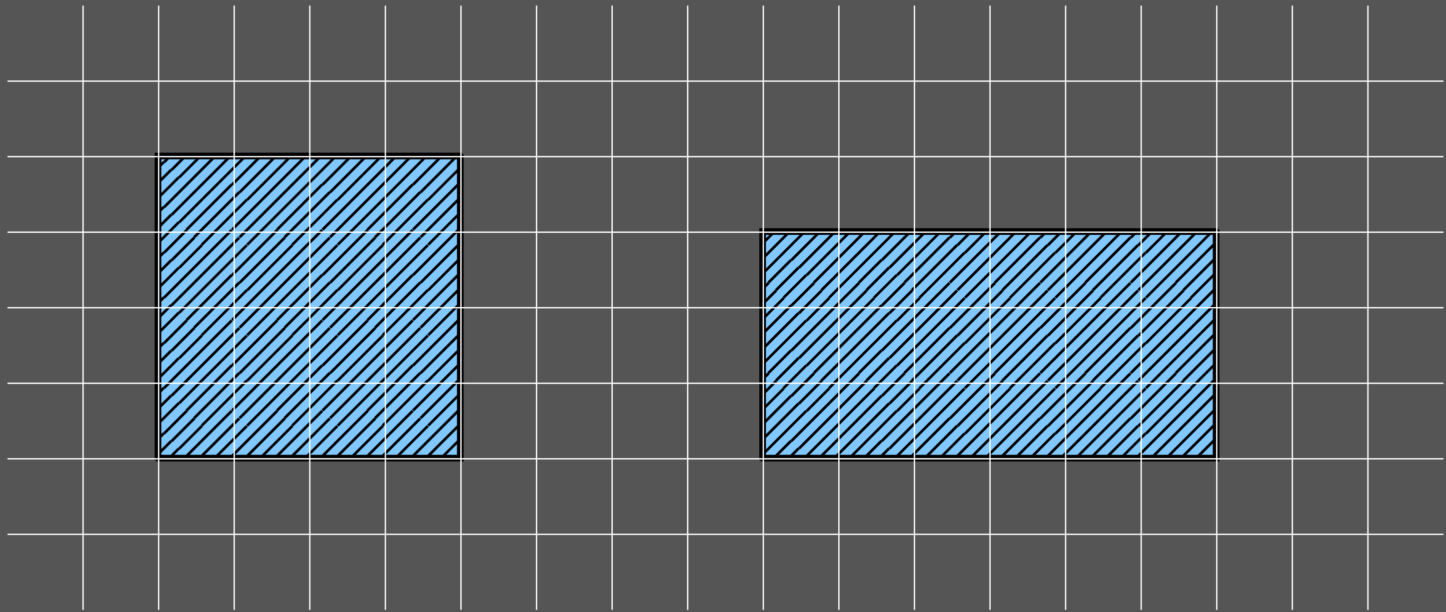
The red dotted line joins the smallest and the largest of each set of three numbers. As you can quickly see, there's only one group of three numbers where the smallest is 10 less than the largest. So now we have our answer:

Ravi's three numbers were 6, 8 and 16

ans 23 **matching numbers**

Here are a couple of suggestions :

A rectangle 4 cm x 4 cm has area 16 cm^2 and perimeter 16 cm



A rectangle 3 cm x 6 cm has area 18 cm^2 and perimeter 18 cm

ans 24 six-a-side !

- The one-hole shape uses 6 tiles, the two-hole shape uses 10 tiles and the three-hole shape uses 14 tiles.

- As you can see, Mark has just been adding 4 tiles each time, so the four-hole shape would use 18 tiles.

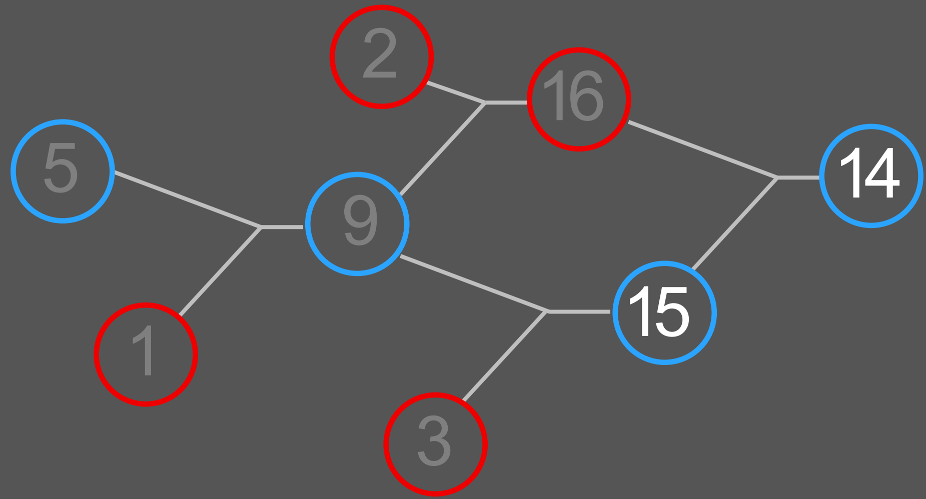
- We can think of the pattern this way : We begin with 2 tiles, then add 1 lot of 4 tiles to complete a 1-hole shape, add 2 lots of 4 tiles to complete a 2-hole shape, add 3 lots of 4 tiles to complete a 3-hole shape and so on. So, we have a pattern which is just :

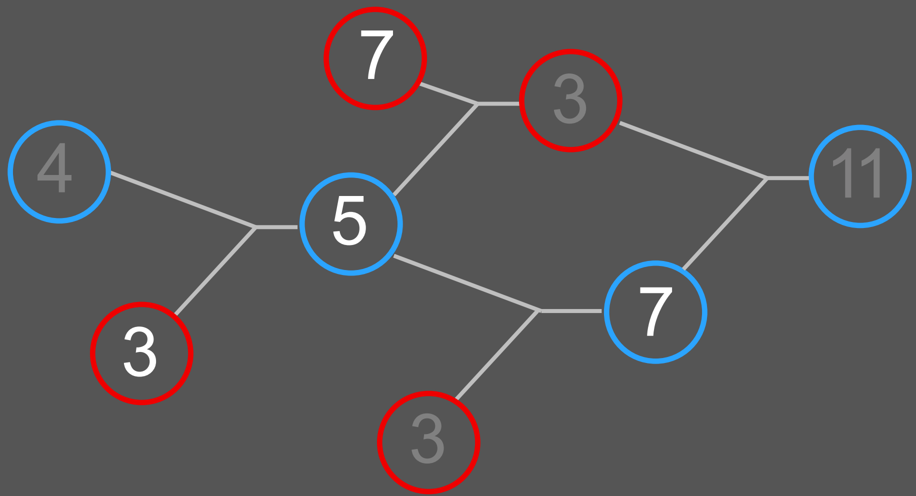
  times the number of holes by 4 and add 2.

  So, if Mark used 150 tiles, he must have used 37 lots of 4 tiles (because 37 x 4 = 148) to begin with and then added 2 tiles to complete the pattern. Which means that it's a 37-hole pattern!

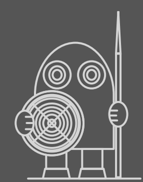
ans 25 **mapping webs 1**

Remember, the rule for this mapping web is : to combine two numbers, just double the 'blue' number and then subtract the 'red' number. For the web on the right, you simply have to work forwards; to complete the one below, you'll need to do some working backwards. Here are the answers :





answers for missing numbers shown in white



ans 26 three brothers

One way of handling this questions is :

- make a list or chart which shows all the possible combinations, that's to say teacher / seaside, teacher / country etc
- put S, J and P in each
- go through the seven pieces of information given in the question and cross out any initials ruled out eg 'John is not a teacher' lets us cross out all the Js in that row
- finally, just three initials remain (one on each row) so there are your answers!

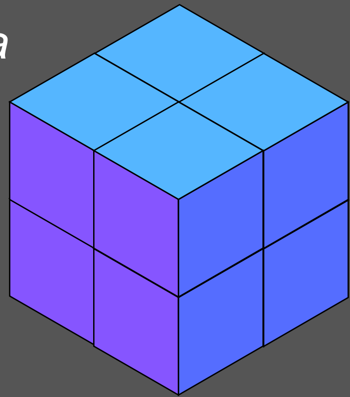
	seaside	country	town
teacher	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ ~~P~~	S ~~J~~ ~~P~~
fireman	~~S~~ J ~~P~~	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ ~~P~~
builder	~~S~~ ~~J~~ ~~P~~	~~S~~ ~~J~~ P	~~S~~ ~~J~~ ~~P~~

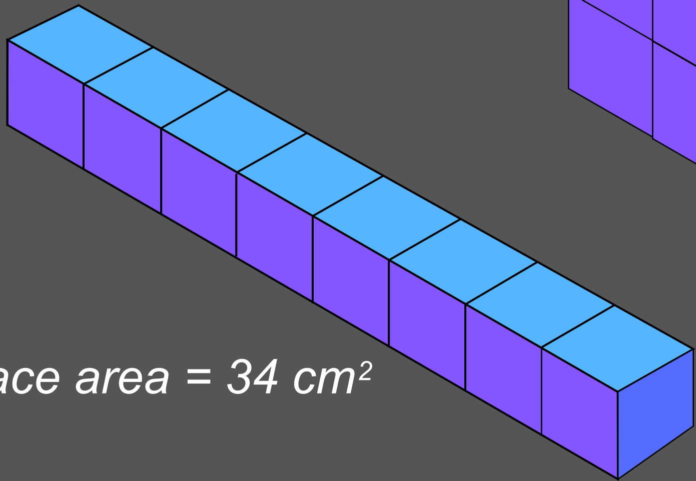
conclusions :

1 John is the fireman
2 the builder lives in the country
3 Simon is a teacher
4 John lives by the sea

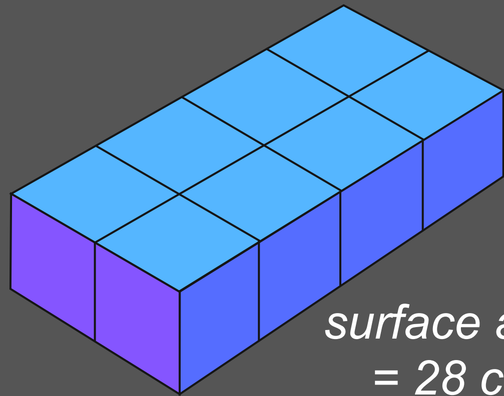
ans 27 cube surfaces

surface area
= 24 cm²



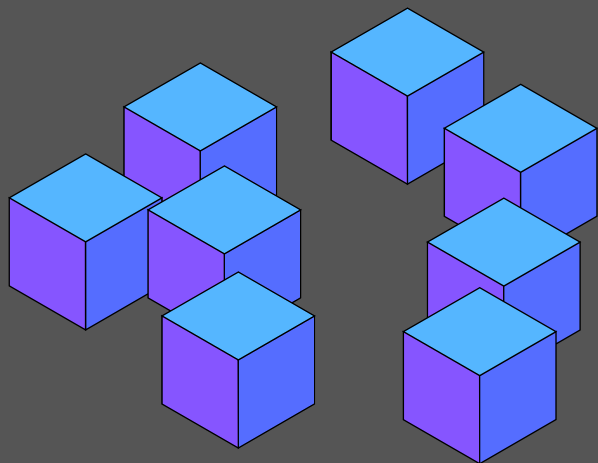


surface area = 34 cm²



surface area
= 28 cm²

total surface area
= 8 x 6 = 48 cm²



ans 28 a grandson called Ben

- The second piece of information gives us a good place to start. Because, if the digits of the number add up to 8, then it must be one of these :

