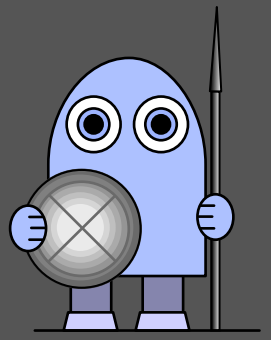
no problem !

book 1

problem book

four winds



four winds maths

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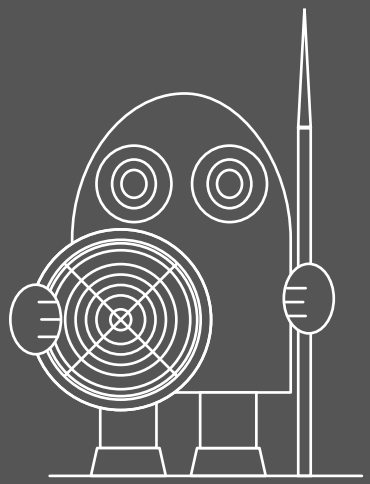


list of contents

01 your questions answered

Instead of a normal 'introduction' to the book, we've listed here answers to many of the questions you might ask at the beginning.

02 list of problems

This is a listing of all the problems, colour-coded to show you which area of the subject each problem focuses on.

03 problems 1 - 50

This is the main part of the book : here are the problems themselves, starting with a few easier ones and then going on to the harder challenges, before finishing with a small number of really quite difficult ones.

04 how to get started

This section has some helpful suggestions for those times when you're really stuck and just can't get going on a problem. Here you'll find a number of different ideas you can try, plus an easy way to remember them. Read this section before you start – or if you'd prefer, just dive straight in to the problems and come back to this when you need to.

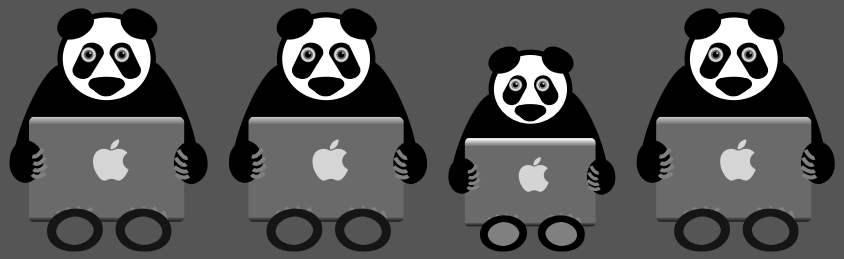
05 answers 1 - 50

Here are full answers and explanations for all the problems. There's no right way of solving any of them and so for many we show you different ways of tackling them. This is purely for your interest of course ~ we know that for any of these problems, you might well have a better way (one which works well for you).

your questions answered . . .

where do all the problems come from ?

Here at Four Winds we have a dedicated team, working night and day to produce maths books – books of problems, books about maths etc. The team members live on bamboo shoots and take a short break each day to do the Times Crossword. They live happily together in the Four Winds HQ on the edge of the Berwyn Hills in Wales. Below is a picture of the four team-members who have worked on this book :



These four have quite different jobs : The poser loves making up interesting problems, whilst the checker likes to check the answers and the explanations. The artist (called El Nepalo) does all the graphics. And last of all, the team leader puts everything together . . .

Are the questions all of the same difficulty?

The first ten or so are a little easier (to get you started) and the last ten are rather more challenging.

What are the problems about ?

Well, as you'd expect, there are all kinds of problems about number. Then there are logic problems, as well as problems about probability and statistics. And of course, there are problems about shape. The book gives you a good selection of all these topics.

Is there one definite answer to each problem ?

Some of the problems have a definite answer, while others have a number of right answers. One thing is for certain, though – there's no one right way of doing any of them! Your way might be quite different from anything we've put in the answers section. But that's quite ok !

Do you have to do these questions on your own?

Some problem-solvers say they prefer working alone. Others say they prefer working in pairs because it makes them feel more confident. So, you choose! Work on your own or work with someone else – do what suits you best !

Do you have to do the questions in order ?

It's completely up to you !

How long should you spend on a problem ?

Again, it's up to you! If you enjoy problem-solving, then just keep going until you've found a solution. But if you've done lots of working-out and you're still lost, just leave the problem alone for the time being and come back to it later. Or perhaps try explaining the problem to someone else to see if they have any ideas. And if you just don't know where to start on a problem, do have a look at the 'how to get started' section : it has some useful suggestions.

Is it a good idea to get help if you're really struggling ?

If you've been working hard on a problem and you're getting nowhere, it's a good idea to explain the problem to someone else (mum, dad, brother, sister or whoever) to see whether perhaps they can come up with something new. BUT it's important to tell them that you really want to solve the problem using the maths you already know. Suppose, for example, someone says to you, 'I would use algebra for this problem'. The best thing in this case is for you to say, 'thanks, but I need to do it using the maths I already know.' And surprisingly, it often happens that while you're explaining a problem to someone else, you suddenly realise what it is you need to do to solve it.

Do you need algebra for any of these problems ?

No, you really don't, not for any of them! You can do all the problems using the maths you already know.

Have these questions been tried out on anyone ?

Yes, of course! A number of willing volunteers cheerfully worked their way through the questions and afterwards shared their impressions with the Four Winds team. The suggestions they came up wtih helped us to make improvements to some of the questions and answers.

If you've got the right answer, is there any point reading the explanations ?

Yes, it's always good to see how someone else has tackled the problem, in case you come across new ideas. However, we don't ever want to give you the impression that our way of solving a problem is the only proper way, or even the best way. If you were going to drive from London to Manchester, you might look at a map and see that there are several different routes you could take (motorway, country roads or whatever). You'd choose a route which suited you. If someone then told you that a particular route was 'the only way' or 'the right way', you'd think they were either stupid or mad (or both). The same thing applies here.

Did the volunteers who worked through the questions have any advice for others ?

Yes, actually. They had various suggestions but the one thing they all agreed on was 'have a go!' – by which they meant don't be afraid to tackle any of the questions and even if at first you don't seem to be getting anywhere, do keep trying. Good advice!

list of problems

1 sports day
2 time's up
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PTO

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45 alfred & betty

46 a tale of two flutes

47 cutting corners

48 odd shops

49 wrekin ride

50 the blue cube

KEY

problems on logic, sets, combinations, permutations, probability, statistics

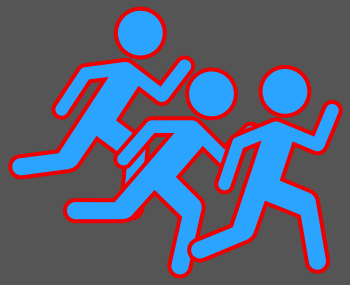
miscellaneous number problems (based on pre-algebra skills)

problems involving various aspects of shape (2-D and 3-D)

Sports Day

On Sports Day, the runners in the juniors' 200m race were Anne, Jeremy, Kate, Mary and Sanjit. This is how they finished :

- Kate came in second
- Mary was just behind Sanjit
- Anne was last



Write a list showing exactly who came where in the race.

2 time's up!

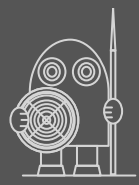
Navya arrives home late one evening after a birthday visit to the cinema. Tired but happy, she brushes her teeth and heads for her bedroom. As she jumps into bed, Navya notices that the digital clock on the bedside table is showing 23:56

'That's a lucky sign,' she says, 'The four separate digits add up to 16 and today happens to be my 16th birthday!'

$$2 + 3 + 5 + 6 = 16$$

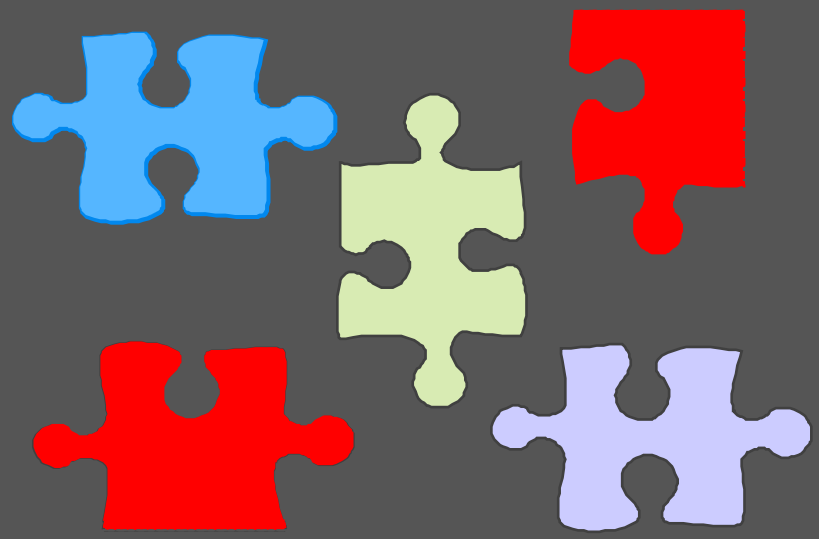
Thinking just of the 24-hour digital clock,

- Write down all the times where the digits add up to 24
- Write down all the times where the digits add up to 23
- Write down all the times where the digits add up to 22
- Write down all the times where the digits add up to 21
- Without writing them down, say how many digital times would give a total of 20



3 an edgy problem

Syed sits down one rainy day to solve a jigsaw puzzle which his uncle has given him. The finished puzzle is in the shape of a rectangle and all the pieces are roughly the same size. On the long side of the finished puzzle, you would count 54 pieces and on the short side of the finished puzzle, you would count 27 pieces.



1 How many pieces are there altogether in Syed's jigsaw?

2 As you can see, the corner pieces and the pieces along the edges have either one or two straight sides; these pieces are called 'edge pieces'. But there are also pieces which don't have any straight sides at all; these pieces are called 'inner pieces'. Just how many inner pieces are there in Syed's puzzle?

4 easy rider

Jamie works in a car factory in Detroit (that's in Michigan, USA); he likes to spend his weekends riding his scrambler bike in various off-road events around Detroit. Jamie's own bike is a Herald Rambler 250 but what he'd like to own one day is a Ducati Scrambler.

Jamie has never actually won an event but this season he's been doing pretty well. In the last seven club events his finishing positions have been as follows :

3rd, 2nd, 3rd, 4th, 2nd, 3rd, 4th

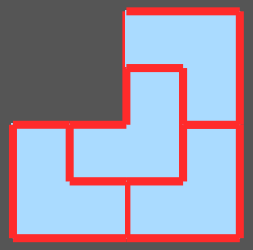
There are eight events in the season, so Jamie is busy training for the all-important last one. If he can end the season with an average finishing position of 3rd or better, he'll get a cash prize of $1200. This could go towards buying a Ducati ! What position must Jamie achieve in this last race if he's to end up with 3rd as his seasonal average ?

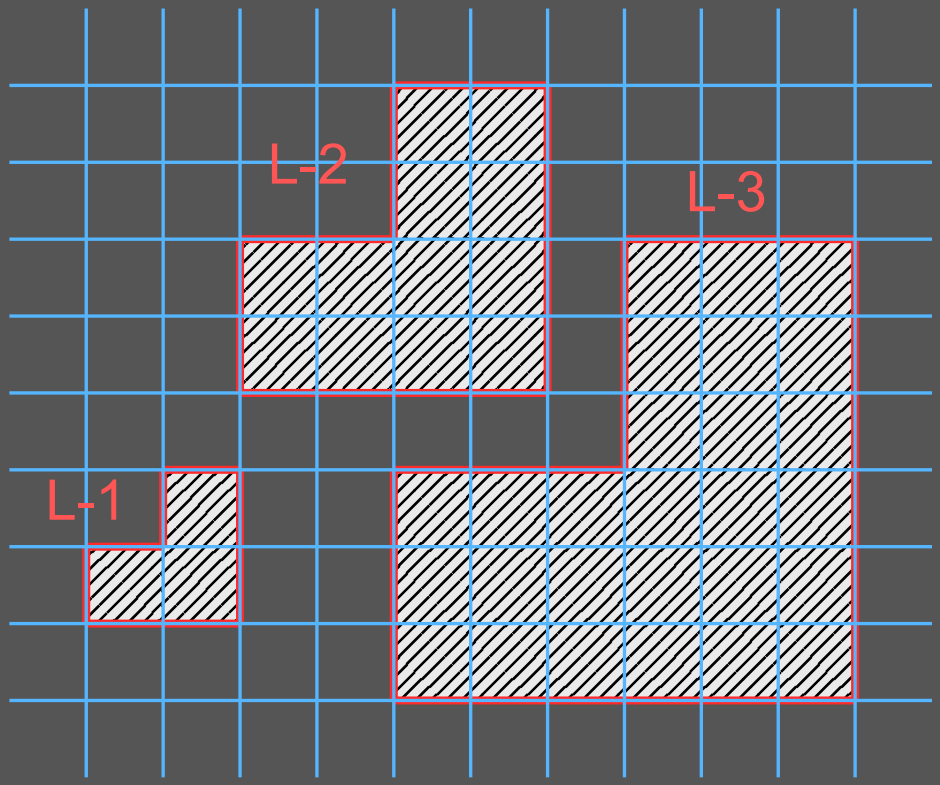
5 L is for learner

You'll need squared paper for this one . . .

Look at the figures on the right. You've got the same shape but in different sizes. Think of the smallest L-shape as a tile. If you have a number of these L-1 tiles you can completely fill a larger L-shape with them; of course, some of the tiles will need to be rotated or flipped over but it can be done.

