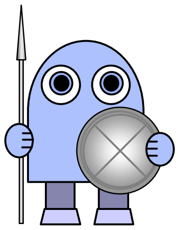


MATHS CHALLENGE CARDS SET D

ANSWER BOOK

four winds maths

1 4 adults, 4 teenagers and 12 children will give you exactly £20.

2 **a** The five tracks cost exactly £5 in all – which means that the average cost per track was £1.

b By the end of the day Daniel had downloaded six tracks. If their average price was 96p, they must have cost £5.76 altogether (96p x 6). That's 76p more than his previous total, so the new track must have cost him 76p.

3 **a** 18 cm^2 **b** 6 cm **c** 6 cm

4 First of all, we know our answer must be an odd number (since it leaves a remainder of 1 when you divide it by 2).

Next, if we write down the numbers which leave a remainder of 4 when you divide them by 5, we get :

9, 14, 19, 24, 29, 34, 49 . . . and so on.

We can see there's a simple pattern here – the numbers all end in either 4 or 9 ! The ones ending in 4 are no good because they're even – and we already know we need an odd number ! This means the number we're after must end in a 9.

So, we only need to look at the numbers :

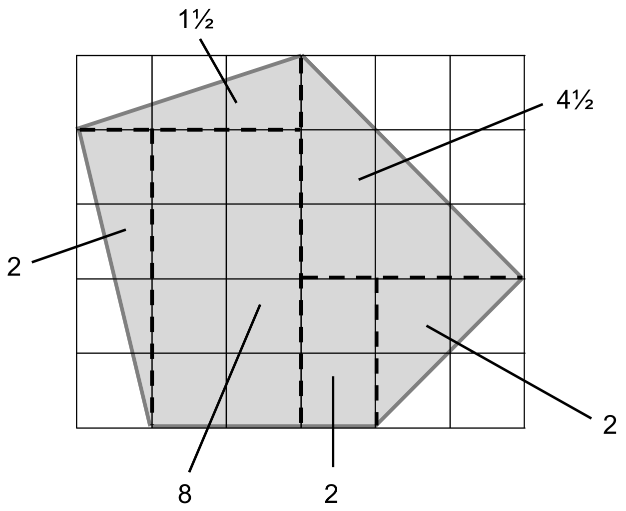
9, 19, 29, 39, 49, 59

(No need to go higher because we know our number is under 60.)

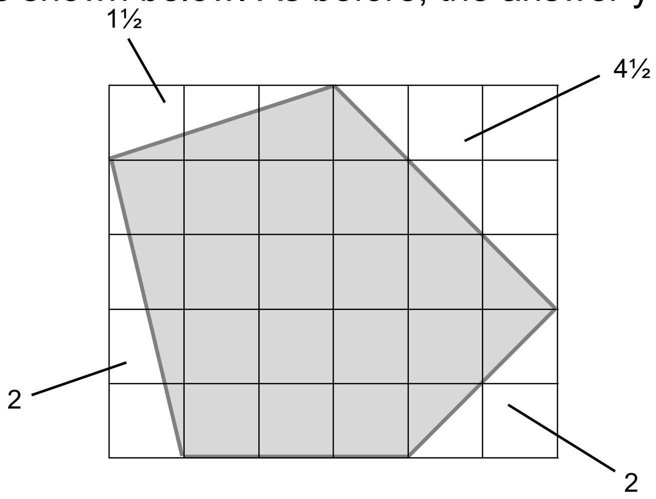
When we look at these numbers to see whether they leave the remainders we want when we divide them by 3 and by 4, we find that only 59 fits the bill.

So, 59 is our answer !

5 One way of going about this is to split the pentagon up into rectangles and right-angled triangles. Adding up the separate areas gives you a total of 20 squares :



Another approach is to take the area of the outside rectangle (30 squares) and then subtract the areas of the four right-angled triangles, as shown below. As before, the answer you get is 20 squares.



There are various ways of doing the second part of the question. Any pentagon is correct as long as it lies inside a 6 x 5 rectangle and has an area of 20 squares, so : just draw your outer rectangle and then find four corner right-angled triangles whose areas add up to 10 squares.

6 The pattern is fairly easy to spot – to get the sum, just double the last term in the sequence and subtract 1.

$$\text{So } 1 + 2 + 4 + 8 + 16 + . . . + 1024 = 2047$$

---

7 The BMW is white – and Sue drives it.

---

8 **a** 3/10 **b** 4/5 **c** 1/2 **d** 3/4 **e** 7/20 **f** 17/20 **g** 1/8 **h** 3/8

---

9 **a** The twins' father is 36 years old.

b In 4 years' time the father's age will be exactly double the sum of the twins' ages. (By then they will each be 10 and he'll be 40.)