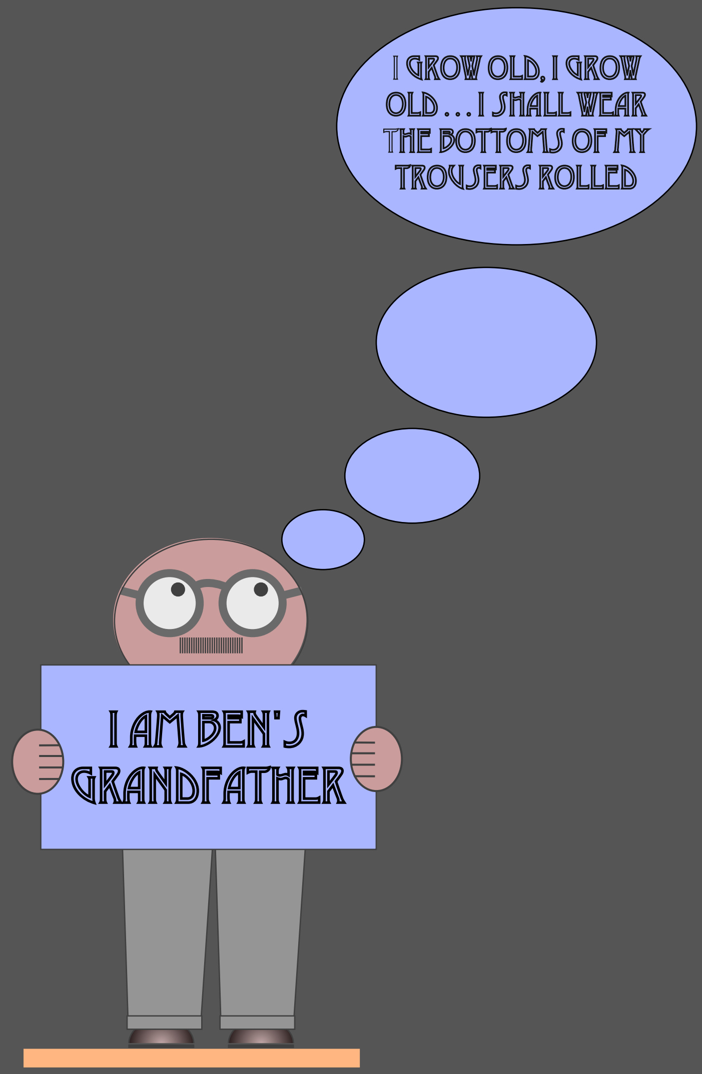
  08, 17, 26, 35, 44, 53, 62, 71, 80

- Since John's parents are each 45 yrs old, the first five of these numbers can be ruled out straight away. This leaves us with just 53, 62, 71 and 80.

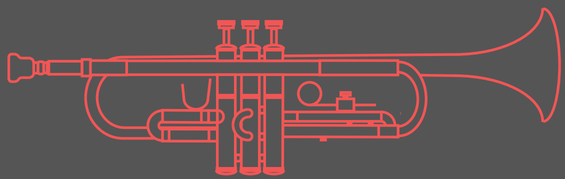
- The first bit of information we have is that our number is a prime number. Obviously 62 is an even number, so it isn't prime . . . and 80 has lots of factors, so it isn't prime . . . so now we're left with 53 and 71 (both prime numbers).

- 53 is ruled out for another reason : If Grandfather really is 53, he must have become a father at the age of 8 - most unlikely!

- final confident answer : Grandfather is 71 !



brass notes



Using U for up and D for down, you could hold the three valves in these different positions :

U	U	U
U	U	D
U	D	U
U	D	D
D	U	U
D	U	D
D	D	U
D	D	D

– and that's 8 different combinations altogether !

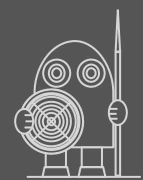
** Of course, a real trumpet can play more than 8 different notes, which tells you that there's more to playing a trumpet than just learning these 8 combinations.*

special note :

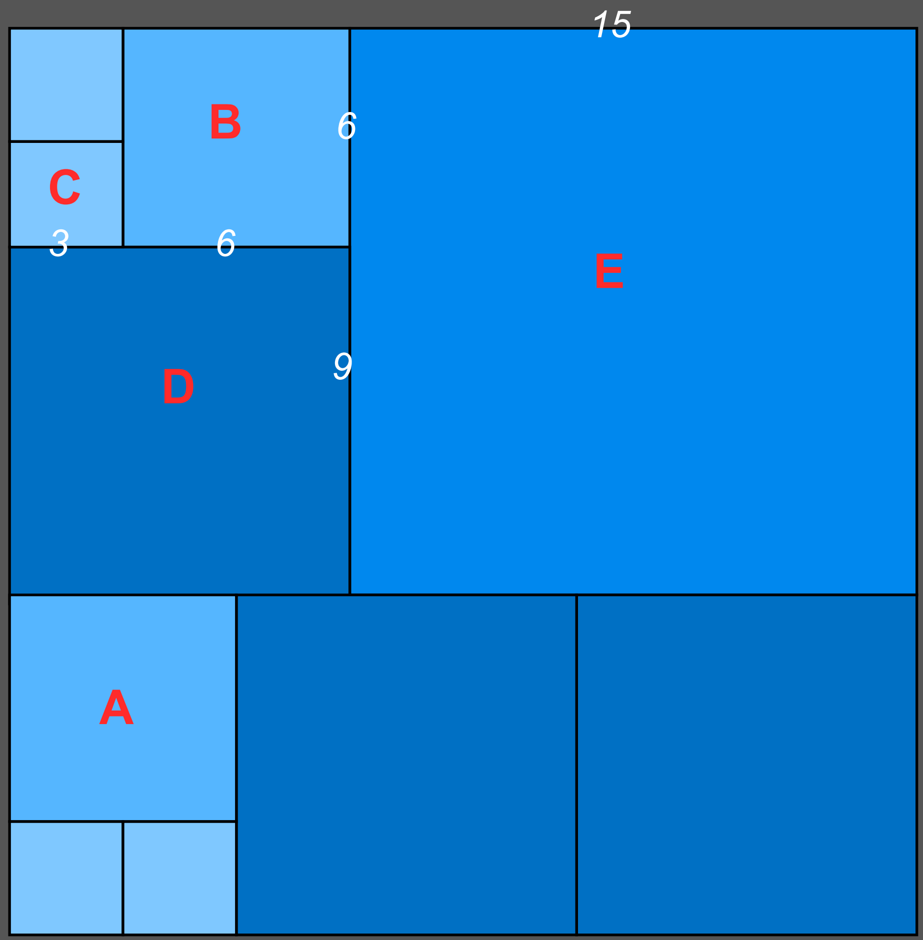
In fact, trumpeters usually call the valves 1,2 and 3 and they list the combinations like this (ps. 0, as you might guess, just means 'valve left open') :

0	0	0
0	0	3
0	2	0
0	2	3
1	0	0
1	0	3
1	2	0
1	2	3

– that's 8 different combinations altogether!



ans 30 all square



If square A has an area of 36, then square B, which is the same, also has an area of 36, which means it has sides of length 6.

Square C obviously has sides exactly half as long as square B's sides, so square C's sides must be just 3.

By simple addition, this gives us 9 for the side length of square D.

Then once again by simple addition, square E must have side length 6 + 9 = 15.

Squaring the 15 gives us the area of E :

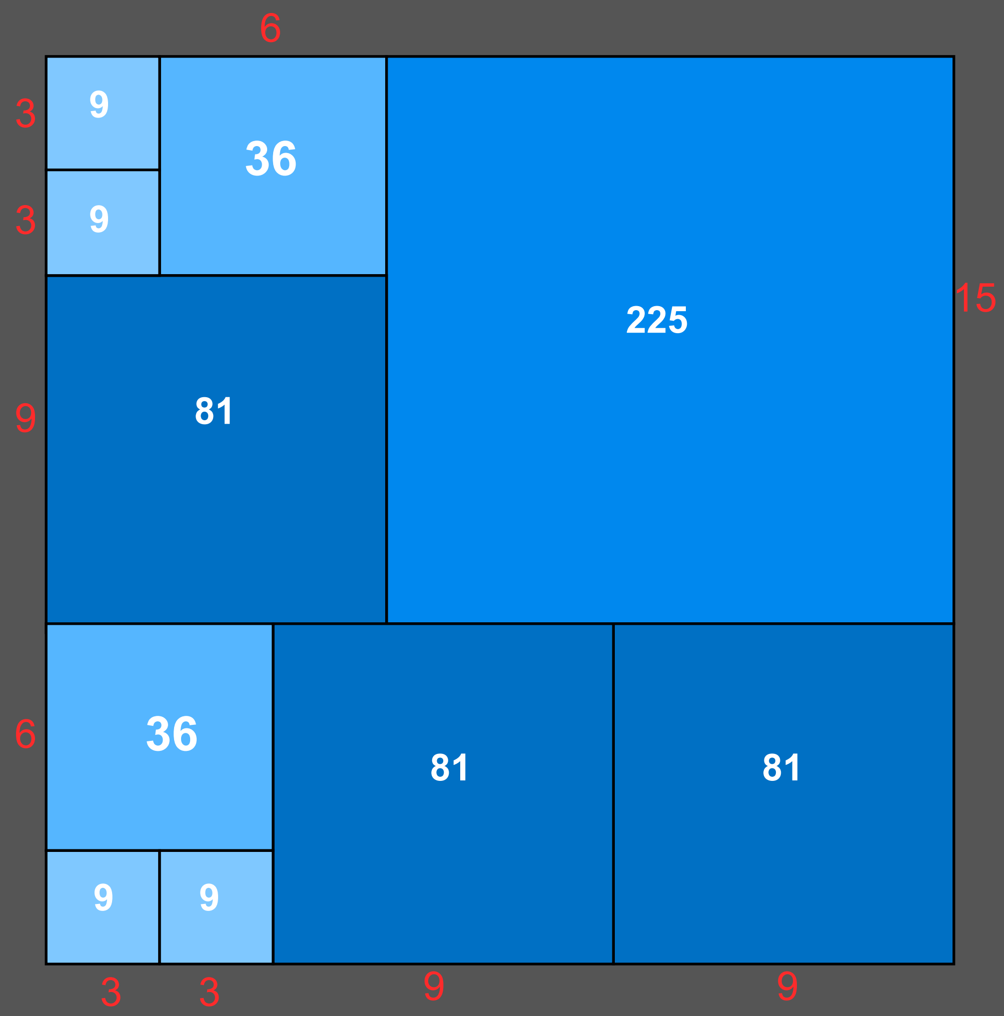
$$15 \times 15 = \underline{225}$$

and that's your answer !

PTO

ANS 30 all square

For your information, here is the same diagram with all lengths and areas shown :



lengths of sides are shown in red

areas are shown in white

ans 31 Mr Average

Where to begin? Well, straight away we know that B weighs 40kg and that this is 80% of D's weight; perhaps you can see (or easily work out) that D must weigh 50kg. What else do we know? Let's use $\square$ to stand for the average (mean) of the group. Now we know three more things : We know that A's weight is $\square$, we know that C's weight is $\square\square$ and we know that the total weight of the whole group is exactly $\square\square\square\square$ (that's to say four times the average).