Here's how for shape L-2 :



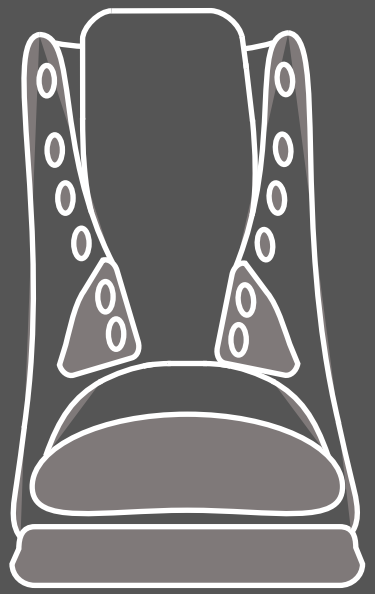


- Now try to fill the largest L-shape (shape L-3) with small tiles. Can you find more than one way of doing this?

- How many tiles were needed for shape L-2? – or for shape L-3? – and how about shape L-4 ? Describe the pattern you've found.

6 Jake's leaflets

Jake is a student and he's also a bit hard up. To make some money, he recently took a job delivering leaflets. It's not a great job : you have to deliver an awful lot of leaflets to earn just a little money but at least it's better than nothing! Or so Jake thought . . .



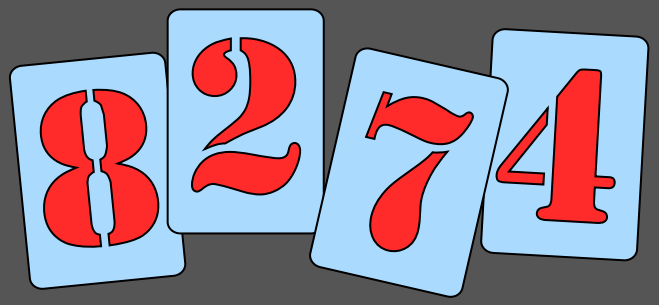
On his first day, Jake's feet hurt terribly. Before long, one of his toes was bleeding and his ankle was raw. But he kept going and kept going . . . until he'd delivered exactly half of the leaflets.

At this point Jake had some lunch (a quick sandwich in his case) and then bravely started off again. But bad luck was coming his way! When he'd delivered just 75 more leaflets, it started to rain, his boots began to leak – and really that was the end for Jake! He gave up (even though in his sack he still had 400 leaflets).

How many leaflets did Jake have at the start of his day?

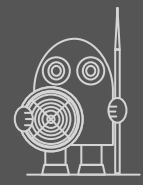
7 getting warmer . . .

Hassan's teacher gives him four number cards : 8, 2, 7 and 4, just like the ones pictured here. The teacher then gives Hassan a challenge : he has to use all four cards to make a number as close as possible to 7500. Hassan shuffles the cards about and finally settles on 7428. 'You can get nearer than that,' says the teacher – and straight away Hassan realises he needs to shuffle two of the cards around. You get the idea . . .



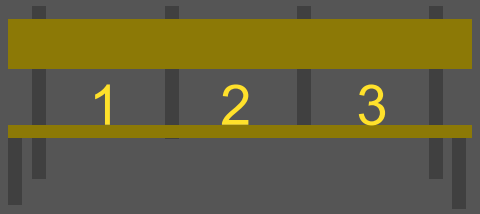
Using the same four cards as Hassan, write down the numbers you can make which come nearest to these :

- 7200
- 2900
- 5000
- 6250
- 4350
- 9270
- 8650
- 8351



8 in the hot seat . . .

Rosie, Sam and Tom are sitting on the short bench outside the headmaster's room. They are in trouble - all three have been late getting to their lessons. There are 3 seats on the bench :



The children have just plonked themselves down randomly. What's the probability that Rosie is sitting in place number 1?

* not everyone has come across probability, so just before the answer to this question, you'll find a simple explanation . . .

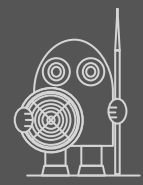
9 overlapping rings

Look carefully at the shape on the right. It's made up of three coloured rings linked together in a certain way. Imagine the rings are made of card and that you're looking at them from above.

- Suppose someone cuts the blue ring and removes it. Will the green and yellow rings still be linked?

Now look at the shape on the left. This one is made up of four coloured rings linked together in a certain way. Again, imagine the rings are made of card and that you're looking at them from above.

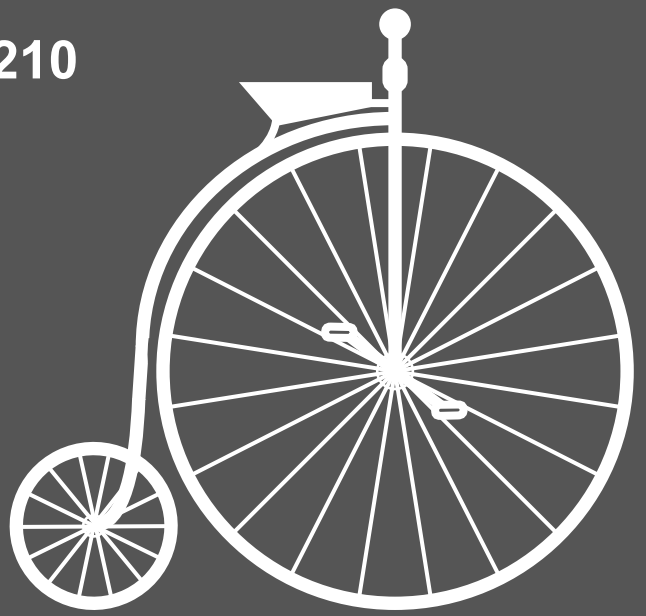
- Suppose someone cuts the blue ring and removes it. What will happen to the remaining three rings?
- Suppose instead someone cuts the blue ring and the red ring and then takes them both away. What will happen to the remaining two rings?



10 The Pedalman

The Maths Detectives are on the trail of a wicked bicycle thief they call The Pedalman. They have tracked him down to a street in London, called Old Montague Street, but they don't know which number house he's staying at. They do know for sure that it's one of these nine numbers :

23 49 62 87 105 121 169 188 210

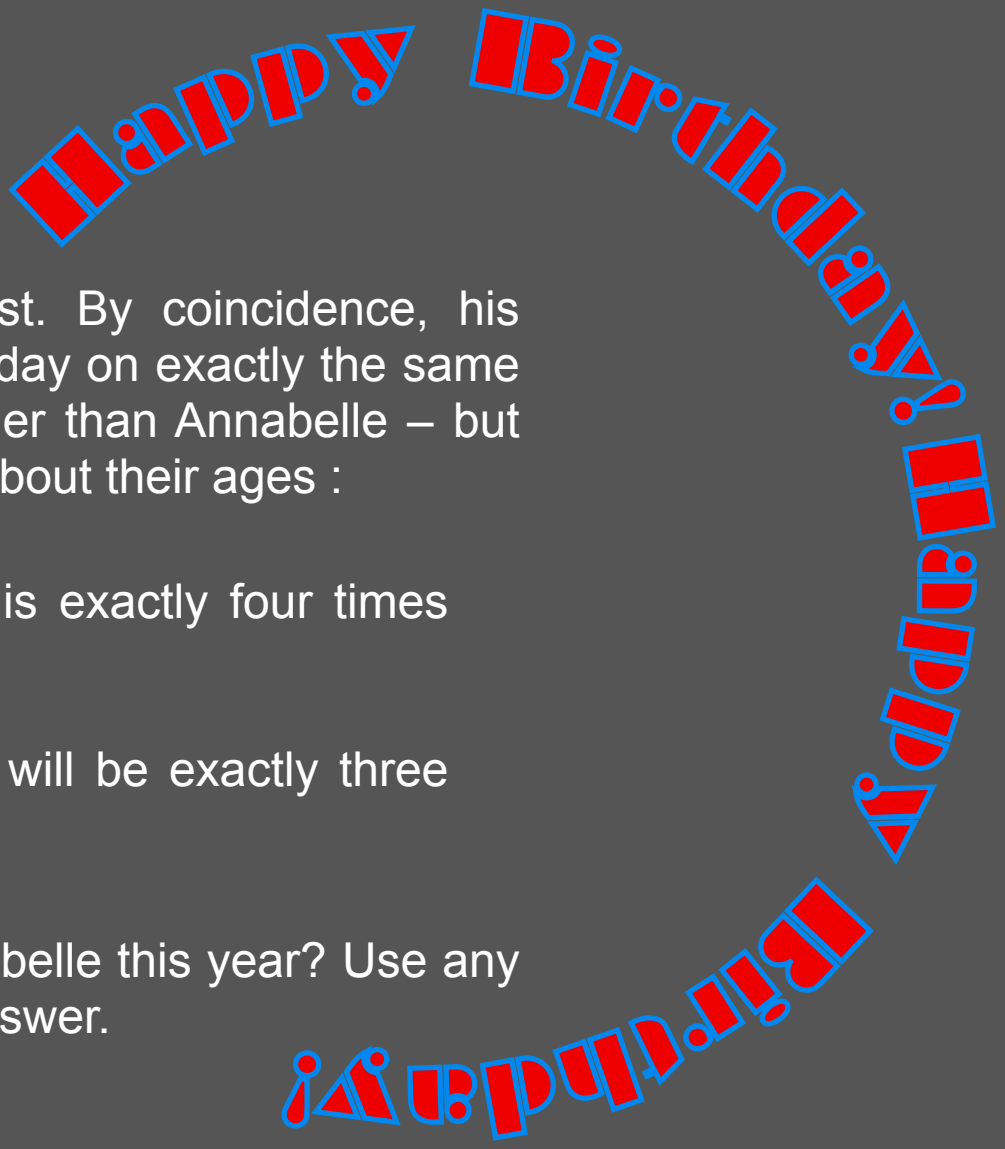


They're also sure of these important facts :

- the number is a not a square number
- the number is not a prime number
- the number is not a multiple of 7
- the number is a multiple of 3

Your problem : What's the number of the house where The Pedalman is staying?

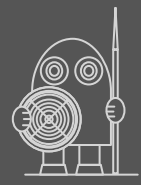
happy birthday, Ben !



Ben's birthday is on June 1st. By coincidence, his cousin Annabelle has her birthday on exactly the same day. As it happens, Ben is older than Annabelle – but here are two interesting facts about their ages :

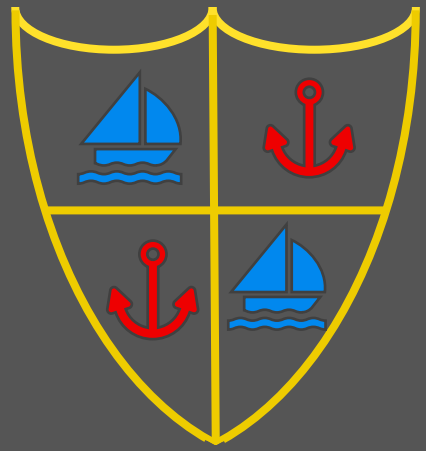
- This year, Ben's age is exactly four times Annabelle's age.
- Next year, Ben's age will be exactly three times Annabelle's age.

So, how old are Ben and Annabelle this year? Use any method you like to get to an answer.



12 sailing by . . .

On the right is the new badge of the Lymington Sailing Club. As you can see, the design uses just three different colours. The Club Committee members like the new design but they're not sure about the colour scheme. They are quite sure that :



- The two boats must be the same colour as each other.
- The two anchors must be the same colour as each other.
- The trim must be yellow.
- There must be three different colours in the final design.

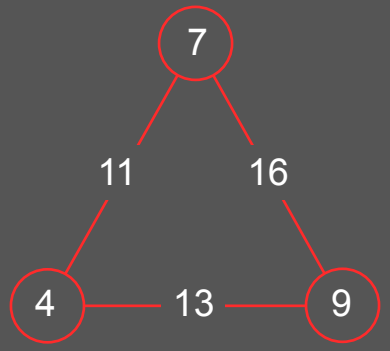
Here are the colours you're allowed to choose from :

How many different colour arrangements are possible ?

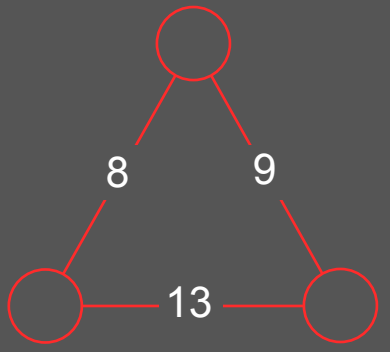
special note :
We're counting eg boats blue combined with anchors red as a completely different arrangement from boats red combined with anchors blue . . .

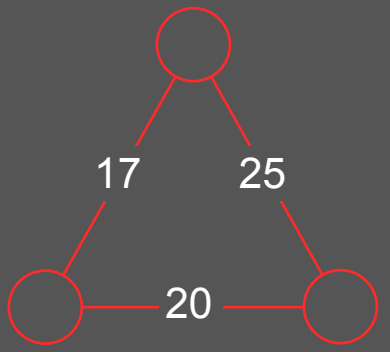
13 number triangles

Look at this 'number triangle' : as you can see, the numbers at the end of each line add together to give you the number between them.