---

10 If you lower Ben's guess by 50%, you get 30 – and if you increase Sally's guess by 50%, you also get 30. So 30 is the number you're looking for !

---

11 It's tempting just to think : with average speeds of 20 km / hr for the first stage and 30 km / hr for the second stage, the overall average must be 25 km / hr. But look more carefully . . .

During the first stage, Andy travels 60 km at an average speed of 20 km / hr – so that's 3 hours of travelling.

During the second stage, Andy travels 60 km at an average speed of 30 km / hr – so that's 2 hours of travelling.

This means that for the journey as a whole, Andy travels 120 km in 5 hours – which gives you an average speed of 24 km / hr !

---

12 The five numbers are : 2, 3, 6, 9, 10

13 It's worth looking at a few examples here to see how things work :

	middle number	how many houses
1, 3, 5	3	3
1, 3, 5, 7, 9	5	5
1, 3, 5, 7, 9, 11, 13	7	7
1, 3, 5, 7, 9, 11, 13, 15, 17	9	9

The pattern in these sequences is easy to see : to get the middle number in the sequence, you just add 1 to the last number and halve the result. Another way of looking at this is to say you are finding the average of the first and last numbers in the sequence.

When it comes to finding the number of houses in one of these sequences, you can just rely on the fact that it's the same as the middle number. Another way of doing things is to find the size of the 'gap' between the smallest and the largest numbers in the sequence – then divide this number by 2 (since each individual 'jump' is 2) to see how many 'jumps' there are between the numbers; just add 1 to this to find how many numbers there are.

Here are the answers to the questions :

a This is a straightforward sequence, so we can say there must be 39 houses – and the middle number is also 39.

b Things are not quite so easy with this one. One thing we can do is subtract 29 from 125, giving us 96. This tells us that the 'gap' between the lowest and highest numbers is 96, or in other words, 48 'jumps'. So there must be 49 numbers. Another way of doing this is to look at the whole sequence 1, 3, 5 . . . 121, 123, 125. This must have 63 numbers in it. Then we can look at the sequence 1, 3, 5 . . . 25, 27. This has 14 numbers in it. The difference between 14 and 63 gives us the answer we're after, that's to say 49 numbers.

Perhaps the easiest way to find the middle number is just to work out the average of 29 and 125; this gives us the answer 77.

(*You will have noticed that in this question we've only looked at sequences with an *odd* number of terms in them – otherwise there isn't really a 'middle number'. If you have time, you can investigate what happens when you have an even number of houses in the list – or how things change when you look at the even-numbered side of the road.)

14 At first you might think you haven't been given enough information here. However, you just need to remember the important fact that in a 3 x 3 magic square, the magic total is always three times the middle number . . .