Of course, the total weight of the whole group is made up by adding together the four weights of A, B, C and D. We can show this as follows :

Mr A's weight = $\square$

Mr B's weight = 40kg

Mr C's weight = $\square\square$

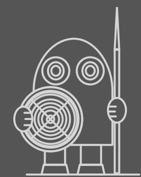
Mr D's weight = 50kg

$\rightarrow$ $\square + 40 + \square\square + 50 = \square\square\square\square$

$\square + 40 + \square\square + 50 = \square\square\square\square$

So, what does this mean? You might recognise it as an equation but all that matters is that everything on the left comes to the same as what's on the right). And what this amounts to is just this :

$\square\square\square + 90 = \square\square\square\square$



PTO

ans 31 Mr Average

So, we're left with : 3 lots of $\square$ plus 90 must be the same as four lots of $\square$

and this tells us that $\square$ must be worth 90 : $\square$ = 90

And now it's not very hard to work out the weights of all four athletes :

Mr A's weight is 90kg

Mr B's weight is 40kg

Mr C's weight is 180kg

Mr D's weight is 50kg

At last we have answers to our two problems! First of all, Mr D weighs 50kg and secondly, the four men together weigh 360kg – exactly at the safety limit !

ans 32 the long bench

1	2	3	4
X	R	S	T
X	R	T	S
X	S	R	T
X	S	T	R
X	T	R	S
X	T	S	R
R	S	T	X
R	T	S	X
S	R	T	X
S	T	R	X
T	R	S	X
T	S	R	X

One pretty standard way of handling problems like this is to begin by making a list of all the equally likely possible seating arrangements. You can see such a list here on the right :

Next, we identify those arrangements which have Tom sitting in place number 1. As you can see straight away, out of the 12 possible arrangements, there are just two which have Tom in place number 1. So, we can write

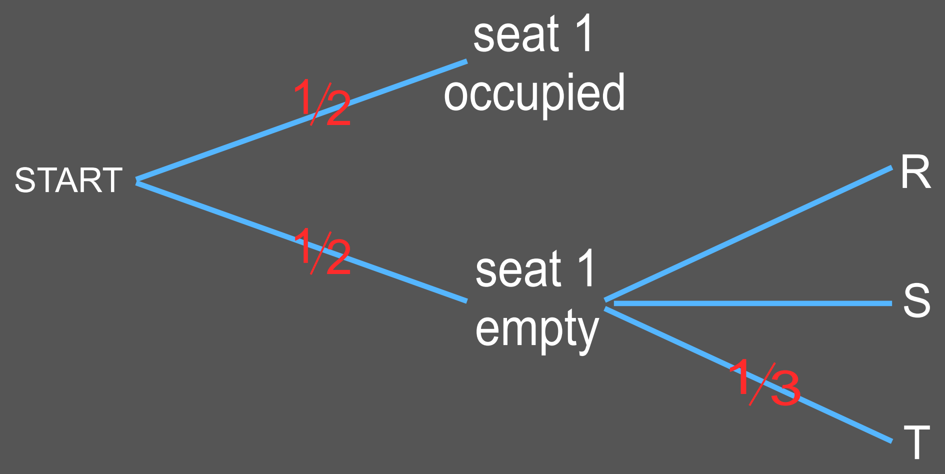
probability (Tom sits in place 1) = 2/12 = 1/6

– and that's our answer! Probability of Tom sitting in place 1 = 1/6

PTO ▶▶

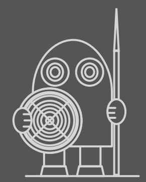
the long bench

Some people prefer a 'tree diagram' method of looking at problems like this one. Such a diagram for this problem is shown on the right :



You can see how the diagram works : To begin with there's an equally likely probability of either seat 1 or seat 4 being occupied when our three arrive. We show this on the diagram with two branches, each marked '1/2'. Then when seat 1 is empty, there's a 1/3 probability that Tom will sit in it. We need to work out 1/3 of 1/2, which of course is the same as 1/3 x 1/2, or 1/6. And so we have our answer :

probability of Tom sitting in seat number 1 = 1/6



PTO

ans 32 the long bench

Here's a slightly different way of looking at the problem (you might see it as a more helpful way of explaining the tree diagram on the previous page) :

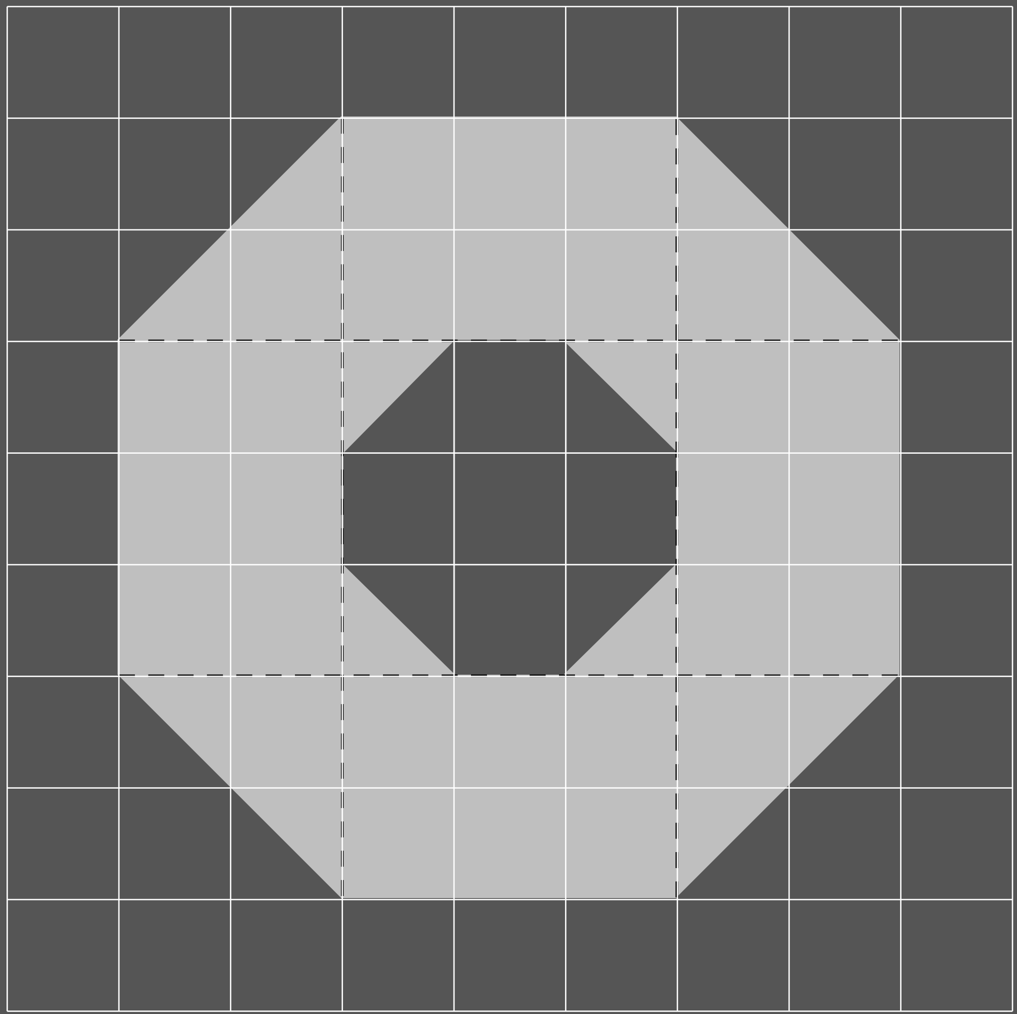
As the friends arrive, the probability that seat 1 is already occupied is 1/2 . . . and so the probability that this seat is empty is also 1/2.

So now we can say that there's only a 1/2 chance of any of the friends taking seat 1. But if seat 1 is empty and it's there to be taken, there's only a 1/3 chance that it's Tom who gets it.

Which means that Tom has a 1/3 of a 1/2 probability of sitting down in place number 1. And as you probably know, 1/3 of 1/2 just means 1/3 x 1/2, which is of course 1/6.

answer : probability of Tom sitting in place 1 = <u>1/6</u>

ans 33 an octagon ring



This is the same diagram as in the question but here we've drawn the gridlines over the ring. Now it's easier perhaps to see that the ring is made up of two different kinds of shape, that's to say, triangles and rectangles. Altogether we have :

4 triangles, each of area 2 squares

4 triangles, each of area 1/2 square

plus

4 rectangles, each of area 6 squares

– which means that the total area of the ring must be exactly 34 squares.

PTO

ans 33 an octagon ring

. . . or of course you might have done this a different way, that's to say by starting with a square and subtracting :

first of all, the large square around our shape has an area of 7 x 7 = 49 squares

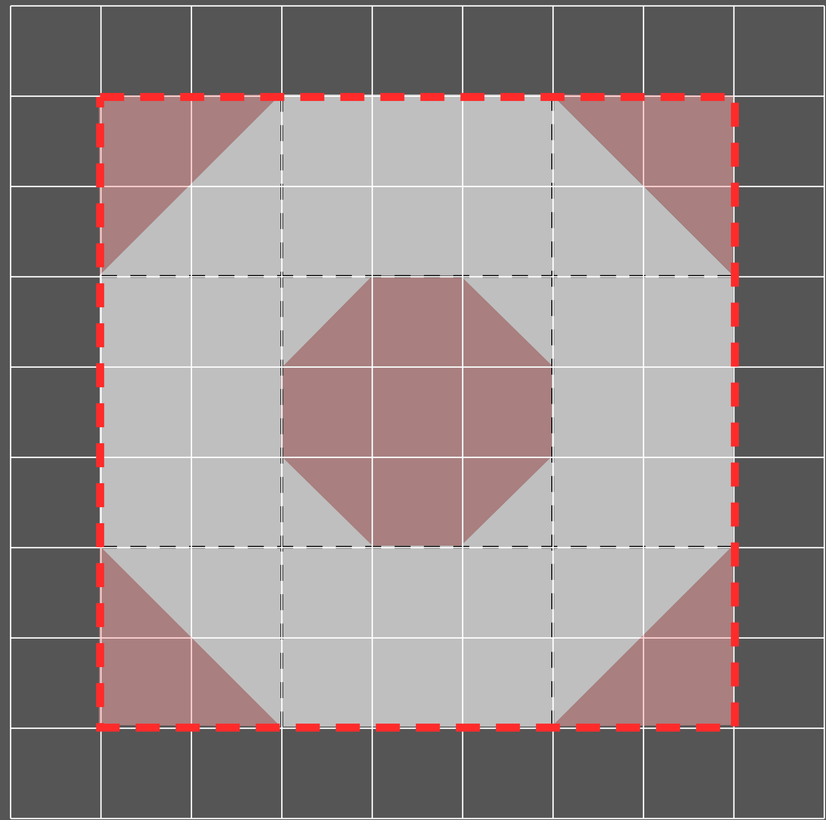
Now take away from this

4 corner triangles, each of area 2 squares

and an octagon in the middle, area 7 squares

– which gives us a final result of

49 – 15 = <u>34 squares</u>



ans 34 you're my number wall !

◎ No great challenge with this one! It takes only a moment's thought to see that the missing number on the bottom row must be 3 – and then it's just a matter of adding pairs of numbers to complete the wall . . .

	40		
	19	21	
10	9	12	
4	6	3	9

◎

	48		
	10		
3	6	4	15

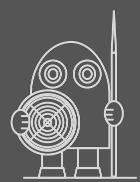
. . . but this one's harder! Obviously, the two missing numbers on the bottom row must add up to 10. We have no idea what they could be, so let's just try 6 and 4 – and see what happens.

We now have a bottom row which reads :

3, 6, 4, 15

Adding upwards, we can easily fill in the blanks (we've used red numerals for these). And straight away we can see that this works! But is this the only possible answer?

	48		
	19	29	
9	10	19	
3	6	4	15



PTO ▶▶

ans 34 you're my number wall !

Let's try a different pair of numbers : 2 and 8. These numbers also add up to 10, so let's see what happens. Adding upwards, the blanks are soon filled in, as shown on the right – and, as you can see, once again everything works! So now we have another solution to the problem.

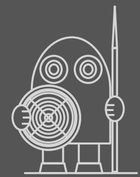
	48		
	15	33	
5	10	23	
3	2	8	15

1 and 9 are two more numbers adding up to 10 and once again, starting with these on the bottom line and filling in the blanks, reveals another arrangement which works well.

	48		
	14	34	
4	10	24	
3	1	9	15

In fact, it turns out that you can fill in the blanks on the bottom line with any pair of numbers you like, as long as they add up to 10.

By the way, this answer is another example of the value of experimenting when you're not sure how to begin; we just tried some numbers and we quickly solved the problem! We also found that this was yet another example of a problem having more than one solution . . .



ans 35 remainders

The numbers which divide exactly by 2 are 2, 4, 6, 8 . . . So the numbers which divide by 2 and leave a remainder of 1 are all of these same numbers but with 1 added to each of them, in other words 3, 5, 7, 9 . . .

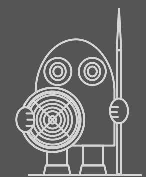
In the same way, the numbers which divide exactly by 3 are 3, 6, 9, 12 . . . and so those which divide by 3 and leave a remainder of 2 are just the same multiples of 3 with 2 added to each one, that's to say 5, 8, 11, 14 . . .