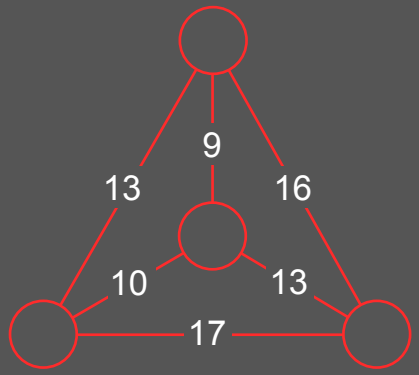


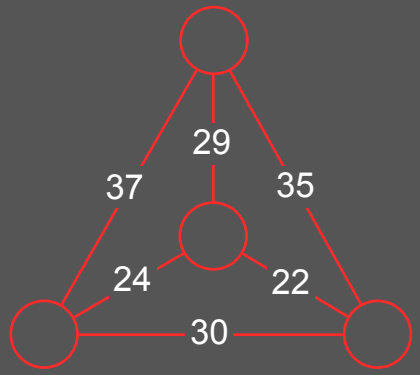
Now here are two more number triangles. This time, the 'totals in-between' are there but the numbers at the end of each line are missing. Try to work out what these missing numbers must be.

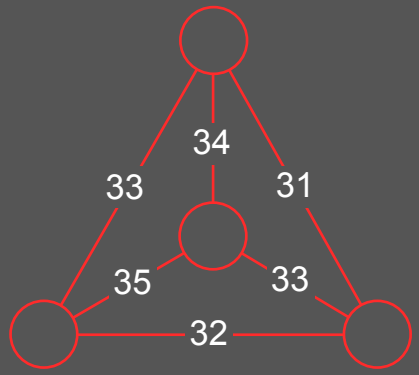


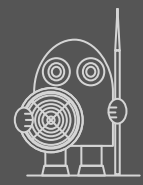


And to follow, something a bit harder! Try to work out what numbers should go into the circles.



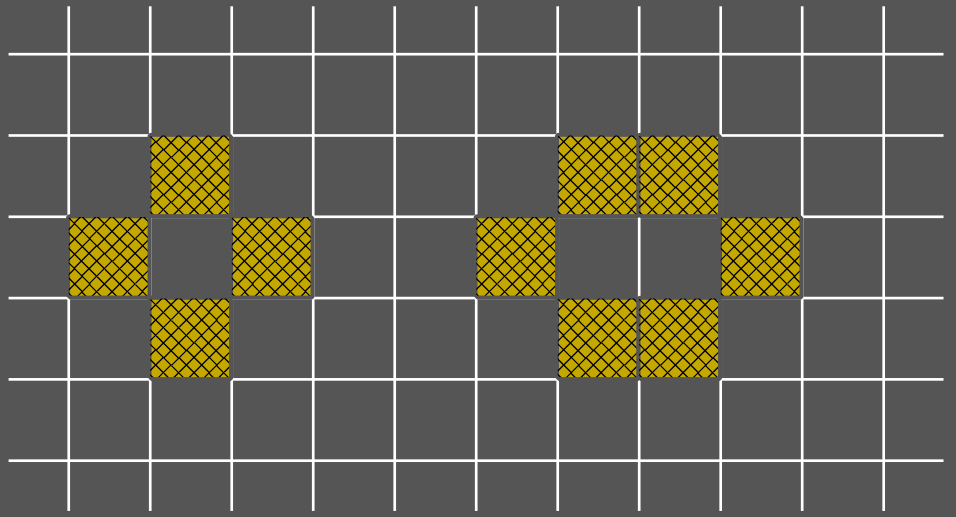






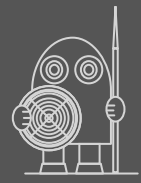
14 penning sheep

On MacHenry's Farm they sometimes need to make temporary pens for holding sheep. The sheep pens are made using square bales of hay. There are just two ways allowed for putting hay-bales together : they can be joined all along one side or they can be joined corner-to-corner, as in the diagram on the right.



As you can see from these two arrangemens, 4 hay-bales will let you enclose an area of 1 square and 6 hay-bales will let you enclose an area of 2 squares . . .

Now for your problem (and remember, you **must** follow the MacHenry's Farm rules): what's the **maximum** area you can enclose if you've got 9 bales of hay?



15 parcels-to-go

There are two parcel delivery firms operating in the North-West of England : there's 'ParcelDrop' and then there's 'Parcels-to-Go'. These two firms are in fierce competition but they don't quite go for the same size of parcel; you can tell this from their delivery charges, which are as follows :

ParcelDrop . . . £2 basic charge plus 50p per kg

Parcels-to-Go . . . 75p per kg (no basic charge)

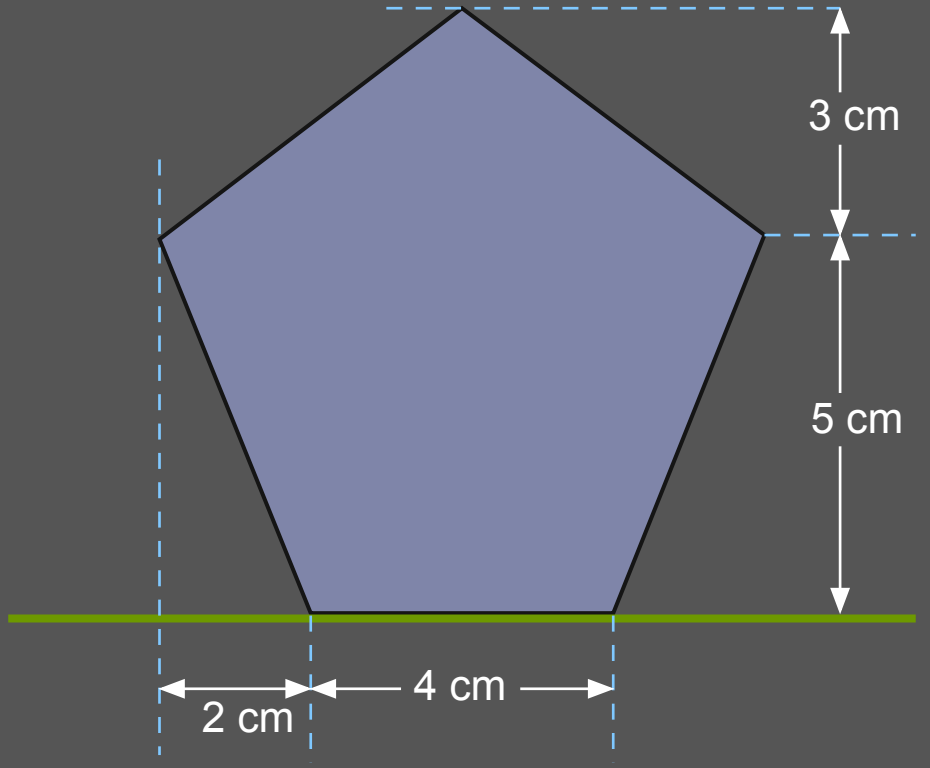
- Which of the delivery firms is cheaper for small parcels? (Take 'small' to mean just 1 or 2 kilograms.)

- There's a certain size of parcel which costs the same to send whichever of these two firms you decide to use. What size of parcel is this?

16 five-a-side

Look at the pentagon on the right. It's not a **regular** pentagon, as the angles are not all the same - but it has got **bilateral symmetry** (that's to say its left and right sides are mirror-images of each other). Here's your problem :

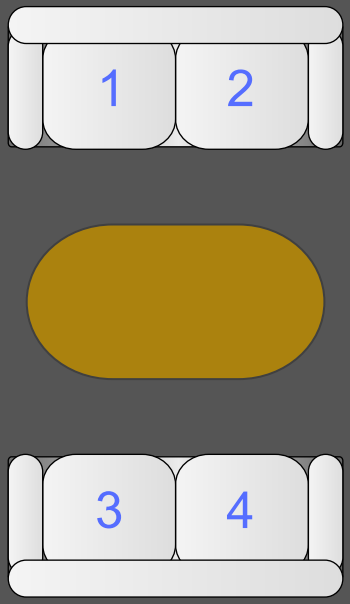
Using any method you like, find the exact area of this pentagon.



** You might find it easier to do this problem if you first copy the pentagon onto squared paper.*

17 Alice's party

It's the first day of the school holidays and Alice has decided to invite three of her friends round to drink coffee and talk about the cycling trip they are planning. Mum says they may use the small sitting room, where there are two sofas on either side of a coffee-table. Alice knows she must think hard about who sits where if the afternoon is to go well. She draws a simple diagram, like the one on the right, with the places numbered from 1 to 4. There are four in the group (Alice, Ben, Charlie and Debbie) and these are the conditions which Alice knows she has to remember :



1 *Debbie must have seat number 4 as it's nearest to the telephone and she's expecting a call.*

2 *Alice must never be seated next to Charlie, as they always quarrel.*

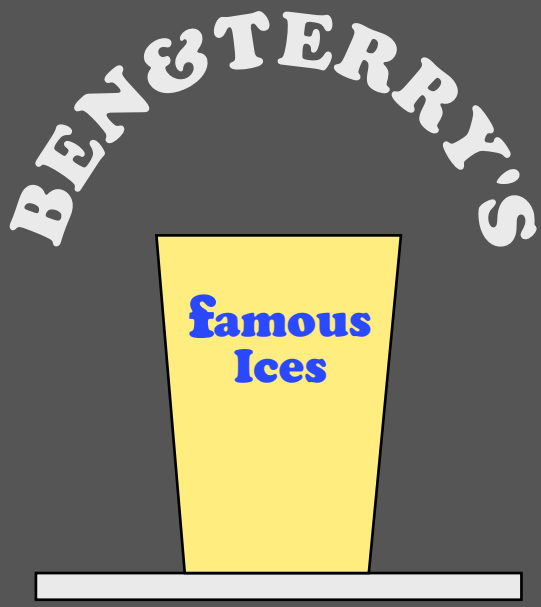
3 *Debbie will happily sit next to Alice or next to Ben but she will not sit next to Charlie.*

4 *Charlie must not sit facing Alice as he always pulls faces and makes her giggle.*

Your problem : who sits where ?

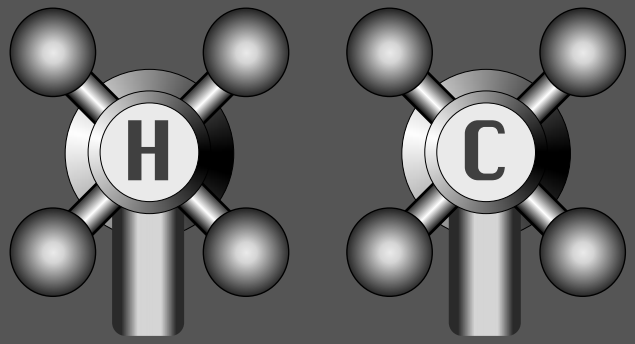
18 Ben and Terry . . .

This question is about a father-and-son ice-cream business based in the seaside town of Eastbourne. At one end of the beach, Ben is selling his ice-creams at the rate of one every 10 minutes. At the other end of the beach, his son Terry is selling ice-creams at the rate of one every 5 minutes. Between the two of them, how many ice-creams per hour are they selling? Working this out is proving to be a hard problem for Ben and Terry . . .



19 . . . and Sally

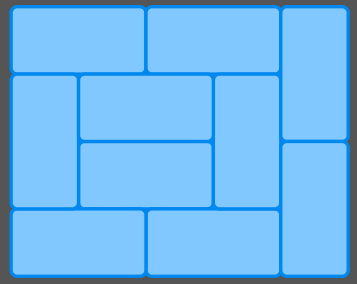
The last problem was about Ben and Terry. . . whereas this problem is about a girl called Sally, who loves to measure things. One day she times how long it takes for her bath to fill using just the hot tap; she finds it takes 12 minutes. When she uses just the cold tap to fill the bath, it takes only 6 minutes. If she were to switch on both taps together, how long would it take her to fill the bath?



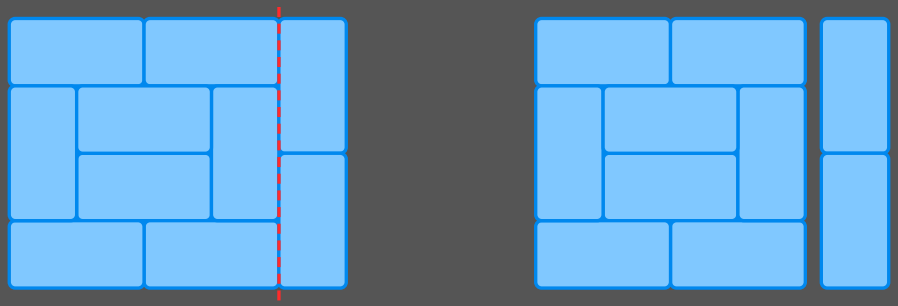
hint *: There's a link between this question and the last one. Take a hard look at how you solved the last question and then ask yourself : does that question suggest a different way of looking at the facts you're given here?*

20 whose fault is it?

Suppose you start off with a 5 x 4 rectangle and you want to tile it completely with 2 x 1 tiles. You'll obviously need 10 tiles and you'll probably find different ways of doing it. Here's one way :



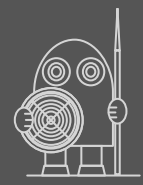
Something you might notice about this way of tiling the large rectangle is that it has a *fault line* in it. We'll show this a bit more clearly here :



A *fault line* is a line which runs right across the rectangle from one side to the other. In a way the fault line divides the rectangle into two separate parts.

Mathematicians have proved that you just can't tile a 5 x 4 rectangle with 2 x 1 tiles without getting a fault line somewhere. But you can tile a 6 x 5 rectangle completely with 2 x 1 tiles without getting a fault line. Can you find a way of doing this?

special note : You'll find turned-over dominoes make good plain 2 x 1 tiles if you want to solve this problem in a practical way – which is probably the best way of doing it.