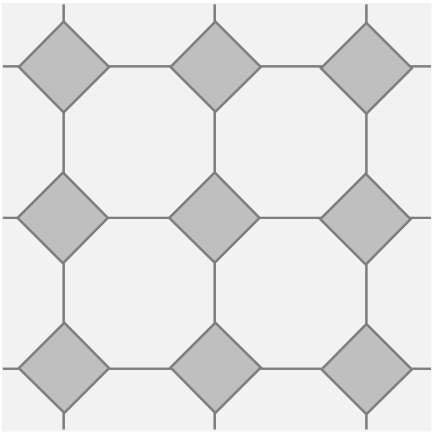
1	24	**14**
26	13	**0**
12	**2**	**25**

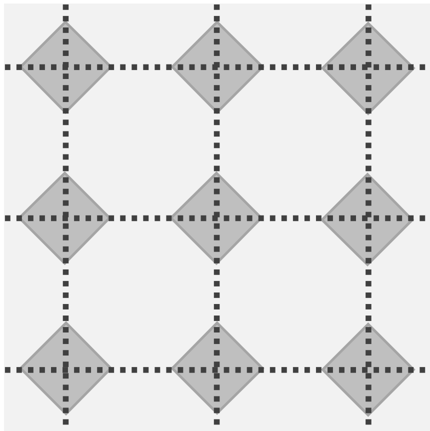
** magic total = 39*

14	**4**	27
28	15	2
3	**26**	**16**

** magic total = 45*

15 On the left is our original tessellation of octagons and squares :

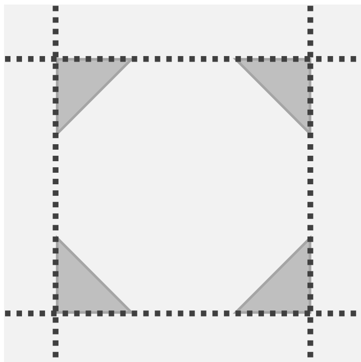




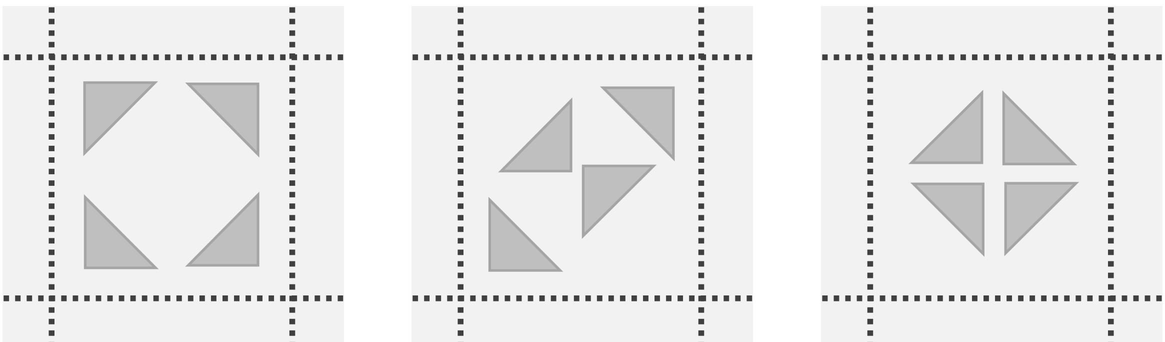
On the right you can see we've put a square grid over the original pattern – so now we can think of it as a tessellation of squares.

If we look at any one of these squares, what exactly have we got?

Each 'tile' we're looking at now is a 10cm x 10cm square with four right-angled triangles at the corners. The area of the whole square is 100 cm^2, so to find the area of the octagon part, we just need to subtract the total area of the triangles from 100 cm^2.

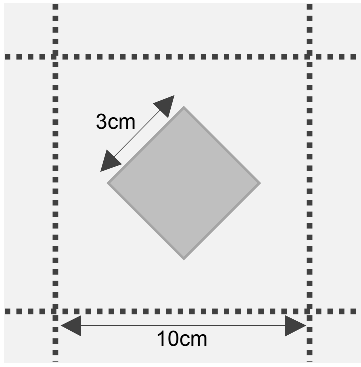


Finding the total area of the four triangles is easy – because if we put them together, they make a 3cm x 3cm square :



area of small square = 9 cm^2

– and this means that in each of our 10cm x 10cm squares, the octagon part has an area of 100 – 9 = 91 cm^2.



In other words, 91% of each square is the octagon part. So any large space tiled with the original pattern of octagons and squares will have 91% of its area covered by octagons.

---

16 **a** Let's look at 64 as an example :

64 = 2 x 2 x 2 x 2 x 2 x 2 (which we usually write as 2^6)

This can be re-written as (2 x 2 x 2) x (2 x 2 x 2), or in other words, 8 x 8 – which is a square number.

Or it can be re-written as (2 x 2) x (2 x 2) x (2 x 2), or in other words, 4 x 4 x 4 – which is a cube number.

You can do the same sort of thing with 729 – or indeed with any number which is something to the power of 6.

b 1 is the only number under 50 which is a square and a cube.

c The smallest power of 10 which is both a square and a cube is 10^6, or in other words, 1 million.

---

17 The easiest way into this one is to look at *how many shirts per hour* are finished by Bob or by Emma or by both of them together.

- Bob takes 10 minutes to finish a shirt – so he finishes 6 shirts per hour.
- Bob and Emma working together finish 15 shirts per hour.
- The difference between 6 and 15 is 9, so Emma must be contributing 9 shirts per hour – and there's your answer !

---

18 To make things easier, let's call the rats A, B, C, D and E.

We know the rats always go out hunting in pairs, so here are all the possible pairs :

AB			
AC	BC		
AD	BD	CD	
AE	BE	CE	DE

The first thing we're told is that B and D must never be together – either in or out. This rules out four of the pairs straight away :

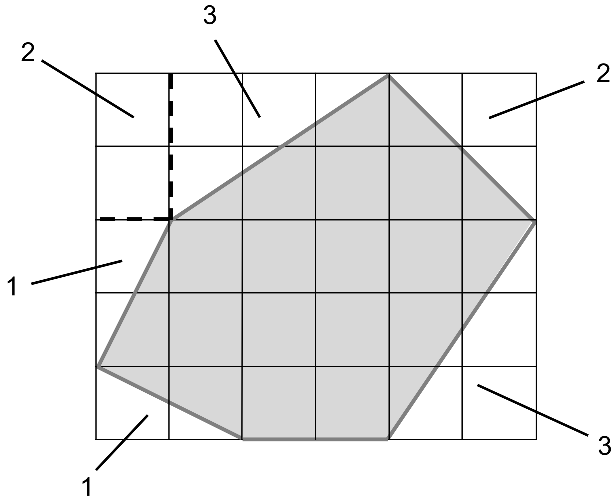
AB			
~~AC~~	BC		
AD	~~BD~~	CD	
~~AE~~	BE	~~CE~~	DE