You can handle dividing by 4 and dividing by 5 in just the same way. On the right is a short version of the four lists you get from these calculations :

÷ 2 gives rem 1	÷ 3 gives rem 2	÷ 4 gives rem 3	÷ 5 gives rem 4
3	5	7	9
5	8	11	14
7	11	15	19
9	14	19	24
11	17	23	-
13	20	-	-
15	-	-	-
-	-	-	-
-	-	-	-
-	-	-	49
-	-	51	54
-	53	55	59
55	56	59	
57	59		
59			

59 is the first number to appear in all four columns – although you do have to work out a whole lot of rows to get there. To save space we've left out a lot of the middle rows and instead we've just shown the beginning and end numbers in the four sets.

So, 59 is the answer to our challenge !



ans 36 parking mad . . .

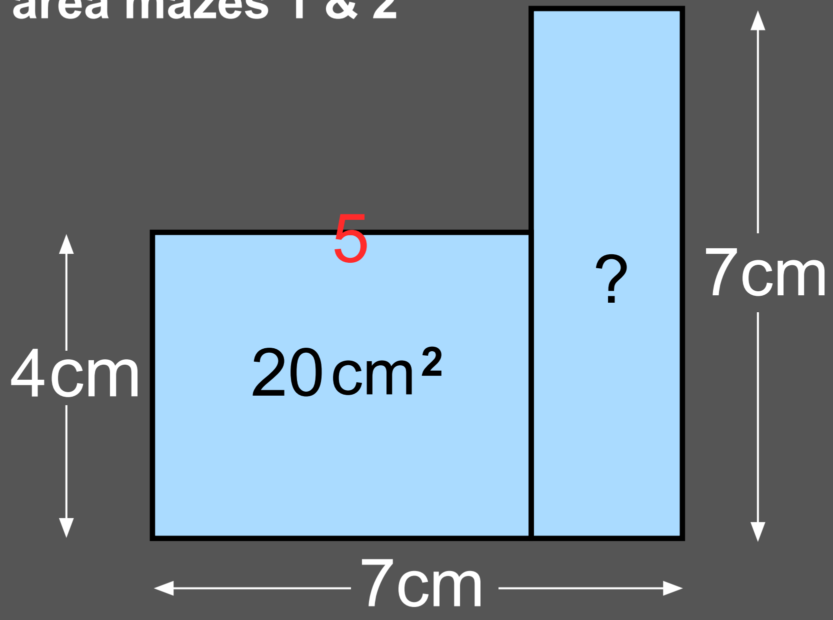
Here are the three cars : a sports car, a saloon and an estate car . . .

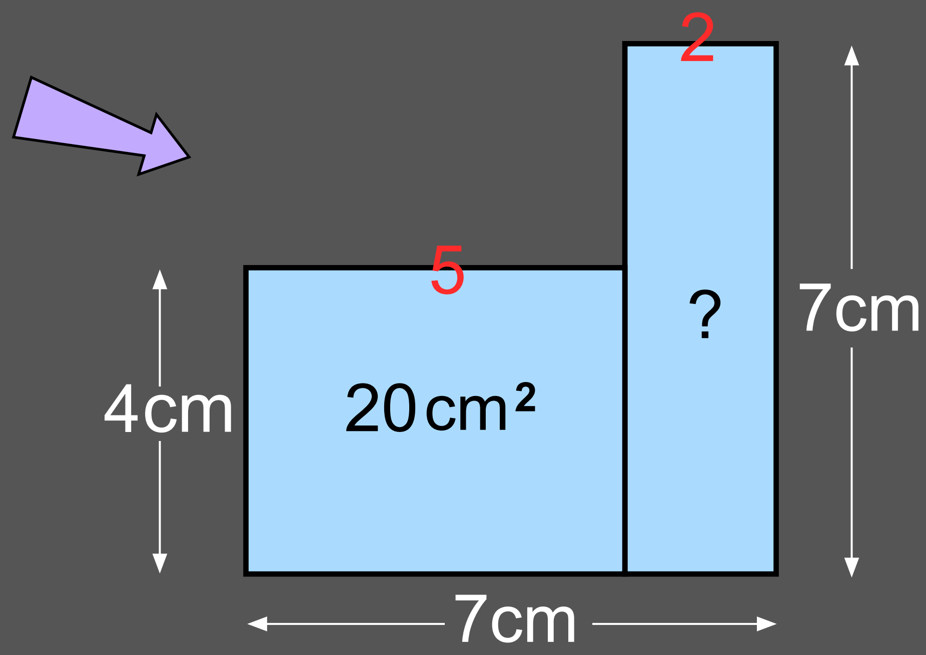
The colours are easy : we're told that the sports car is blue and that the family saloon is red – so the estate car must be yellow . . .

And from what we're told, Dr Brown's car is obviously the red family saloon – and Miss Green's car is not blue, so it must be the yellow estate car – leaving the blue sports car for Mr Smith . . .

answer : Mr Smith owns the sports car.

(a)





First of all, look at the rectangle on the left. Its area is 20cm² ; one side is 4cm and so the other side must be 5cm.

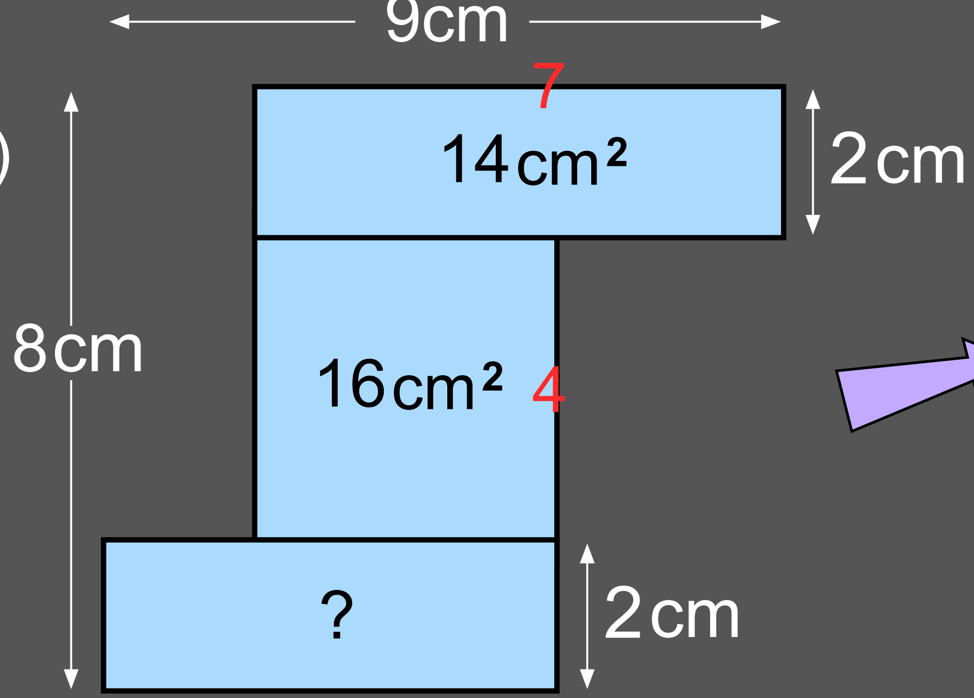
This means that the rectangle on the right must be 2cm wide . . .

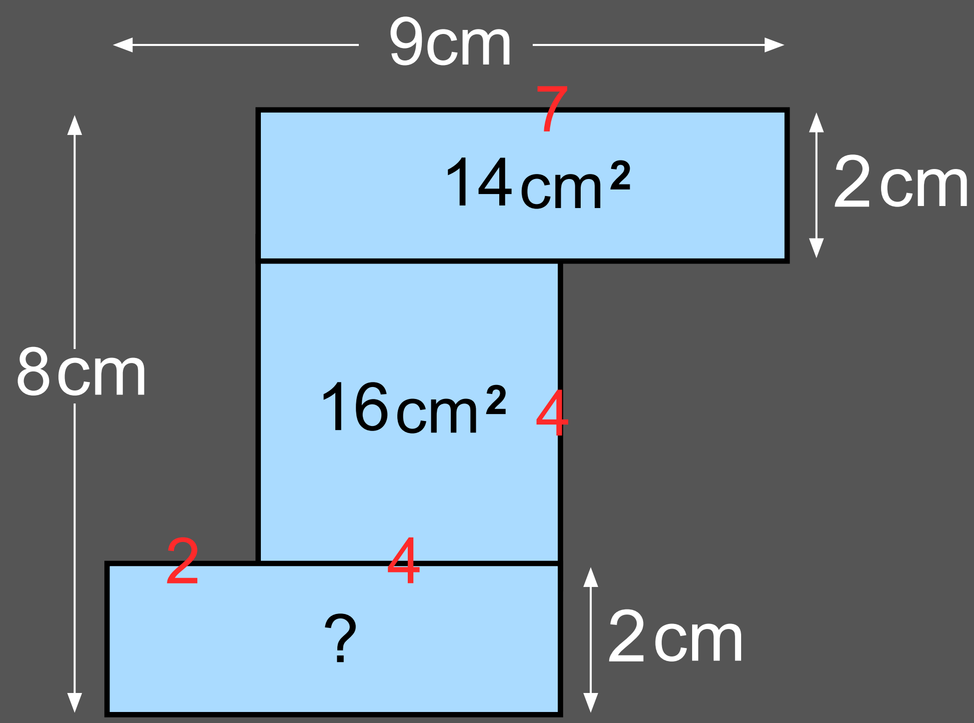
. . . and so the area of this rectangle must be 14cm²

PTO

ans 37 area mazes 1 & 2

(b)





To begin with, notice that the top rectangle has an area of 14cm² and one of its sides is 2cm; so the other side must be 7 cm. Also, the figure as a whole is 8cm deep and so by subtraction (taking away the two lots of 2cm), we get 4cm for one side of the middle rectangle.

With an area of 16cm² and one side of 4cm, the other side of the middle rectangle must also be 4cm. And, by subtraction, the remaining length of the bottom rectangle must be 2cm. So its two sides must be 6cm and 2cm, giving an area of 12cm²

PTO

ans 37 area mazes 1 & 2

These two small problems are examples of an interesting type of puzzle called the 'Area Maze'.

With each area maze you are asked to find just one thing, which will be either a length or an area. To solve any area maze problem, you need only one bit of maths knowledge : how to find the area of a rectangle. Also (you may be pleased to know), all of these problems can be solved without using fractions or decimals, so there's just one rule you have to follow : stick to whole numbers! It's this rule which means that some area maze problems will send you quite a long way round to get to the answer – just like finding your way round a real maze . . .