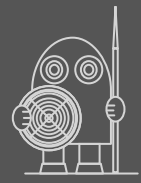
21 mr owl ate my metal worm

You know what a *palindrome* is : it's a word or a number which reads the same way from left to right and from right to left. In the world of numbers, 757 is an example of a *palindrome number* and 48084 is another one; and if it's years you're thinking of, then 1331 is an example of a *palindrome year*.

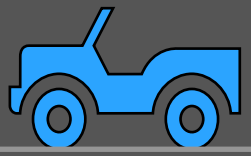
As you might know, there are *palindrome words*, like radar, rotor or reviver (though of course they don't all have to start with an *r)*. And there are even *palindrome sentences* where the letters are the same left to right and right to left; the title of this problem is one example (although here the grouping of letters is not symmetrical).

- Write down all the palindrome numbers between 100 and 2000 where the digits of the number add up to 8.

- 2002 was a palindrome year. When will the next one be ? And when was the last palindrome year before 2002 ?



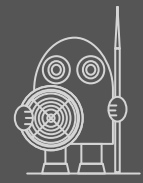
22 Ravi's mean question



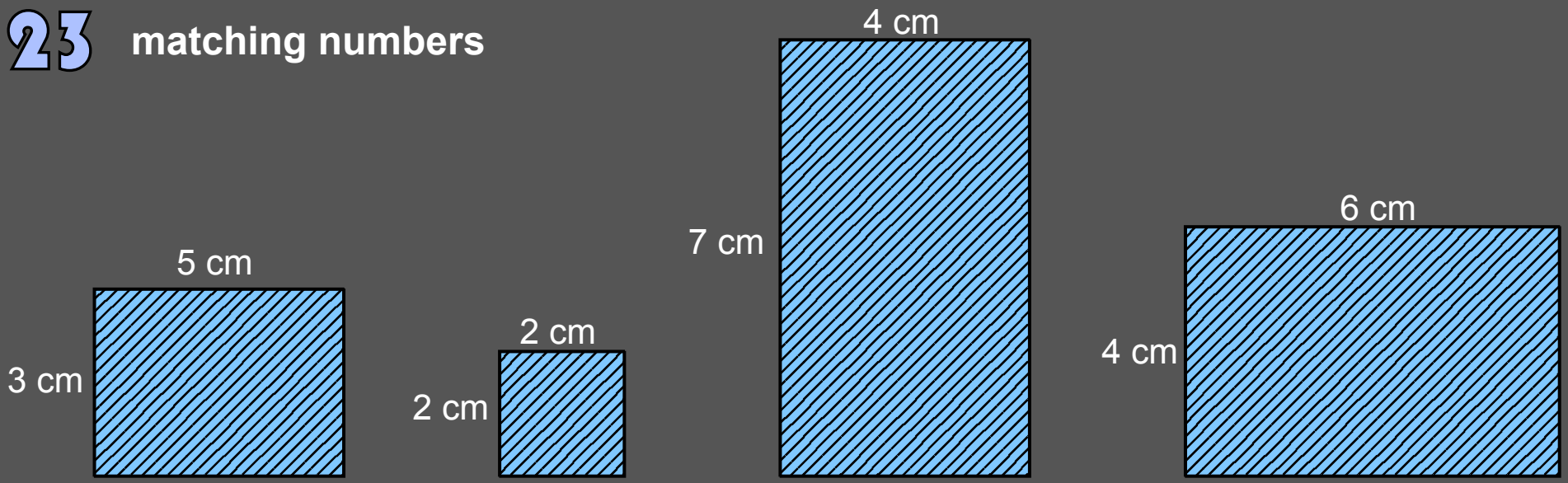
Ravi's teacher asks him to think of three numbers, 'Any three numbers', says the teacher, 'but the average of the three numbers must be 10 exactly!' Ravi does as the teacher asks, and he then gives the class these three pieces of information about his numbers :

- The mean of the three numbers is 10. (We knew that already!)
- The smallest number is 10 less than the largest number.
- The largest number is exactly double the middle number.

What are Ravi's three numbers?



23 matching numbers



area = 15 cm^2
perim = 16 cm

area = 4 cm^2
perim = 8 cm

area = 28 cm^2
perim = 22 cm

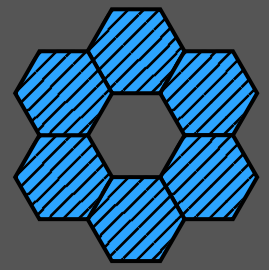
area = 24 cm^2
perim = 20 cm

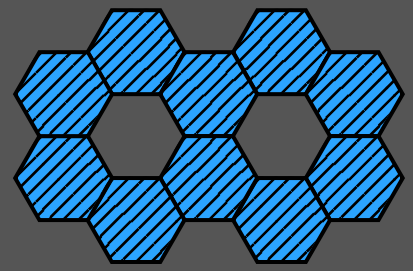
If you look at the rectangles above, you'll notice that for each rectangle the figure for the area and the figure for the perimeter are different. But there are rectangles where the area and the perimeter show the same number! Try to find two different rectangles where this happens. Keep to whole numbers for the lengths of the sides.

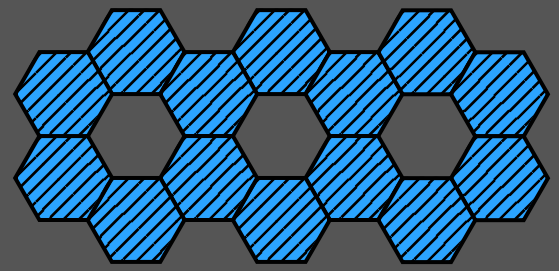
hint : Perhaps one of the rectangles you're looking for is a square. Remember, a square is a kind of rectangle.

24 six-a-side !

Mark has made the shapes on the right by putting together hexagon tiles. The tiles are completely symmetrical and each of their sides is exactly 1cm.



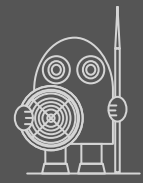




- How many tiles has Mark used in the first shape (call it the 'one-hole' shape)? How many has he used in the next one (the 'two-hole' shape)? And how many has he used in the 'three-hole' shape?

- Without drawing it, work out how many tiles Mark would use for a four-hole shape.

- Mark has a large box of these hexagon tiles and he spends some time making a shape like the others but longer. He uses 150 tiles. By looking at your previous answers, work out how many holes this new long shape must have.

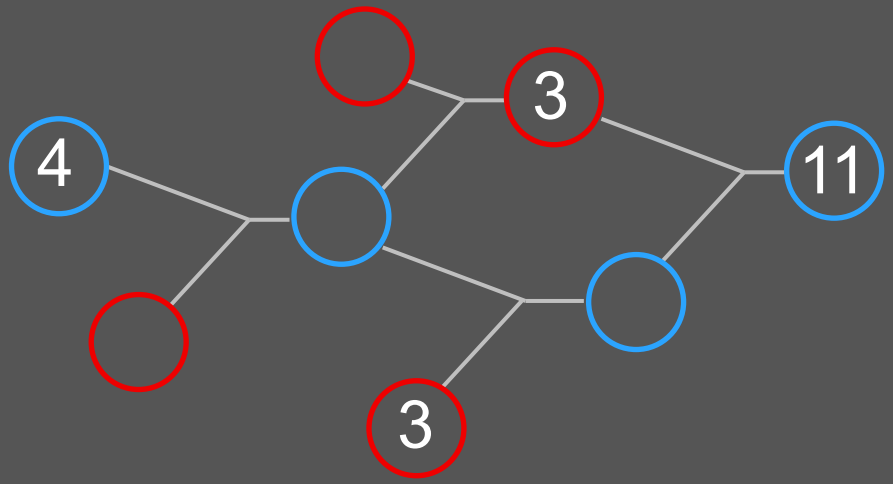


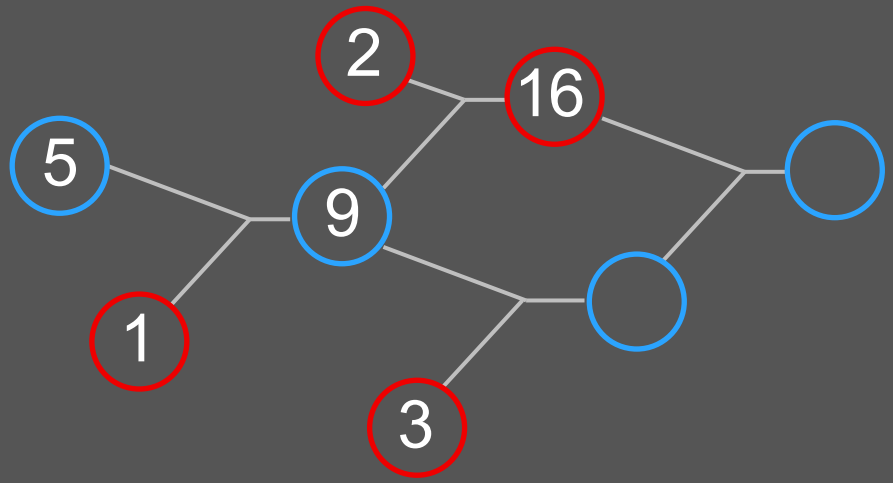
25 mapping webs 1

Perhaps you know what a **mapping web** is : you have a certain rule for combining numbers and using this rule you can build up a web of numbers. Starting with different rules and with different numbers, you get different mapping webs.

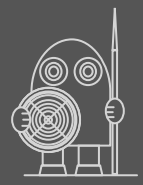
Now to explain the mapping web for this problem : wherever you see lines coming from a blue circle and a red circle and linking to another circle on their right, you just double the number in the blue circle and then subtract the number in the red circle, putting your answer in the circle on the right.

Here's an example :





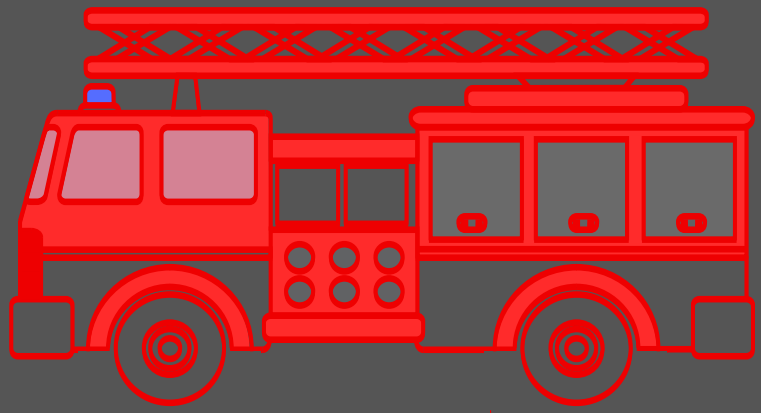
Your problem : make a quick copy of these two mapping webs and work out how to fill in the blanks.



26 three brothers

Simon, John and Peter are brothers. One of them is a teacher, one is a fireman and the third brother is a builder. One of the brothers lives at the seaside, one lives in the country and the other one lives in town. Here's some more information about them :

- John is not a teacher
- Simon does not live at the seaside
- Peter is not a fireman
- the teacher lives in town
- the fireman does not live in the country
- Simon is not a builder
- the teacher is not Peter



Try to work out which brother does which job and where each brother lives and then answer these questions :

1 Who is the fireman?

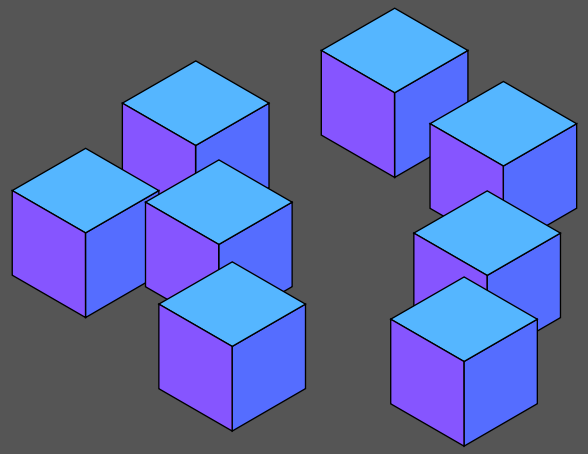
2 Where does the builder live?

3 What is Simon's job?

4 Which brother lives by the sea?

27 cube surfaces

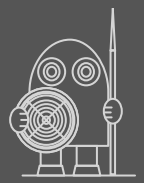
Sajid has some brightly coloured 1cm unit cubes. He finds that there are three different shapes of cuboid which he can make by joining eight of the unit cubes face to face.



- Make a sketch of each of Sajid's three cuboids.
- Carefully work out the surface area of each of the cuboids.
- What's the total surface area of the eight unit cubes before they're glued together?

** A **cuboid** (just in case you're not sure) is a rectangular 3-D shape. You can think of it as a cube that's been stretched. Most cardboard boxes are cuboid in shape.*

** You might not have come across **surface area** before now – but it's not a difficult idea : To work out the surface area of a solid shape like a cube or a cuboid, you just find the area of each separate face – and then add all these areas together. And that's surface area!*

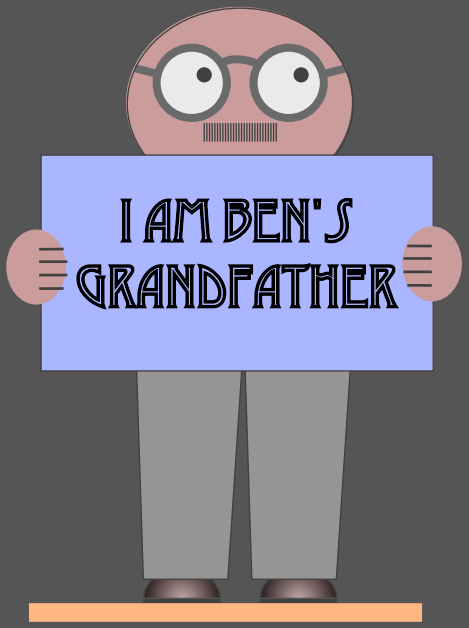


28 a grandson called Ben . . .