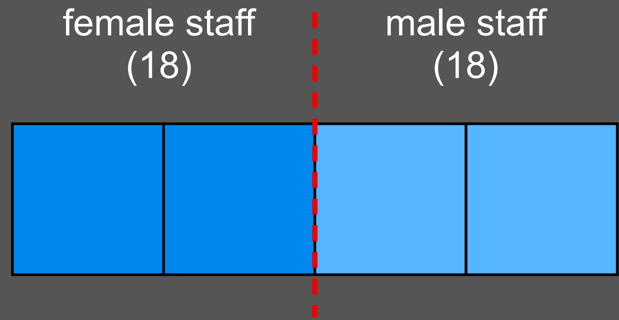
日本

** Area Mazes first appeared in Japan (where they were called 'Menseki Meiro') and they quickly became very popular. They were invented by Naoki Inaba, a man who has thought up lots of different maths problems and puzzles . . . If you enjoy solving these, there are lots of Area Maze books available (but be careful – many of the books jump up rapidly from easy questions to really quite hard ones).*

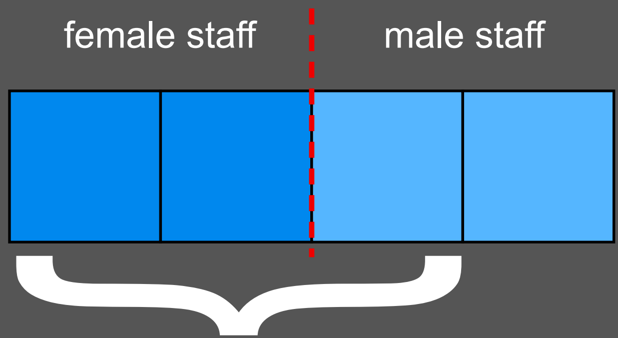
ans 38 teaching equality

There are different ways of going about this problem but one way is to use diagrams to make things clearer:

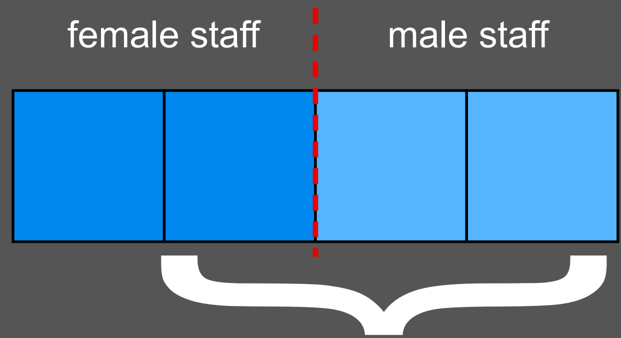
Suppose we use one square to stand for 9 members of staff, then the balance of male/female staff at the School can be shown like this :



On any half-day, three-quarters of the teachers are present. Three-quarters of 36 is 27, or in other words, three full squares on our diagram. And so we can show the largest and the smallest numbers of females present at any one time :



largest no. of female teachers = 18



smallest no of female. teachers = 9

– and so question 1 is already answered : As there are always at least 9 female teachers present, the Inspector can indeed be sure of finding a female teacher there whenever he calls.

ans 39 DIY magic square

PROBLEM 1 : The first thing to do with problems like this is to find the magic total. Adding up the numbers in the right-hand column gives us 24 and so this is our magic total. The middle row has two numbers in it, 15 and 1, which add up to 16. So we need an 8 in the centre to make our required 24. In the same way, we can work out that the two missing corners must be After this, it's easy to fill in the missing numbers in the centre column : they must be 11 and 5 . . . and that's it – problem solved!

3	11	10
15	8	1
6	5	13

MT = 24

PROBLEM 2 : How can we get started on problem 2? Obviously we can just try our numbers in lots of different positions and hope that sooner or later we'll hit on an arrangement which works. But looking for an answer this way might take an awful long time! Maybe we can make things easier for ourselves if we can get straight to the magic total or perhaps to the centre number. On the next pages you'll find some ideas which might help . . .

PTO

ans 39 DIY magic square

finding the magic total . . .

Let's just think about the rows in a magic square. As you already know, each of the three rows in a 3 x 3 magic square must add up to the magic total. So if you add the three rows together, then of course you'll have three times the magic total. But stop and think : what are you adding together when you add up the three rows? Well, obviously you're adding up the three rows. But wait! Each row contains three of the nine numbers which make up the magic square – and so adding the three rows together is just the same as adding together all nine numbers. Thinking of our problem, we know that our nine numbers are 0, 1, 2, 3, 4, 5, 6, 7 and 8. These numbers add up to 36, which means that our three rows must add up to 36. So each row must add up to 12, or in other words,

magic total = 12

If you find this thinking a bit hard to follow, don't worry! It's not an easy idea, so you might need to go over it once or twice before you're sure you really understand it. It's an important idea though, so it's worth the effort!

With magic total = 12, perhaps you can already say what the centre number must be . . .

PTO

DIY magic square

the centre number . . .

You probably remember that in problem 1 the centre number was exactly one-third of the magic total. This connection holds true for all 3 x 3 magic squares. We know that the magic square we're going to construct has magic total = 12 and so the centre number must be 4. This leads us to some more useful facts . . .

these two numbers must add up to 8

We know that 4 is our centre number. This means that whether we're looking at rows, columns or diagonals, the numbers on either side of the centre must add up to 8 (since 12 is the magic total). See the diagram on the left, which illustrates this.

So the pairs on either side of the centre 4 must be : 0,8 1,7 2,6 3,5

All we have to do now is to find how to arrange these pairs . . .

PTO

ans 39 DIY magic square

Well, first of all we found this one way of arranging the pairs. But then we looked again and we found some more ways of arranging the same pairs. In the end we found a total of eight different solutions. You can see them all on the next page. Obviously, they all have magic total = 12 and they all have 4 as the centre number.

3	2	7
8	4	0
1	6	5

MT = 12

Have a look at the next page and compare the eight different answers. Do you think they really should count as different solutions to the problem? Or do you think that we should see them as just one solution rearranged in different ways? This is the sort of thing mathematicians often have to decide.

And – can you describe what you have to do to solution 1 to get each of the others? (For example, you get solution 2 by simply rotating solution 1 clockwise through 90˚.)

PTO

ANS 39 DIY magic square

3	2	7
8	4	0
1	6	5

1	8	3
6	4	2
5	0	7

5	6	1
0	4	8
7	2	3

7	0	5
2	4	6
3	8	1

7	2	3
0	4	8
5	6	1

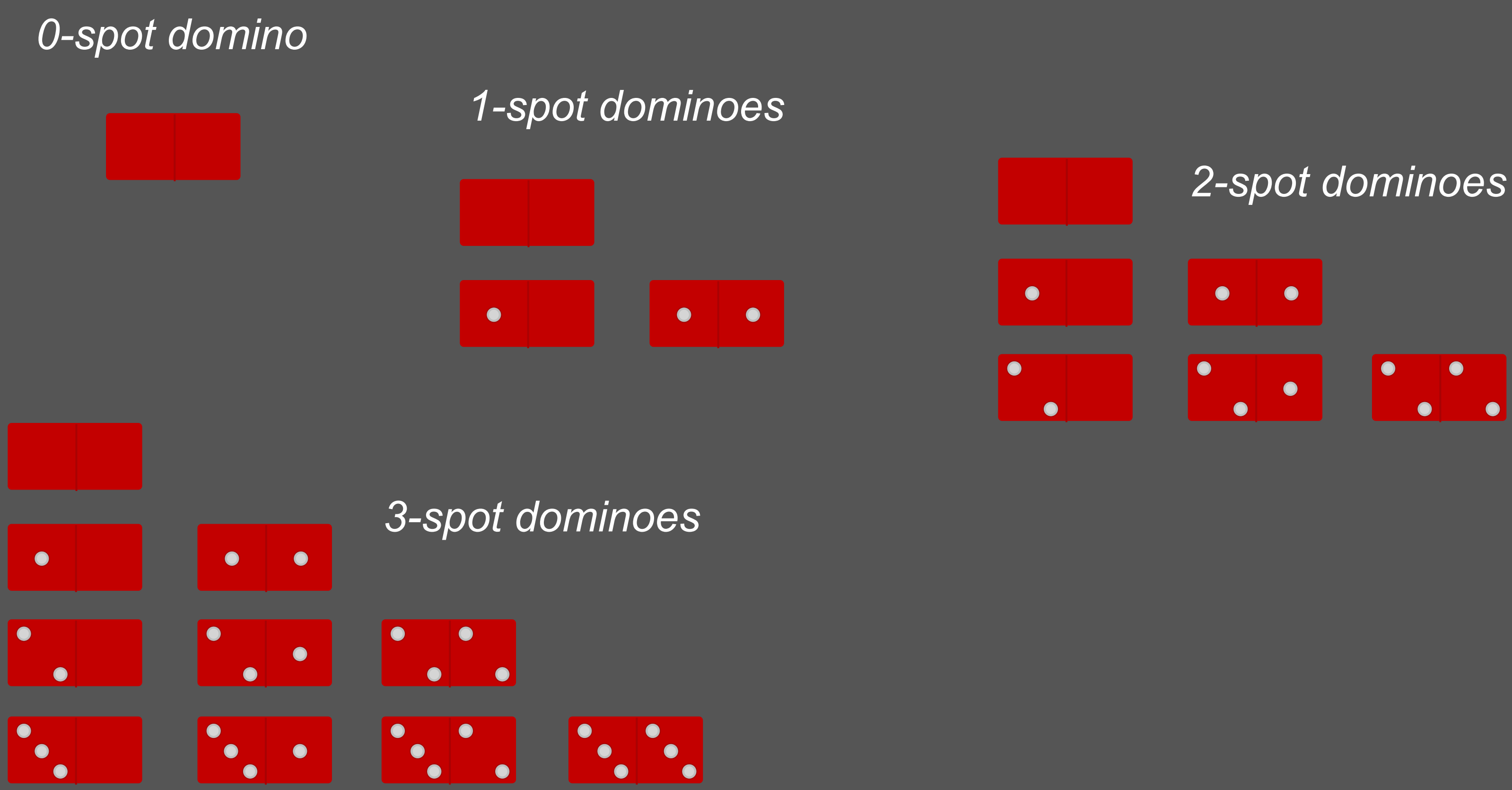
5	0	7
6	4	2
1	8	3

1	6	5
8	4	0
3	2	7

3	8	1
2	4	6
7	0	5

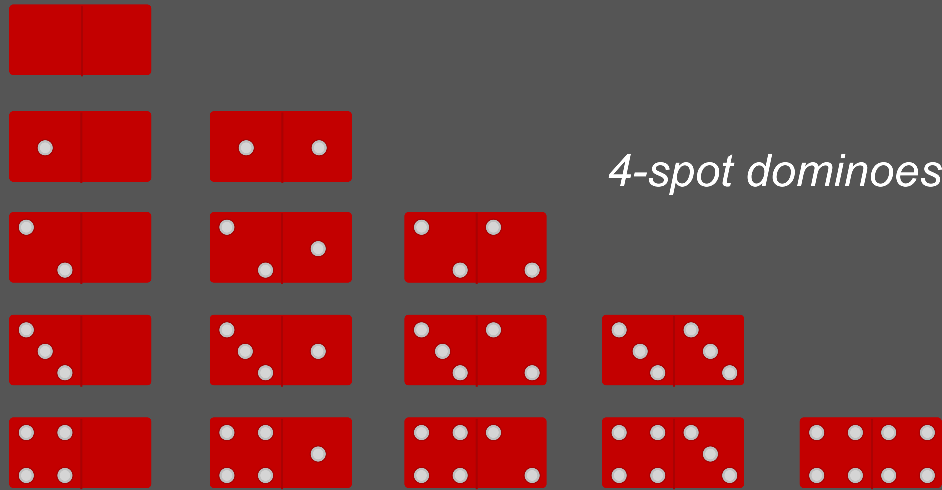
ans 40 domino faces

You'll find the answers to all four questions in the diagrams below and on the next few pages, which show domino sets from 0-spot to 6-spot, as well as a 9-spot set.

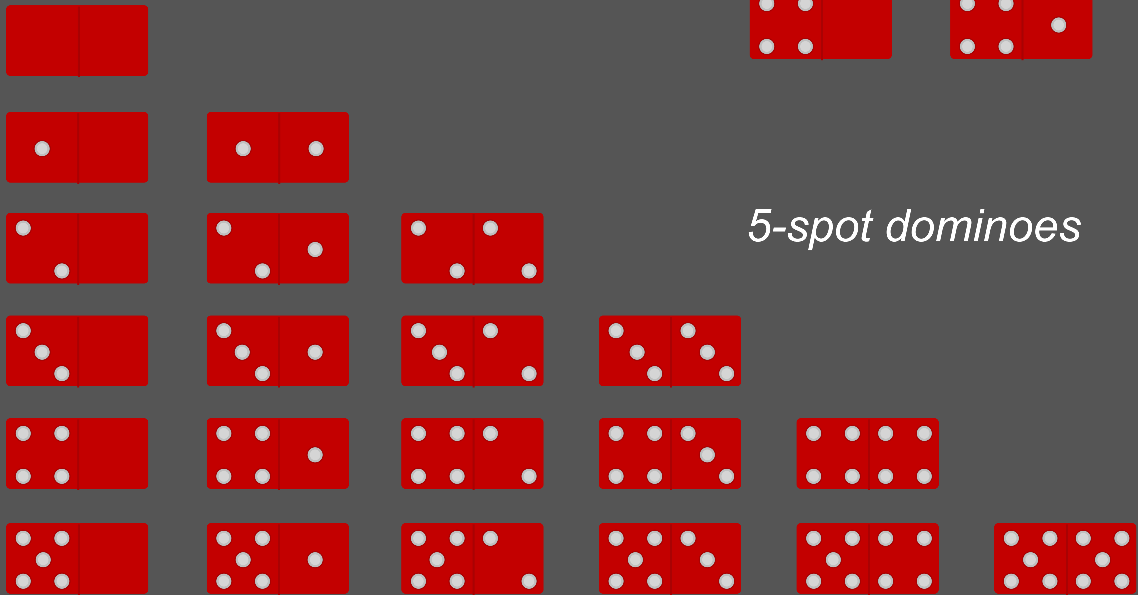


PTO

ans 40 domino faces



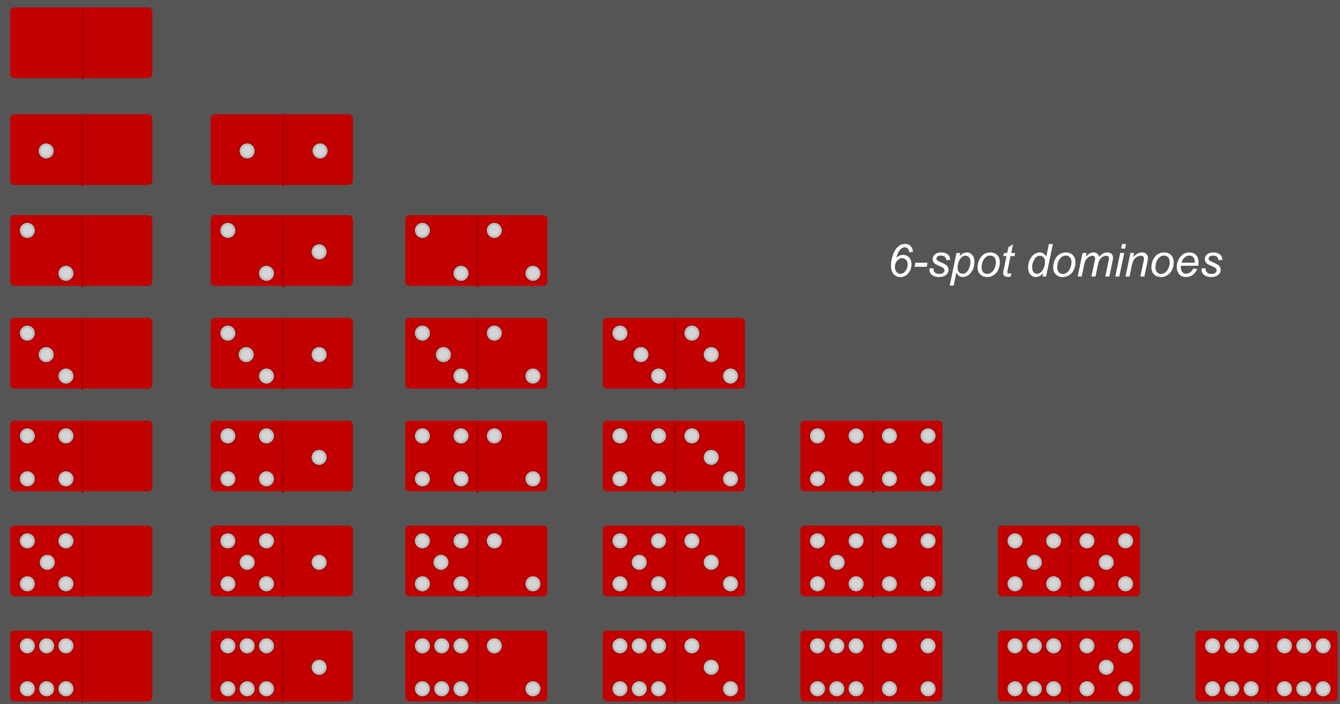
4-spot dominoes



5-spot dominoes

PTO

40 domino faces



6-spot dominoes

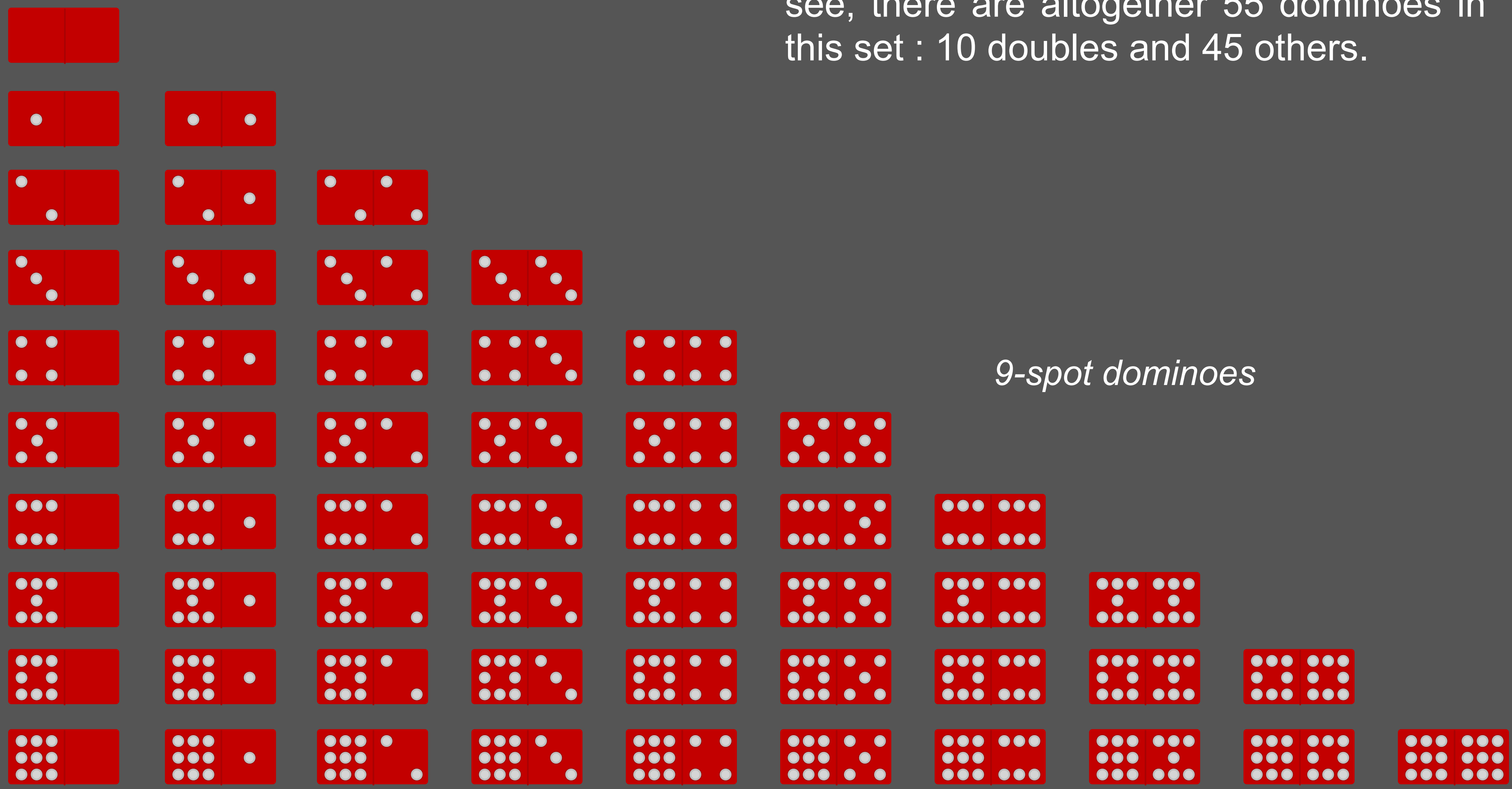
Notice that the number of dominoes in these sets goes 1, 3, 6, 10, 15, 21, 28 . . .

These numbers are of course the triangle numbers! As you probably know, the triangle numbers come up quite often in maths investigations. One simple way to get the triangle numbers is to start with 1, then add 2, then add 3, then add 4 and so on.

PTO

ans 40 domino faces

Finally, the 9-spot dominoes. As you can see, there are altogether 55 dominoes in this set : 10 doubles and 45 others.

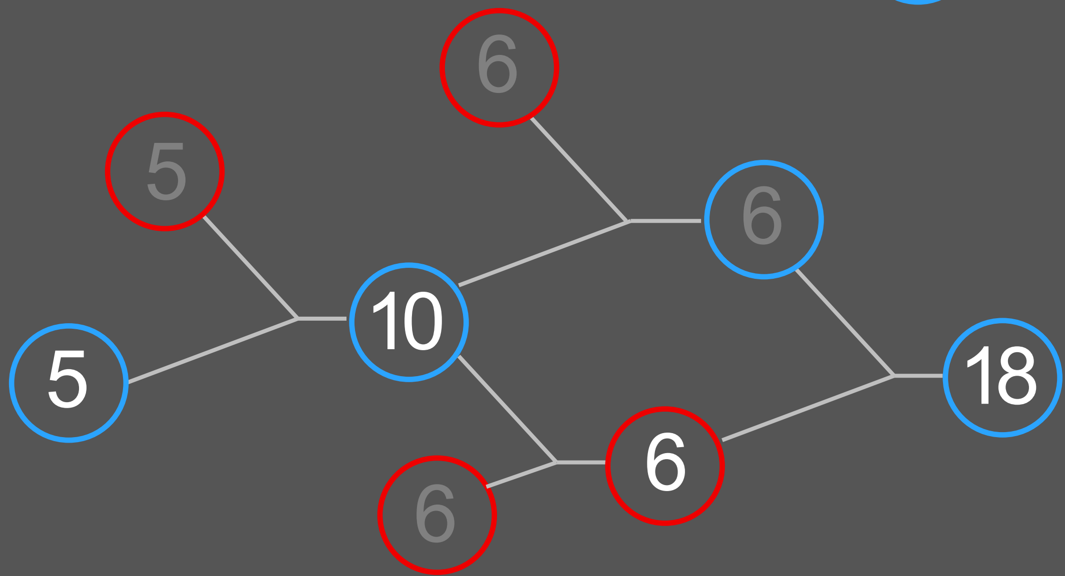


9-spot dominoes

ans 41 mapping webs 2

Remember the rule for this mapping web : to combine two numbers, square the 'red' number and then subtract three times the 'blue' number. As you probably discovered, to complete these two webs, you need to do working backwards as well as forwards.

Here are the answers :



ans 42 cube calendar

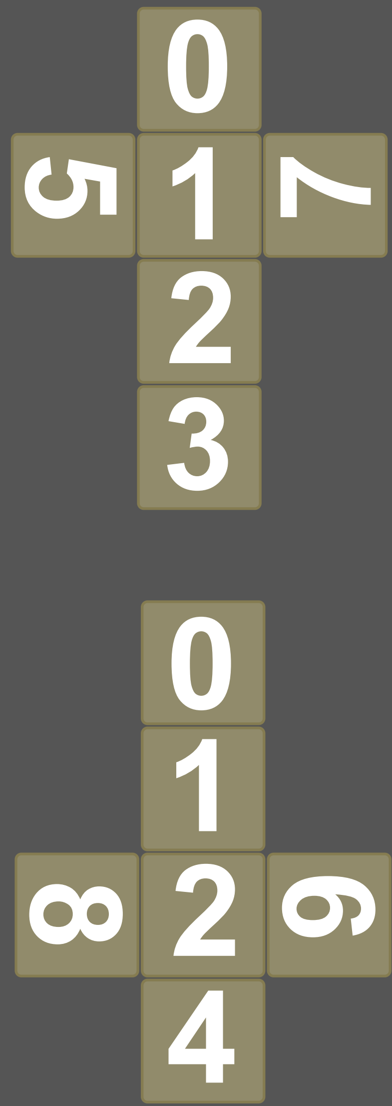
31 is the greatest number of days there are in any month, so we need our cubes to be able to display all numbers from 1 to 31. We can make our life easier by noticing one or two things :

Because of 11 and 22, both 1 and 2 have to be on each cube. Next, because 0 has to be combined with every number from 1 to 9, there must be a 0 on each cube (since a cube has only 6 faces). And because 1 and 2 have to be combined with every number from 1 to 9, there must be a 1 and a 2 on each cube (which we've already proved).