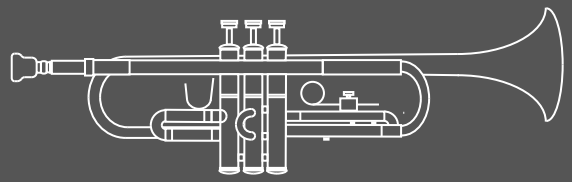
Ben has a grandfather whose house is just a short walk away from Ben's house. Here are some interesting facts about the grandfather's age :

- It's a prime number.
- When you add together the digits of this number, you get 8.
- John's parents are both aged 45.

Ben doesn't know how old his grandfather is. 'He could be any age,' says Ben. But of course, that's not true, is it? Using the information above, work out how old Ben's grandfather is most likely to be.



29 brass notes



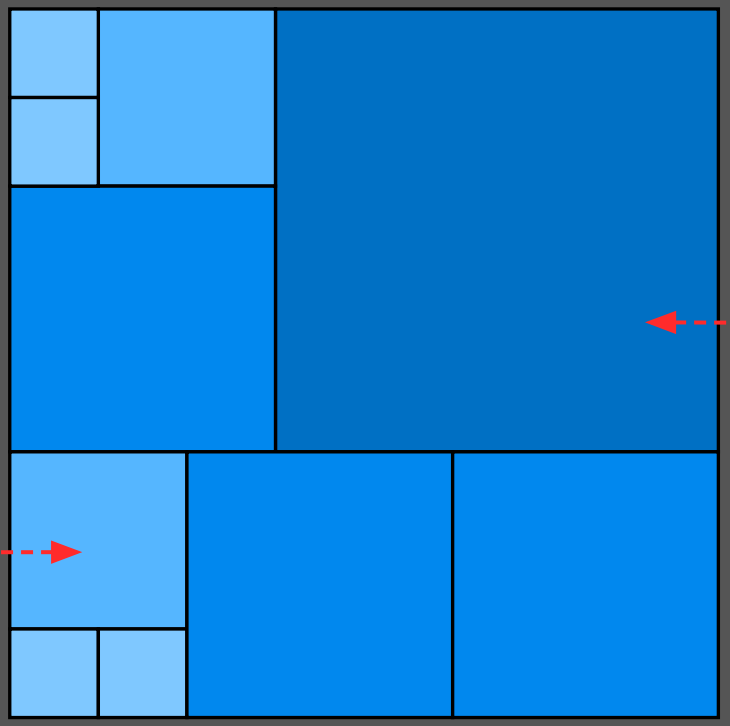
The trumpet has 3 main keys, or 'valves' as trumpeters call them. Each valve can be either up or down. So, how many different positions can you find for these three valves to be in? In other words, how many different arrangements (up or down) of these three valves can you find? Decide on your own way of listing the different possibilities – but do explain it!

30 all square

The larger square shown here is made up of ten smaller squares – and these smaller squares are of four different sizes :

problem :

if this square has an area of 36 square units . . .



. . . what's the area of this square ?

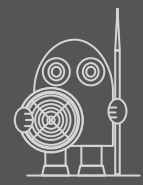
NOTE : squares with the same shade of blue have the same area as each other

31 Mr Average

In the Olympic Village (men's quarters) there's a small lift which takes athletes from the ground floor up to the shower rooms. There's a sign in the lift which says,'Max. Weight 360 kg'. Usually only one or two athletes at a time get into the lift but one morning four men at once get into the lift. This is a maths question, so let's call these four athletes Mr A, Mr B, Mr C and Mr D. Mr A is a shot put champion, Mr B is a long-distance runner, Mr C is a sumo wrestler and Mr D is a hurdles champion. Here's some information about what they weigh :

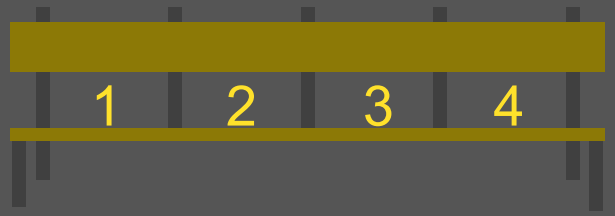
- Mr A weighs exactly the average (mean) for the group
- Mr B weighs 40 kg (and that's 80% of D's weight)
- Mr C weighs exactly double the group (mean) average

Your two problems are these : Firstly, what does Mr D weigh? And secondly, do the four men together exceed the maximum permitted (safe) weight for this small lift?



32 the long bench

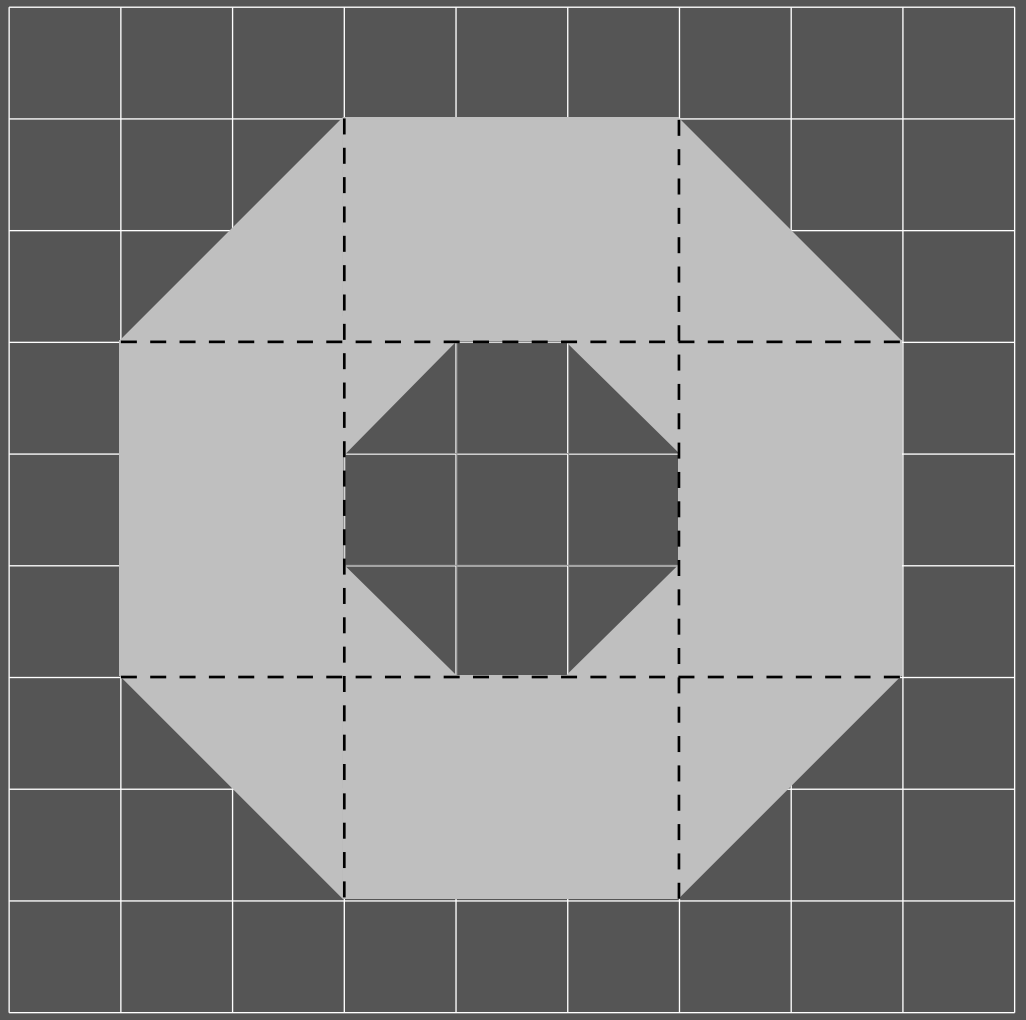
Rosie, Sam and Tom are sitting on the long bench outside the headmaster's room. They are in trouble once again - this time for bringing pet mice to school. There are 4 seats on this bench :



There's already someone sitting at one end of the bench but the children just plonk themselves down randomly on the remaining three seats. Assuming that seats 1 and 4 are equally likely to be occupied already, what's the probability that Tom sits down in place number 1?

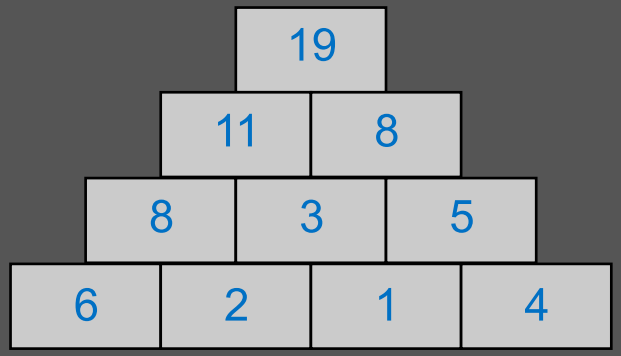
33 an octagon ring . . .

Look carefully at this shape, which has been drawn on a square grid. It's a kind of ring (you could call it an 'octagon ring') and all the corners fall exactly on grid points. Use any method you like to work out the area of this ring. Your answer will be an exact number of squares.

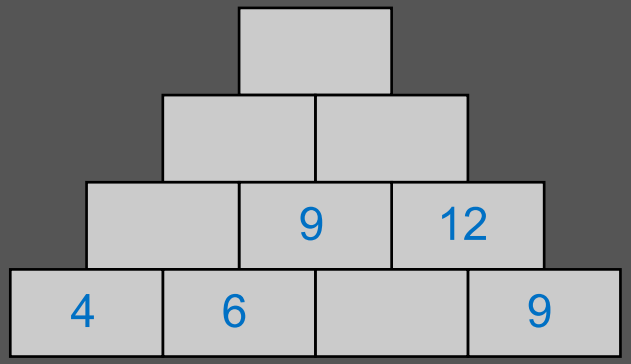


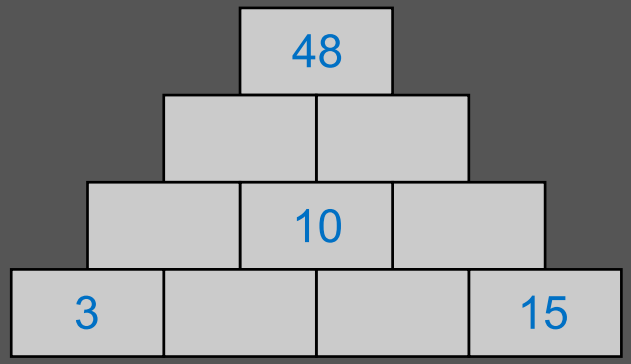
34 you're my number wall !

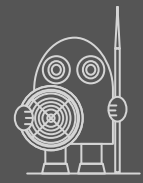
Perhaps you've come across **number walls**? The way a number wall works is this : when you have two bricks next to each other, you add their numbers together – and this total is the number for the brick resting on them. You'll see what this means as soon as you look at the number wall on the right. Easy number wall questions usually give you the numbers on the bottom row and some others above – and then ask you to fill in the gaps. It gets a little harder when there are numbers missing from the bottom row – and it becomes more difficult altogether when most of the mumbers are missing.



Here are your two problems. There's an easy one to begin with! You'll soon work out what the missing numbers must be. The second problem is harder. As you can see, you're given just four numbers and you have to find all the others. At first it looks impossible but – with a little hard thinking – it can be done!







35 remainders

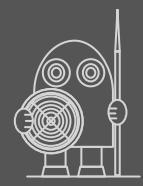
If you divide 11 by 2, you get a remainder of 1; divide 11 by 3, you get a remainder of 2; divide 11 by 4, you get a remainder of 3.

Exactly the same thing happens with 23 : divide it by 2, you get a remainder of 1; divide it by 3, you get a remainder of 2; divide it by 4, you get a remainder of 3.

You'll find you get the same pattern in the remainders if you divide 47 by 2, by 3 and by 4. Just try it!

Now here's the challenge. There is a number under 65 which is even more remarkable. If you divide it by 2, you get remainder 1; if you divide it by 3, you get remainder 2; if you divide it by 4, you get remainder 3; and if you divide it by 5, you get remainder 4.

Can you find this special number?



36 parking mad . . .

There are three cars parked outside Dr Brown's surgery. One of the cars is a sports car, one is a family saloon and one is an estate car. One of the cars belongs to Dr Brown, one belongs to Mr Smith and one belongs to Miss Green. Here's some information about them :

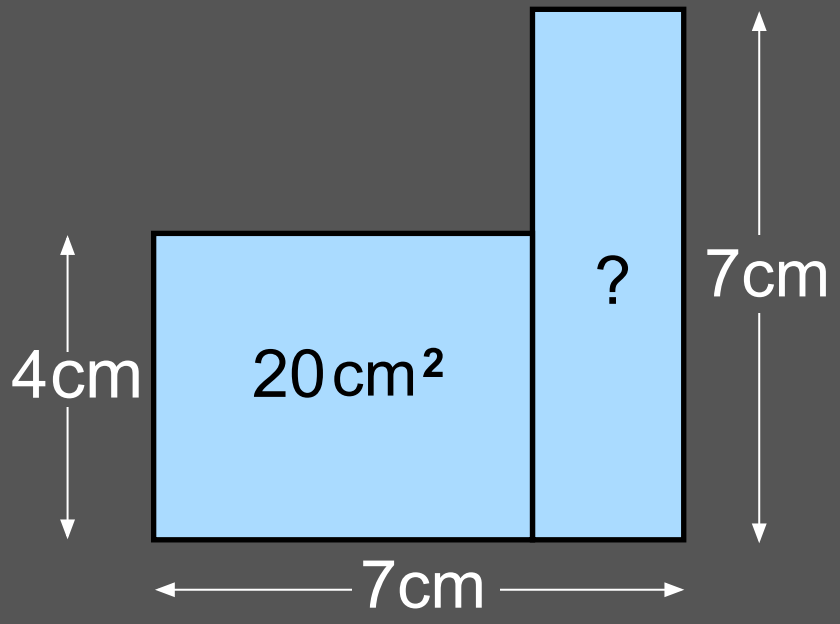
- the sports car is blue
- the red car is a family saloon
- one of the cars is yellow
- Dr Brown's car is not yellow and it's not a sports car
- Miss Green's car is not blue

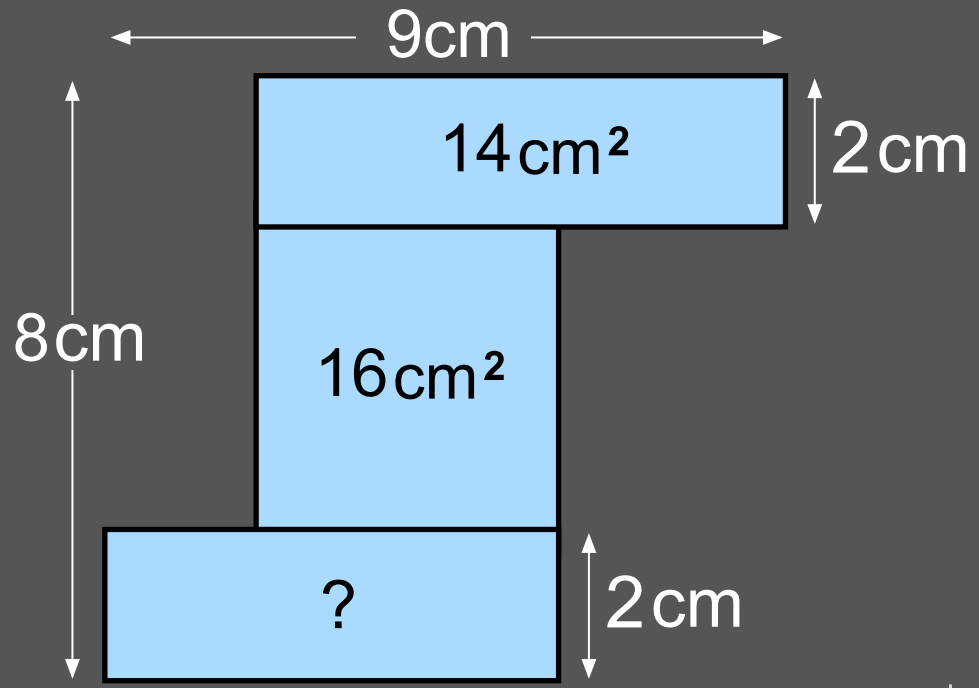
And your problem is : Who owns the sports car?

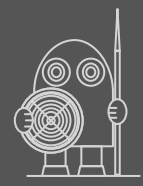
37 area mazes 1 & 2

Look at the two figures below. The one on the left is made up of two rectangles and the one on the right is made up of three rectangles.

Some lengths are shown on these figures and so are some areas. For each of these figures, you have to to work out the unknown area. With each figure, you will need to use the lengths and areas you're given in order to work out other lengths, until finally you have enough information to find the area of the last rectangle.

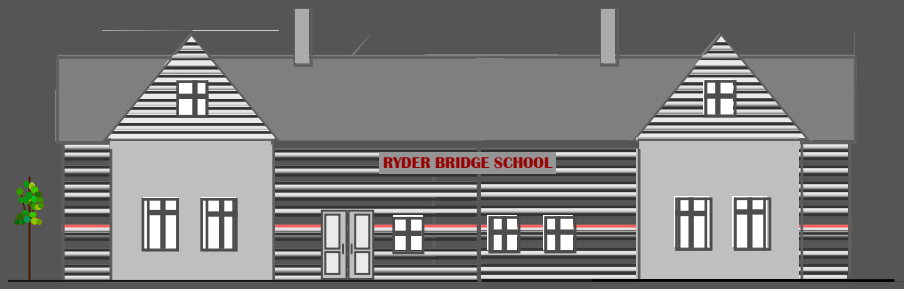






38 teaching equality

Of the 36 teachers who work at Ryder Bridge School, exactly half are men and half are women. The School is open every weekday (that's ten half-days in all) and on any half-day three-quarters of the teachers will be present.



- Mr Gladstone, the Schools Inspector, turns up at Ryder Bridge one thursday afternoon. Can he be sure there will be a female teacher present ?

- What's the largest number of female teachers who might be present on any particular half-day?

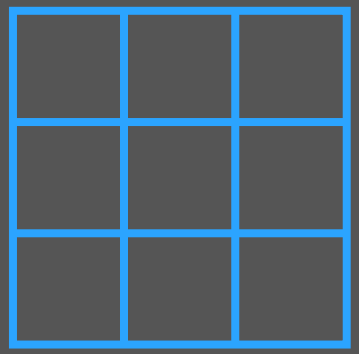
- What's the smallest number of female teachers who might be present on any particular half-day?

39 DIY magic square

PROBLEM 1 : You've probably come across 'magic squares' before. As you know, the numbers along each row and down each column, and along each diagonal, must add up to the same total, which we call the 'magic total'. The square on the right is a magic square but some of the numbers have been rubbed out. Your problem is to fill in the gaps.

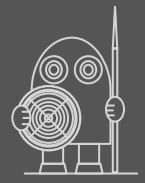
		10
15		1
		13

Once you've completed the magic square, just answer this simple question : what's the connection between the number in the centre and the magic total? The connection you've spotted here is actually the same for all 3 x 3 magic squares !



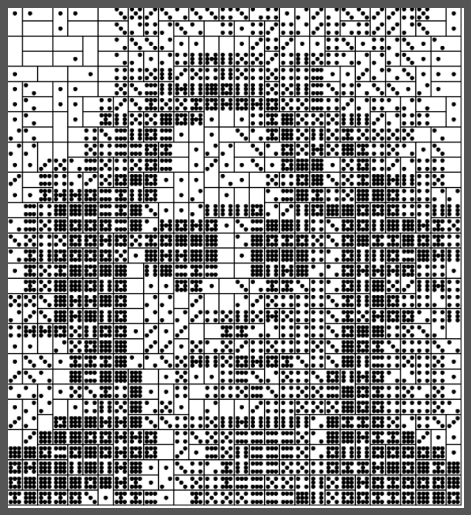
PROBLEM 2 : This problem is a little harder. First, draw yourself an empty square grid like the one on the left. Now try to make up a magic square of your own, using each of the numbers 0 to 8 once and once only. Use any method you like – and remember, if it gets you to a magic square which works, then it's a good method!

hint : if you've spent a long time on this one and you're getting nowhere, look again at the interesting connection you found at the end of problem 1 . . . it might just be useful . . .

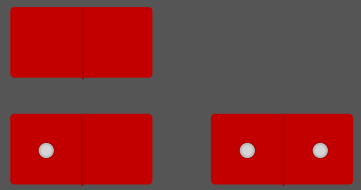


40 domino faces

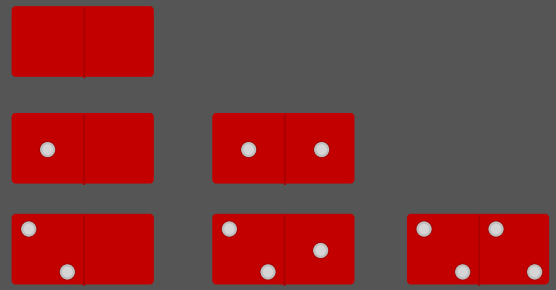
You probably know that an ordinary set of dominoes (sometimes called '6-spot dominoes') has 28 dominoes in it. There are many different games you can play with a set of 6-spot dominoes; you'll find them all on the internet. Of course, 6-spot dominoes aren't the only kind you can buy; some people like to play with 9-spot dominoes. On the right you can see a portrait of John Lennon which mathematics man Robert Bosch made using nine sets of 9-spot dominoes. When you have a few moments to spare, take a look at his website : *DominoArtwork.com*



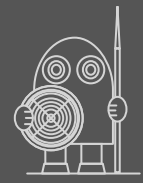
Here's the set of 1-spot dominoes. As you can see, this set has just three dominoes in it.



And here's the set of 2-spot dominoes. As you can see, this set has six dominoes in it.



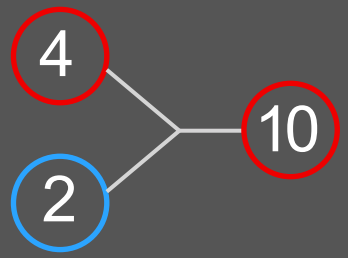
- How many dominoes are there in the 0-spot set?
- Make a quick drawing of the set of 3-spot dominoes.
- How many dominoes are there in the 5-spot set? (Don't draw them!)
- Without drawing them, work out how many dominoes there are in the 9-spot set.

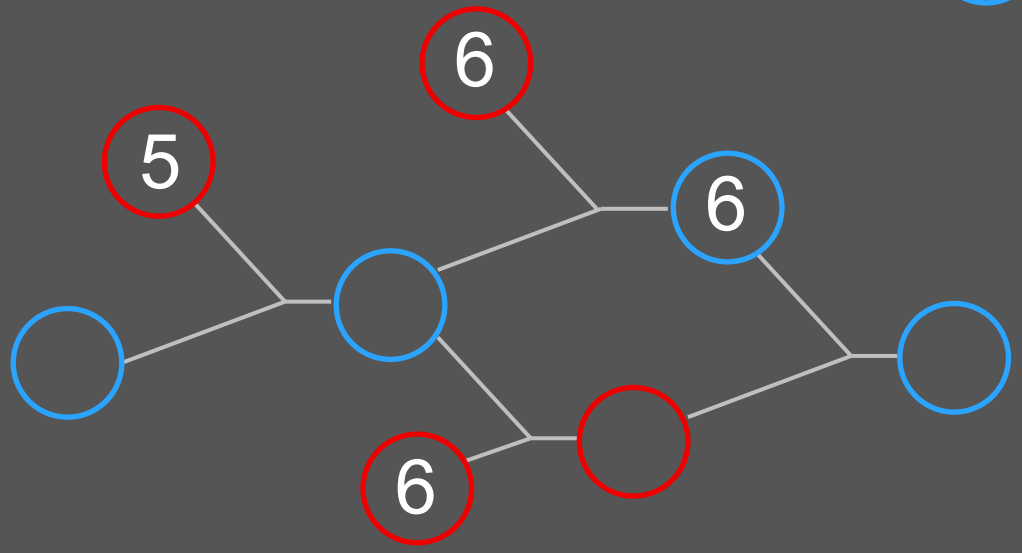


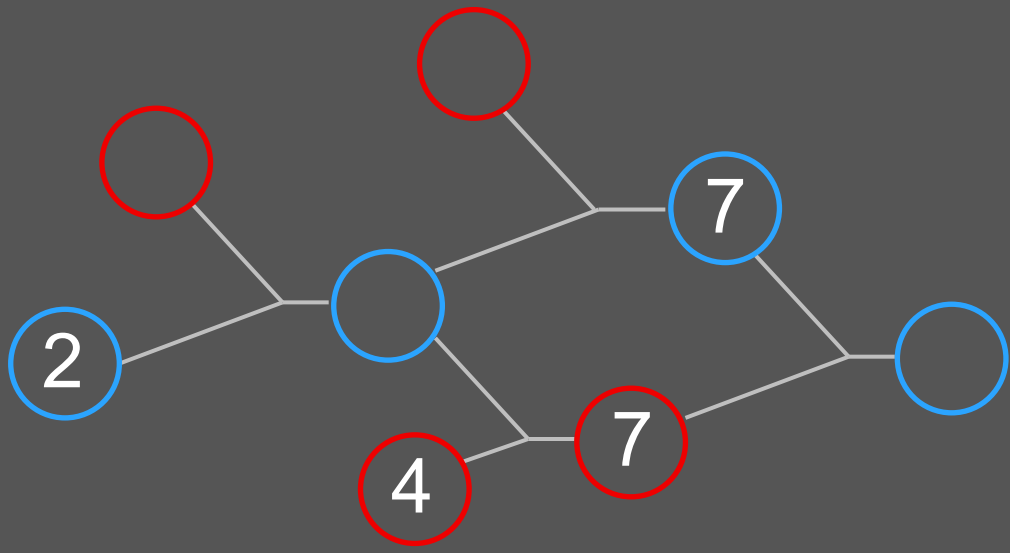
41 mapping webs 2

You've seen **mapping webs** before but here's how this one works : wherever mapping lines come from a blue circle and a red circle, you just square the number in the red circle and then subtract three times the number in the blue circle; the lines lead to another circle on the right and this is where the answer goes.

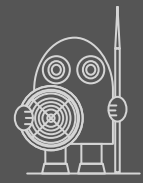
Here's an example :







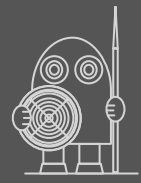
Your problem: make a quick copy of these two mapping webs and try to fill in the blanks.