Finally, which numbers don't we need? We can forget about 10 because that's covered by 01. We can forget about 20 and 30 in the same way. We can also forget about 21 and 31 because they're covered by 12 and 13. And as you never need both a '6' and a '9' on the same date, you can use just one of them to stand for both (by turning one upside-down when you need to).

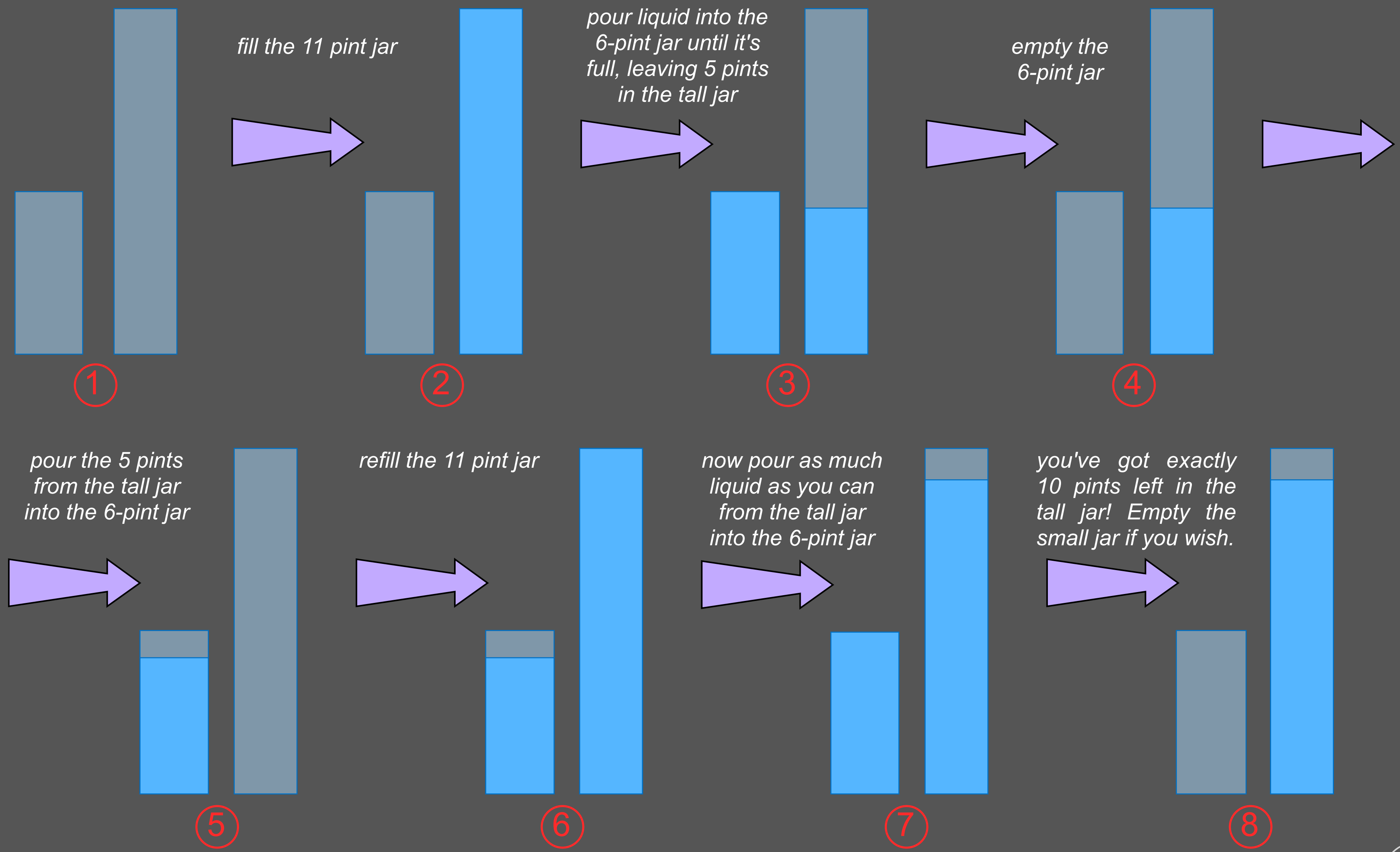
	~~10~~	~~20~~	~~30~~
01	11	~~21~~	~~31~~
02	12	22	
03	13	23	
04	14	24	
05	15	25	
06	16	26	
07	17	27	
08	18	28	
~~09~~	19	29	

And here's one way of arranging the numerals on the two cubes :



ans 43 10-pint target

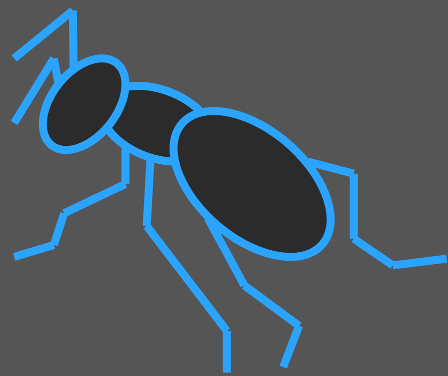
Here's one way of solving the problem :



ans 44 ant and rectangle

With luck, you will have found the rule. It really is a simple one . . .

To find the number of squares your diagonal will cross, just add together the two sides and subtract 1.



For example, if you have a 7 x 9 rectangle, your diagonal will cross 15 squares (that's 7 + 9 = 16, and then 16 minus 1 gives you 15.)

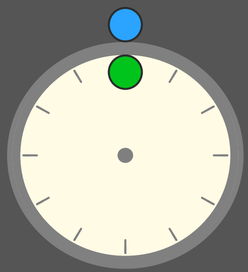
Second example : If you have a 3 x 11 rectangle, your diagonal will cross 13 squares. (Because 3 + 11 = 14 and then 14 – 1 = 13.)

nb Remember, this simple rule applies to prime rectangles only.

ans 45 Alfred and Betty

Instinctively you guess that Alfred will pass Betty somewhere along the way – until you think about it carefully! To make things easier, imagine the two of them cycling round a giant clock face :

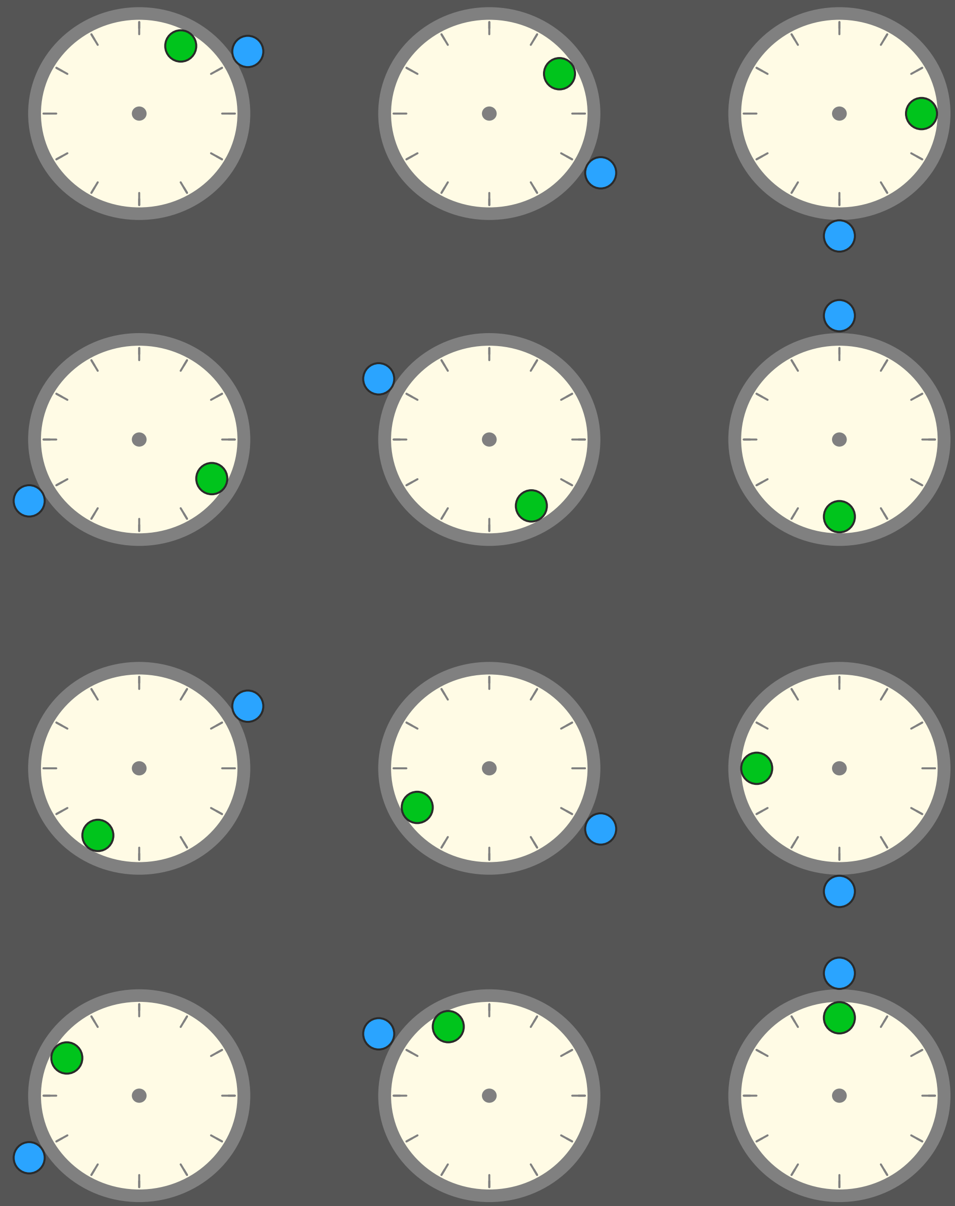
Here's the starting position :



Alfred

Betty

Let's suppose they start at '12 o'clock' (as in the above diagram) – then the diagram on the right shows their positions as Betty reaches 1pm, 2pm, 3pm, 4pm . . . and as you'll see, although they come together again as Betty reaches 12 o'clock once more, Alfred never actually overtakes her!



ans 46 a tale of two flutes

Before we start, it's really important to remember that when you see '25%', it always means 25% of something or other – so you have to be clear what it's 25% of you're after.

How does this affect our problem? Well, we need to remember that making the low bid 25% higher means increasing the low bid by 25% of itself . . . and in the same way, reducing the high bid by 25% means reducing the high bid by 25% of itself.

There's no obvious way of doing this, so perhaps we might use the 'trial and improvement' method here – or in other words, let's try some numbers and see what happens.

So, to begin . . . What should our initial guess be? It's useful if you know all about the price of second-hand flutes but most of us don't. Whatever you guess, you'll home in on the right answer in the end – but of course a good guess gets you there more quickly. If your first guess turns out to be wildly far out, then you can always start again with a new one. Let's start off with a guess of £90 for Roland's target sale price :

1st guess = £90

so, the high bid would have to be £120 (because reducing this by 25% gives you £90)

and the low bid would have to be £72 (because increasing this by 25% gives you £90)

PTO

ans 46 a tale of two flutes

difference between low and high bids = £120 – £72 = £48

£48 is quite a bit too high (we know this difference has to come to £32), so let's make our second guess £75

2nd guess = £75

– and this means the high bid must be £100 and the low bid must be £60 . . . so now the important difference is £40, which is still too high but – we're getting nearer!

3rd guess = £60

Smiles all round! This guess gives you £80 for the high bid and £48 for the low bid - and a difference between the two bids of exactly £32 ! So, we can confidently say :

answer : Roland's target price was £60

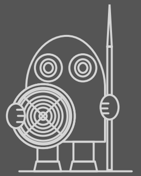
CHECK

If Roland's target = £60, then the high bid must = 4/3 of £60, which is £80.