42 cube calendar

On the shelf behind his desk, Mr Pascal (the maths master) has a daily calendar made of wooden cubes. The numerals on the first two cubes tell you which day of the month it is; and the letters on the remaining three cubes tell you the month. Mr Pascal made the calendar himself, using five plain wooden cubes (taken from his son's toybox!) and some white stick-on pvc letters.

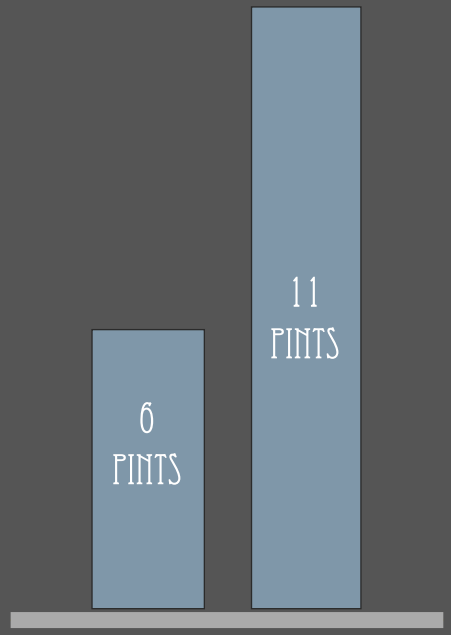
Let's just think about the first two cubes : you have to be able to show all the dates in the month from the first to the thirty-first (no months have more than thirty-one days). To do this, you'll have to think carefully about which numerals you're going to stick onto which cubes. See if you can find a way of doing this so that all dates in the month are possible. Remember, each cube has six faces.



43 10-pint target

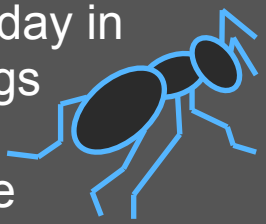
Nowadays we usually measure capacities of containers or volumes of liquid using litres as our basic unit . . . but in the not-so-distant past these things were measured in pints and gallons . . . and so to our problem :

Imagine you have two jars, a 6-pint jar (short and fat) and an 11-pint jar (tall and thin). Using nothing but these two jars, how could you measure out exactly 10 pints of water simply by pouring backwards and forwards? By the way : you are allowed as much water as you like from the tap and you may pour liquid away if you need to.

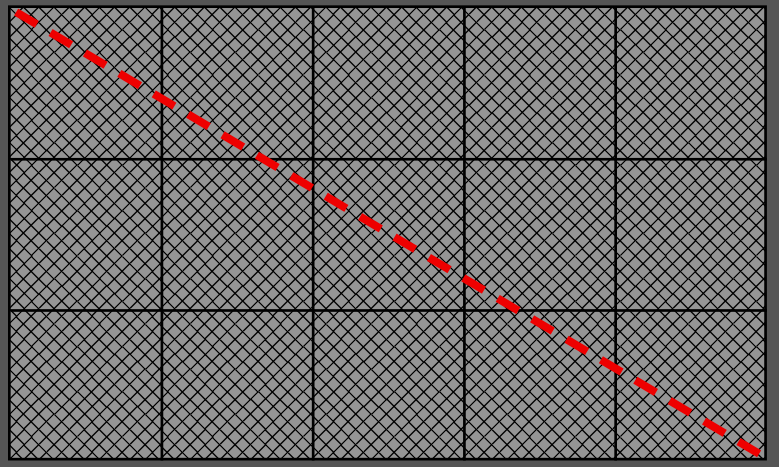


note *This is a very well-known problem. The numbers may be different but the wording is always something like : given an x-pint jar and a y-pint jar, how could you measure out exactly z pints? Of course, the units used might be litres instead of pints.*

44 ant and rectangle



Boris is an ant who loves maths. One day in the maths room he finds some drawings of rectangles, each one made up of squares. He thinks he'll investigate. He goes over to the first rectangle; then, starting at one corner, he walks straight across to the opposite corner, as in the drawing here :

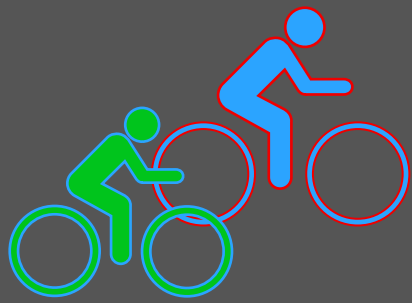


How many squares does Boris cross? You can see that the answer is 7. All the rectangles the children have drawn are *prime rectangles* – this means they are rectangles where the two sides don't have any factors in common (except 1 of course). So, we're looking at rectangles like 7 x 2 or 5 x 3 or 4 x 5 – but not like 6 x 4 (because 2 goes into both 6 and 4) or 6 x 9 (because 3 goes into both 6 and 9).

Draw a few prime rectangles and see how many squares you cross going from one corner to the opposite corner. Can you find a simple rule which lets you work out, from the lengths of the sides, how many squares you'll cross?

45 Alfred and Betty

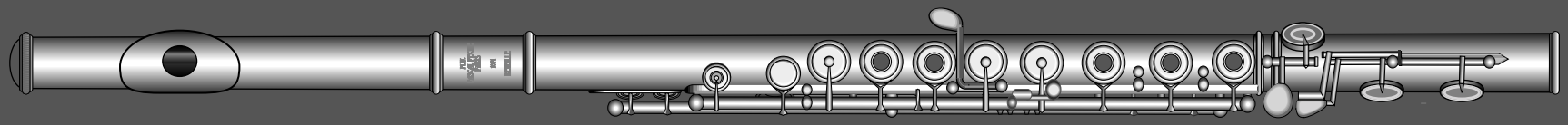
Alfred and his wife Betty, keen to keep themselves fit and healthy, have recently bought a couple of rather handsome mountain-bikes. Alfred's bike actually cost rather more than his wife's but then, as he explained to her, he's a more experienced cyclist and he needs the extra gears. Now, every Sunday morning the two of them go cycling in their local park. There they have a long, smooth, circular path which goes right round the park and which is perfect for cycling.



Alfred (either because he's stronger or perhaps because his bike is lighter) is twice as fast as Betty. In fact, when they cycle round the park, the two of them always start off together and always finish together – except of course that in between Betty has completed one full circuit of the park and Alfred has completed two! It's the same every week!

problem : Where on the circuit does Alfred overtake Betty?

46 a tale of two flutes



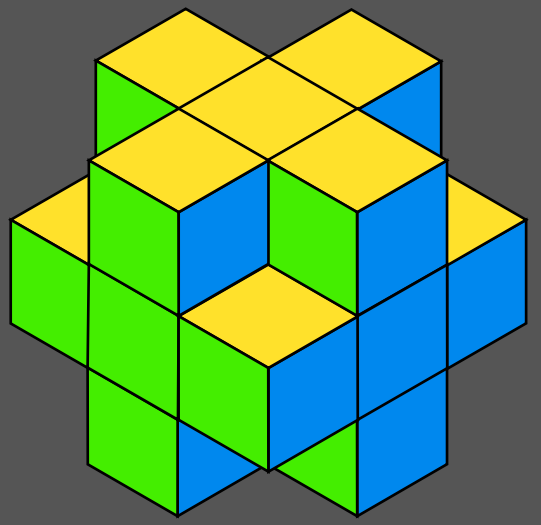
Roland has a silver flute which he plays every day. He also has an old flute and one day, as he's hard-up, he decides to sell this old flute. It's quite tarnished and some of the keys don't work, so he knows he can't ask too much for it. In his mind he settles on a price which he thinks is fair.

So, Roland puts the flute up for sale on eBay – and waits for the bids to come in. By the end of 10 days, there are two bids. One of them is a bit low; in fact, it would have to be 25% higher to equal Roland's target. The other bid is quite high; in fact, you'd have to reduce it by 25% to equal Roland's target. The high bid is £32 higher than the low bid.

What exactly was the price Roland had in mind?

47 cutting corners

Sally had a box of 1cm cubes and she used some of them to make a 3 x 3 x 3 cube. To hold the cube together, she used some blu-tac from her desk. While Sally was out with her friends, her mischievous younger brother Jack went into her room and decided to pull all the corners off Sally's cube. On the right you can see a picture of the shape which remained. Jack thought it was quite an interesting shape.



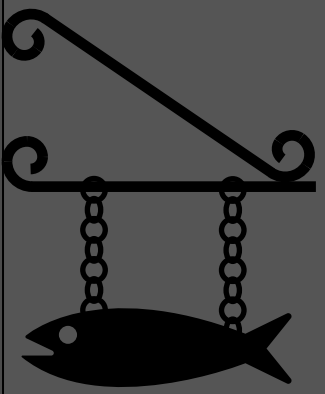
- How many small cubes did Jack remove from the original large cube which Sally had made?
- How many small cubes are there now in this new (more interesting) shape?
- You know what surface area is. So, what's the total surface area of the shape on the right?

** to get the surface area of a shape, you just add together the areas of all the different faces of the shape.*

48 odd shops

At one end of York Road there's a row of five small shops. These shops are numbered 1, 3, 5, 7 and 9. One of the shops is a fish shop, one is a hat shop, one is a baker, one is a cycle shop and one is a shoe shop. Here are some useful facts about the positions of the shops :

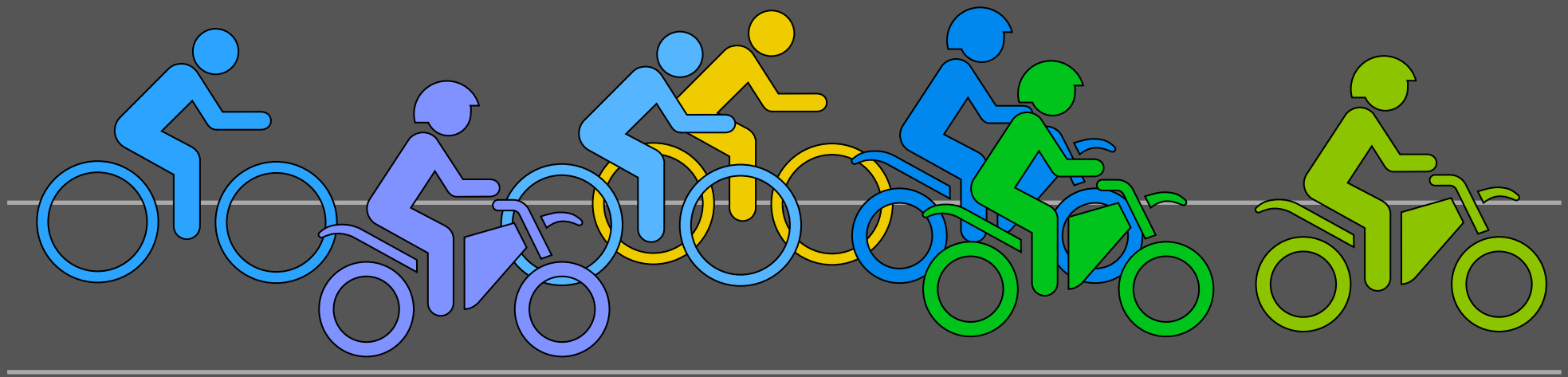
- the fish shop is between the cycle shop and the shoe shop
- the cycle shop is not next to the hat shop
- the baker is not next to the hat shop
- the baker is at one end of the row, at number 1 in fact
- the hat shop and the shoe shop are next-door neighbours



Use this information to work out which shops go with which numbers.

49 Wrekin Ride . . .

On the first day of March each year, a number of cyclists and motorcyclists set off on a hillside ride around the Wrekin, a hill in Shropshire (It's pronounced *ree–kin*). On the last Wrekin Ride, the cyclists and motorcyclists all set off together – but obviously the motorcyclists soon got ahead.

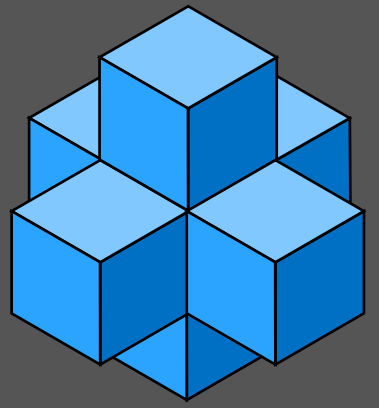


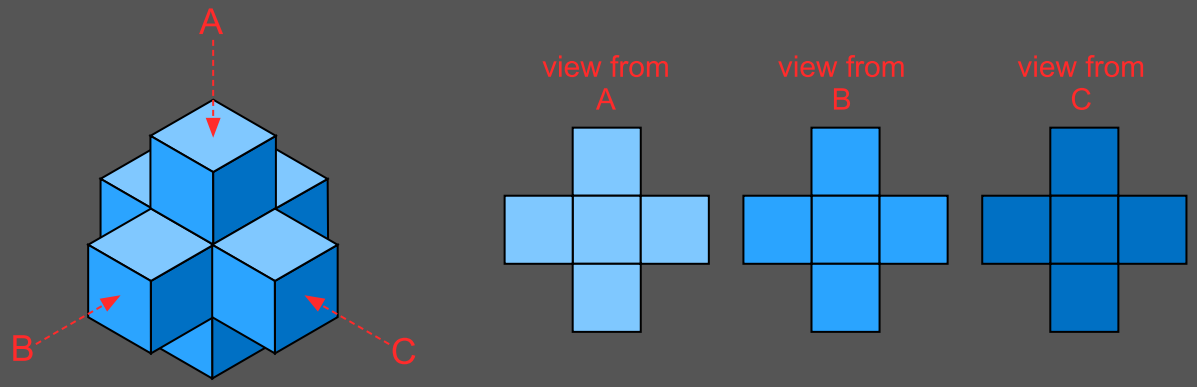
- On the last Wrekin Ride there were twice as many males as females.
- Of the motorcyclists, 14 were male and 13 were female, making 27 in all.
- Among the cyclists, there were three times as many males as females.
- There were more females on motorbikes than on bicycles.