And, with a target of £60, then the low bid must = 4/5 of £60, which is £48.

difference between these two bids is £80 – £48 = £32

check complete √



PTO

ans 46 a tale of two flutes

For those of you who like fractions, another way of looking at things would be this :

The high bid must be reduced by 25% to equal Roland's target – or to put it in fraction-speak, Roland's target is just 3/4 of the high bid. This is the same as saying that the high bid is 4/3 of Roland's target. To save writing things out all the time, let's call Roland's target R and the high bid H. Then what we have is :

$$H = 4/3 \text{ times } R$$

Now let's look at the low bid; we know we must add 25% to this to equal Roland's target – or to put things in fraction-speak, Roland's bid is 5/4 of the low bid. Let's call the low bid L. Then what we have is :

$$L = 4/5 \text{ times } R$$

PTO

ans 46 a tale of two flutes

So we can write the difference between L and H like this :

$$H - L = 4/3\ R - 4/5\ R$$

We need to turn everything to fifteenths to do the subtraction :

$$H - L = 20/15\ R - 12/15R$$

So the difference between L and H is 8/15 of R. But we know this difference is exactly £32. We can carry on like this :

$$£32 = 8/15 \text{ of } R$$

$$\text{so,} \quad £4 = 1/15 \text{ of } R$$

And we're almost home! Because if £4 is 1/15 of R, the whole of R must be 15 lots of £4, or £60.

answer : Roland's target was £60

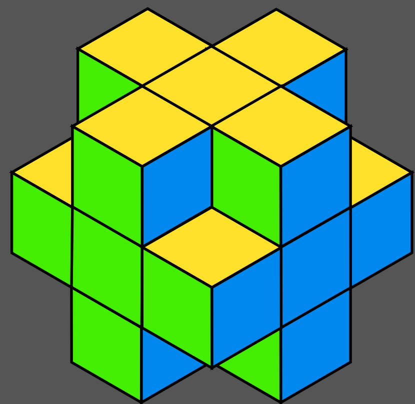
ans **47** cutting corners

- You know that a cube has 8 corners – and if you look carefully, you'll see that Jack has indeed removed 8 cubes from the corners of the original large cube.

- The original 3 x 3 x 3 cube was made from 27 individual 1cm cubes. With the 8 corner cubes taken away, there are now 19 cubes in the remaining shape.

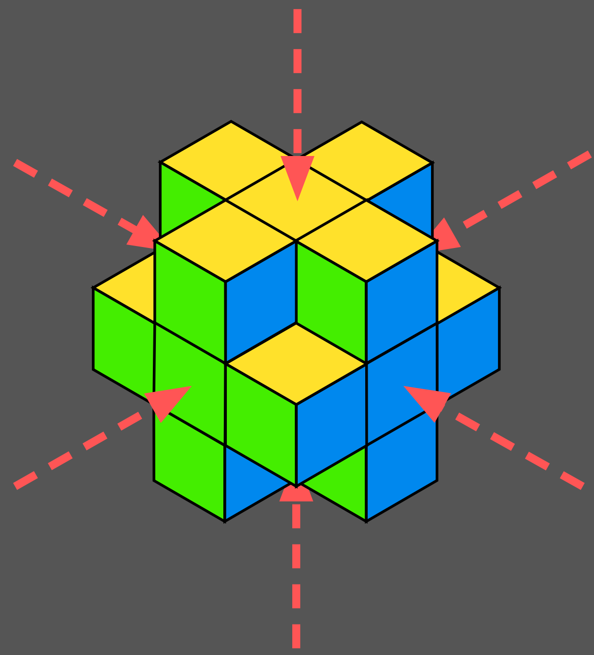
- There are different ways of finding the total surface area of the shape which Jack made. Here are just a couple of ways :

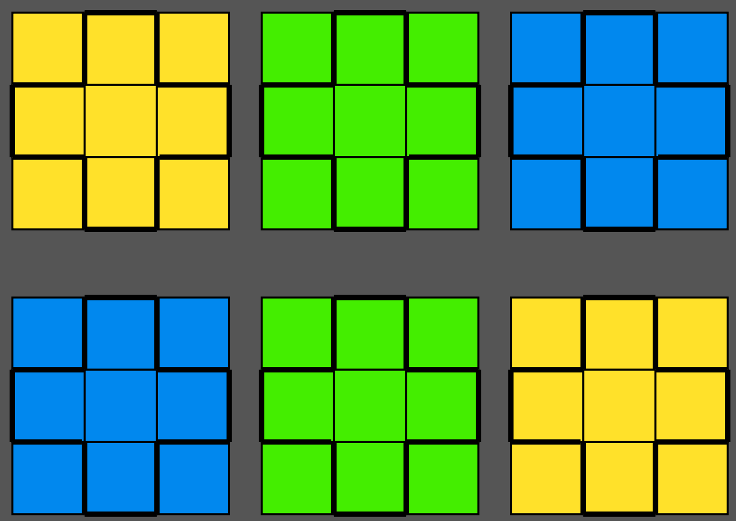
1 There are 6 faces to the 'altered cube' and the middle cube in each face is showing just a $1cm^2$ area. So that's $6cm^2$ to begin with. Then each of the other cubes is joined on by 2 of its faces – this means each of these other cubes is showing 4 faces : there are 12 of these 'stuck-on' cubes and $12 \times 4 = 48cm^2$.
So, total surface area = $6cm^2 + 48cm^2 = 54cm^2$.



PTO

2 Another way is by looking at the cube straight on from one direction eg looking along the line of one of the red arrows (see the diagram on the right). From each direction you'll see a total of 9 cubes facing you :





That's 6 straight-on views, each one showing 9 separate $1cm^2$ surfaces. So, total surface area = 6 x 9 = $54cm^2$.

ans 48 odd shops

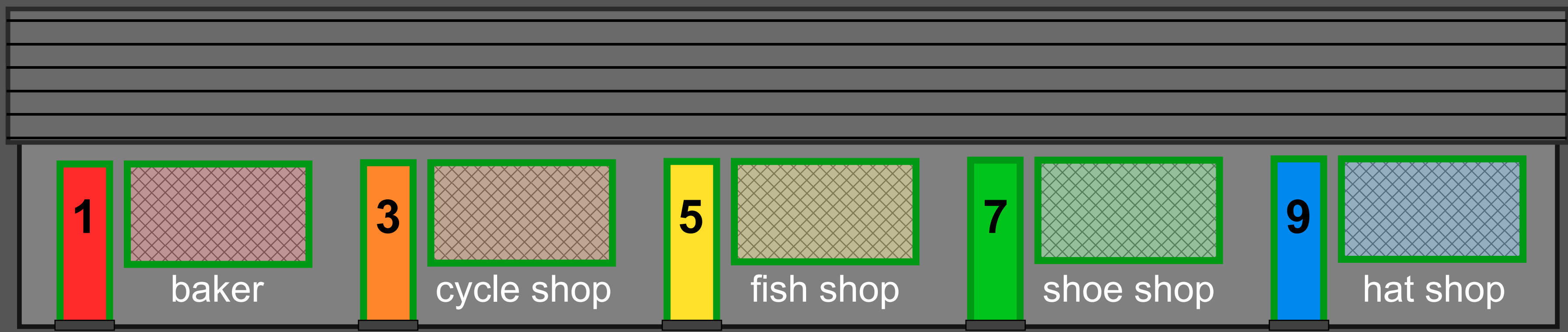
1 The baker is at no 1

2 The fish shop is between the cycle shop and the shoe shop – so we know these three shops must be together, arranged either CFS or SFC

3 So, remembering that B is at no 1, we could have : BCFSH or BSFCH

4 But we're told that C is not next to H, so that leaves just : BCFSH

5 We're also told that H and S are next-door neighbours; we haven't needed this extra bit of information – but it confirms our answer !



ans 49 Wrekin Ride

Looking at motorcyclists, we know that at the start there were 14 males and 13 females, as we show here :

			TOTALS
m	14		
f	13		

But of course we don't know how many cyclists there were – and we don't know how many were male and how many were female . . .

. . . except that the number of females was lower than 13. What to do next? Well, we could try some numbers and see the results . . .

Let's start with a low number. Suppose there were just 2 female cyclists; then there must have been 6 male cyclists, so now our table looks like this :

			TOTALS
m	14	6	20
f	13	2	15

Interesting – but as you can see when you look at the totals, not really what we want. We need the final total of males to be double the final total of females . . . So, the next thing is to try different numbers for female cyclists; let's try 2, 4, 6, 8 and so on.

PTO

ans 49 Wrekin Ride

		TOTALS
14	6	20
13	2	15

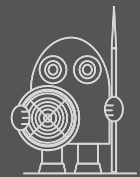
		TOTALS
14	12	26
13	4	17

		TOTALS
14	18	32
13	6	19

		TOTALS
14	24	38
13	8	21

		TOTALS
14	30	44
13	10	23

		TOTALS
14	36	50
13	12	25



PTO

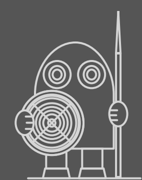
ans 49 Wrekin Ride

And there we have it ! Finally when we try 12 as the number of female cyclists, we get final male / female totals which work : overall number of females = 25 and overall number of males = 50. Here's the result from the last table shown again :

			TOTALS
m	14	36	50
f	13	12	25

– which means that in answer to the two problems we were set, altogether 25 females started the race and 12 of them were cyclists.

Of course, this isn't the only way to solve the proble, so you might have found a different way of getting to the answers . . .



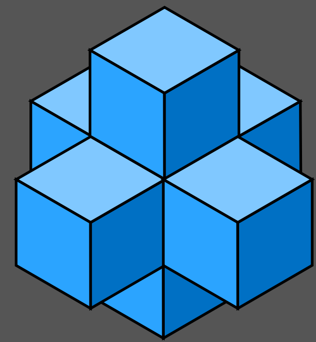
PTO

ANS 49 Wrekin Ride

Looking for patterns :

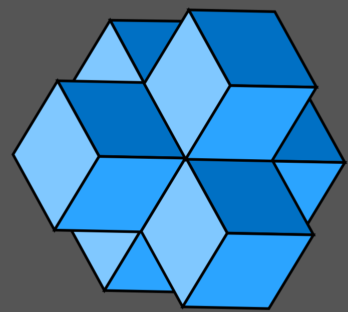
Just looking at totals, we're told that the female total should be half of the male total. In other words, what we need is to have 2 x female total = male total. If you go back to the page where we tried out different numbers of female cyclists, you'll see that as we tried out 2, 4, 6 . . . for the female cyclists, we got: 15, 17, 19 . . . for the female totals. Doubling these totals gave you numbers which were: 10 too high, then 8 too high, then 6 too high . . . So there we have a pattern – and we would expect that carrying on in this way for three more steps, would give us totals which would be 4, 2 and finally 0 too high. The 0 result happens when we've hit just the right number for the female cyclists, which of course is 12 – meaning that 25 females started the race.

ans 50 the blue cube

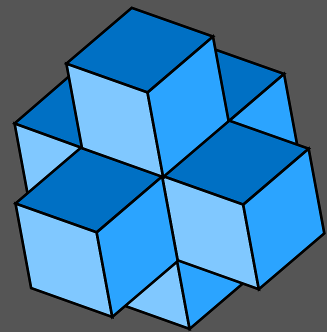


There are different ways of going about these problems.

1 With the new shape, you've got a 'very middle' cube with another cube stuck onto each of its 6 faces. So the new shape must have 7 cubes in it altogether. As a 3 x 3 x 3 large cube, the original shape must have had 27 1cm cubes to start off with. So Jenny must have removed 20 small cubes from that shape.



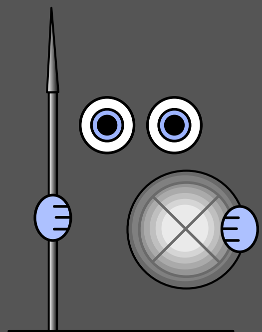
2 Jenny's new shape has 7 cubes in it. You can see this by looking at it – or you can reason it out like we've done above in answering question 1.



3 The new shape is completely symmetrical – it has 6 cubes sticking out from a 'very middle' cube. As you can see, each of these 'sticking-out' cubes is showing 5 faces to the world, and each of these faces has an area of 1cm^2. So, the new shape must have a surface area of 30cm^2. Another way of doing this is to notice that viewed directly from 6 different angles the shape always has 5 small squares facing you; so the total surface area must be 5 x 6 = 30cm^2. (Notice that, compared with the original large cube, the new shape has lost about three-quarters of its cubes but it still has more than half the original surface area.)

no
problem!
book

answer book



four winds