Altogether, how many female riders took part? And how many of them were cyclists?

50 the blue cube

Look at this interesting shape from Jenny's collection. The shape started life as a blue 3 x 3 x 3 cube, made from individual 1cm cubes – but then Jenny decided to pull off every small cube except the middle one of each face. The shape Jenny finished up with is quite symmetrical, that's to say : you can look at it straight on from six different directions and it always looks the same.





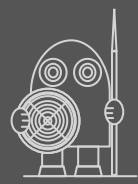
. . . and of course for each of these you can look from exactly the opposite direction – which gives you a total of 6 different views, in three different colours but all the same in shape . . .

Here are three problems for you :

1 How many cubes did Jenny take from the original cube?

2 How many cubes are there in Jenny's new shape?

3 What's the surface area of this new shape?



problem-solving
~ how to get started

Sometimes you look at a difficult problem and think, 'I really don't know where to start on this one!' Many people imagine that a real mathematician can look at any problem and know what he must do to solve it. But that's not how things are! Real mathematicians look at all sorts of problems and very often say to themselves, 'I don't know where I'm going to start on this one!' The interesting thing is searching for a way to unlock the problem and at times that can be very difficult. It can also be really interesting – as you try different things which might help you, you often discover new connections and new patterns. That's mathematics!

dog on the wing

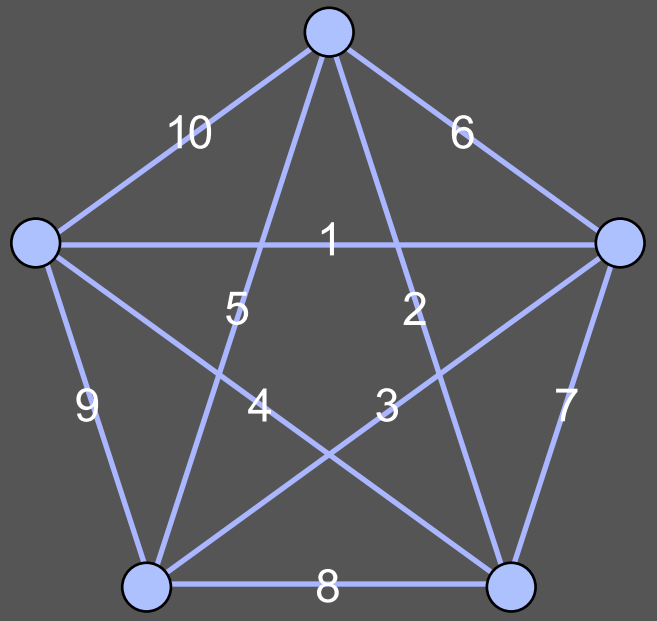
Most people have never seen a dog playing football. But a few years ago in Battersea Park, South London, you could have seen such a dog. It's a large park, and daily there's lots going on: cycling, strolling, dog-walking, pram-pushing and games of all sorts.

But on Sunday mornings the big thing is football.There are serious games being played on the big pitches, with local teams and supporters, plus real referees to keep order and to ensure fair play. And there are also many smaller games going on in various locations around the park. It's in one of these smaller games that every week you'd have been able to see a dog, not a very big dog, playing for one of the sides. He didn't really have many football skills (he couldn't keep the ball in the air using his head) but he was very fast as he scampered down the wing, pushing the ball ahead of him with his nose and quickly finding a way around almost anyone who tried to take the ball from him. As you can guess, quite a crowd would gather to watch 'Dribbler', as he was called . . .

And 'dribbler' turns out to be a good way of remembering our useful tips for problem-solving (they're on the following pages). If you can just remember the word 'dribbler', then you've got the first letters of the important key words . . .

draw

Sometimes a problem you're given is based on a drawing or diagram. But there are other times when drawing something yourself suddenly makes the problem a lot more straightforward. What you draw doesn't have to be a work of art – it just has to make the problem clearer for you. You'd be surprised how many times a simple drawing or diagram makes it easier to see what's in a problem and perhaps makes it easier to solve.



no. of lines = 10

and so

no. of handshakes = 10

information

You've probably been told lots of times, 'read the question!' . . . There is some sense in this because the problem as you read it contains the information you'll need to solve it. Let's hope we're agreed on that but – as you start working on the problem, there are some questions you need to ask yourself :

- Are you using all the information you've been given? Sometimes you're stuck and it's just because there's a bit of information you do need in order to solve the problem and you've overlooked it.

- Have you been given more information than you need? If you're given lots of information, then it's tempting to dive in straight away and start doing some working-out but – it might just be that if you take a moment, you'll find that you really only need certain key facts.

- Can you perhaps look at the information in a completely different way? For example, suppose you've been told that a hot tap takes 6 minutes to fill a bath and a cold tap takes 12 minutes. You might not be sure how you can use this information as it stands but try looking at it this way : *the hot tap can fill 10 baths an hour and the cold tap can fill 5 baths an hour.* Suddenly, the answer is just around the corner!

bit-by-bit

Often you'll start working on a problem and find that you really can't think of any way of getting directly to the answer. But not all problems have to be solved by a clever method or by a smart way of going straight to the answer. At times a bit-by-bit approach is the way forward. Perhaps in arithmetic you've already come across the bit-by-bit way of working some things out. For example, if you had to work out 35% of £64, you might know that you could try to work out 35/100 of £64 but – it might be a lot easier to jot down 10%, then 20%, then 5% and then just total the results to get your final answer. Instead of trying to aim for an answer in one go, you take a bit-by-bit approach and you get there more easily. You can take the same approach with some difficult maths problems.

like

Sometimes you can look at a problem and you know straight away you've already solved another one just like it. If you can remember how you solved that problem, it might help you to solve this one.

But even if you haven't solved something like this before, you might be able to make up a problem that's just like this one but much simpler. For example, looking at a problem about inner and outer pieces in a large jigsaw, you might feel unsure about how to handle the big numbers involved. So, make a quick drawing of, say, a simple 7 x 5 jigsaw (rectangles will do for pieces, no need to put in the curly bits). Now you can easily see how the numbers work out when you want to find things about inner and outer pieces or whatever. Next, you can apply what you've discovered to the dimensions of the larger jigsaw and perhaps feel more confident about the answers you get.

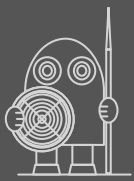
experiment

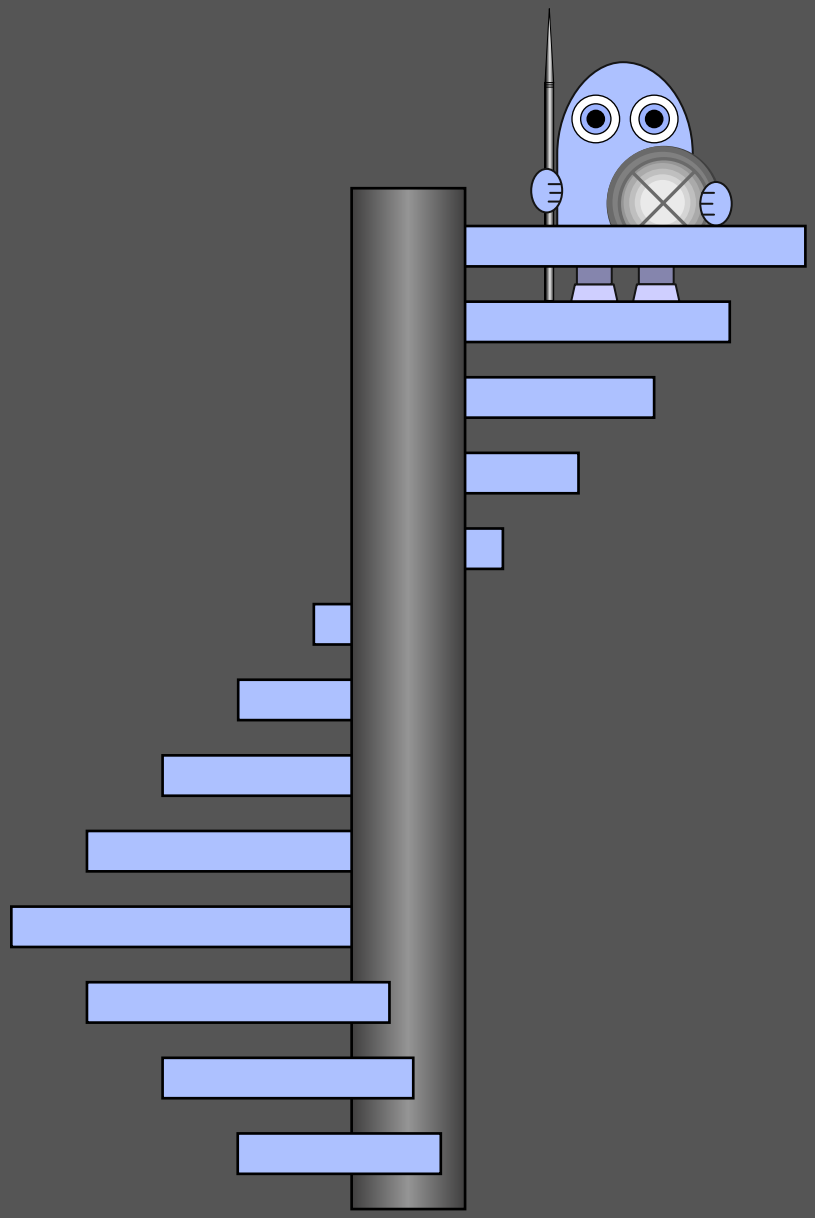
Remember, there's nothing to stop you trying out an answer to see whether it works . . . and if it isn't quite right, then think how best to change it – and try again! Look at this problem, for example :

Peter and Sue are brother and sister; Peter is 5 years older than Sue. Their two ages add up to 27. How old is Sue?

Let's experiment! If Peter and Sue were twins, then they would each be 13½ . But they're not twins, are they? In fact, Sue is younger. So let's start with a try of, say, Sue = 9. This would make Peter 14 and their total ages 23. That's too low, so let's try Sue = 10. With this try, Peter (5 years older) = 15 and the total of ages = 25. Too low again! Putting Sue = 11 makes Peter 16 and the age total = 27. Success! So our problem is solved : Sue is 11 yrs old.

You can use this method in many maths problems. Ignore anyone who says this is just guessing – because it's much more than that. Remember, when we tried an answer which didn't work, we didn't just wildly guess at another one, we stopped and thought – and then we made an intelligent correction (alteration) and tried it out. You could call this method the 'Estimate & Adjust' method if you like. (Doesn't this sound a whole lot better than 'guessing'?)





realistic

Well, when you've worked on a problem and finally got an answer, is it time to sit back and feel pleased with yourself? Not quite! There's actually one more important thing to do. Take a hard look at your answer and ask yourself: Is my answer reasonable . . . is it realistic? Say it was a question about the ages of different people in a family and you worked out that grandfather must be 15 years old. Is that a reasonable answer? Well, not really . . . most grandfathers are a good deal older than that. And, as you probably know, no chicken weighs 300kg, no girl can cycle at 85km/hr and no 10 year old boy is likely to have a height of 14cm – and yet these are all answers which have been given by real pupils.

. . . and this gives you quite an easy way to remember how to get started on a problem if you're really stuck. Just remember the name 'dribbler' and you've got the first letters of the important key words :

** the one-stop chart is on the next page -->*

draw

information

bit-by-bit

like

experiment

reasonable

information

are you using all the information you've been given? can you look at the information in a different way?

experiment

can you try out an answer and then improve it?

draw

can you draw something which will make the problem clearer?

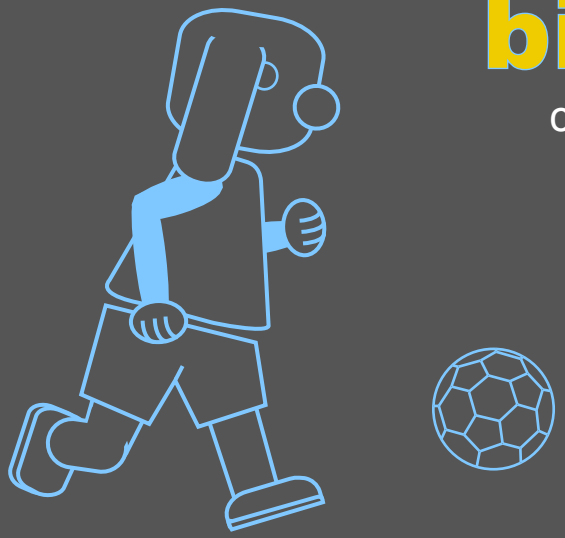
dribbler

reasonable

and when you've found an answer : is your answer reasonable? is it realistic?

bit-by-bit

can you work out the answer in easy stages?



like

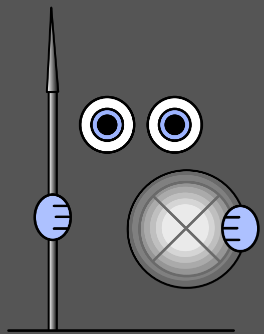
is the problem like something you've seen before – or can you make up a simpler one just like it?

no

problem !

book

problem book



four winds