Next, we're told that no two females may go out hunting together, so this rules out the pair AD. We also know that E can't be left at home with Charlie, ruling out the pair AB. So finally we have a total of six pairs which are not allowed :

~~AB~~			
~~AC~~	BC		
~~AD~~	~~BD~~	CD	
~~AE~~	BE	~~CE~~	DE

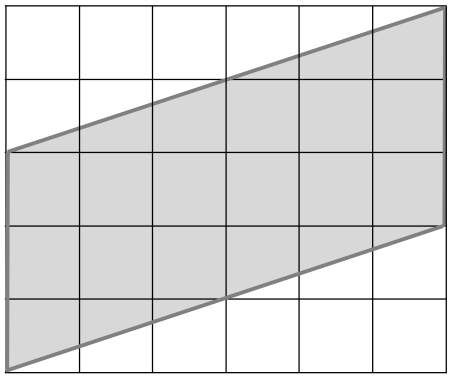
At a glance you can see that A (Annabelle) is the poor rat who never goes out.

19 You can split the hexagon into rectangles and right-angled triangles if you wish to, but it's probably easier just to add up the outer areas and subtract the total from 30 :



This gives area of hexagon = 30 – 12 = 18 squares.

Different answers are possible for the second part of the question but here's one :



20 **a** A 4½ cm x 8 cm rectangle has an area of 36 cm^2 and a perimeter of 25 cm.

b A 1½ cm x 16 cm rectangle has an area of 24 cm^2 and a perimeter of 21 cm.

21 On every chime, Romeo jumps 2 steps upwards and Juliet jumps 3 steps downwards – or in other words, on every chime they move 5 steps nearer together. When the bell starts to chime, the gap between the two frogs is 197 – 22 = 175. Since 5 goes into 175 exactly, you can be sure that at some point the two frogs will land on the same step.

---

22 **a** There are 8 red M&Ms in the bag, out of a total of 20. So, probability of picking red = 8/20 = 2/5.

b Once Rebecca's had her M&M, there are 19 M&Ms in the bag, of which just 7 are red (and 12 are not-red). So, probability of picking not-red = 12/19

---

23 **a** 5 ○ 7 = 100 – 35 = 65

b 10 ○ 10 = 100 – 100 = 0

c 5 ○ (9 ○ 10) = 5 ○ 10 = 50

d 2 ○ (9 ○ 9) = 62, so n = 9

---

24 **a** 16 of the seats are chairs (that's 64 legs) and 12 of the seats are stools (that's another 36 legs).

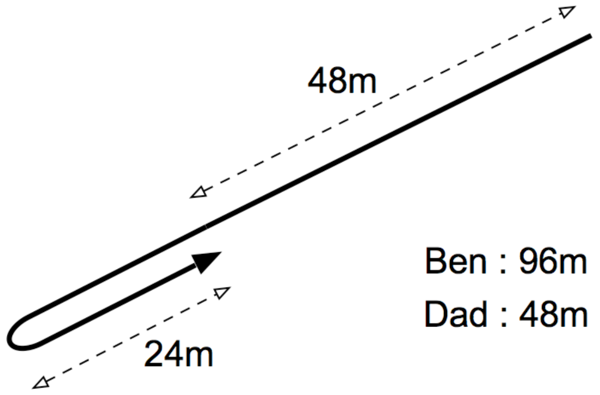
b The smallest number of seats with 42 legs is 11 (2 stools and 9 chairs) – and the largest number of seats with 42 legs is 14 (14 stools and 0 chairs).

---

25 **a** If you increase 5m by 20%, you get 6m. So the new flower-bed measures 6m x 6m = 36 m^2.

b Area of original flower-bed = 25 m^2. Enlarging the flower-bed produces an increase in area of 11 m^2. So, percentage increase in area = 11/25 x 100 = 44%.

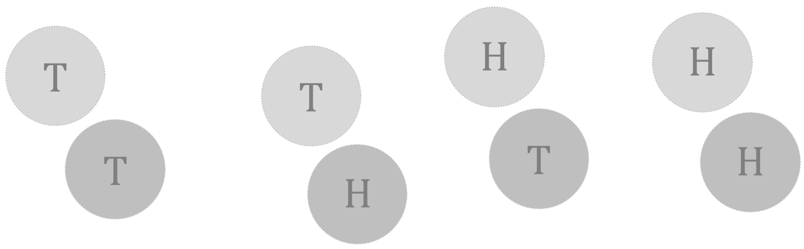
26 **a** When Ben meets his dad again, he's cycled 96 metres altogether and his dad has cycled 48 metres :



b They meet again 8 metres from the end.

---

27 **a** There are four possible ways in which these two coins could land :

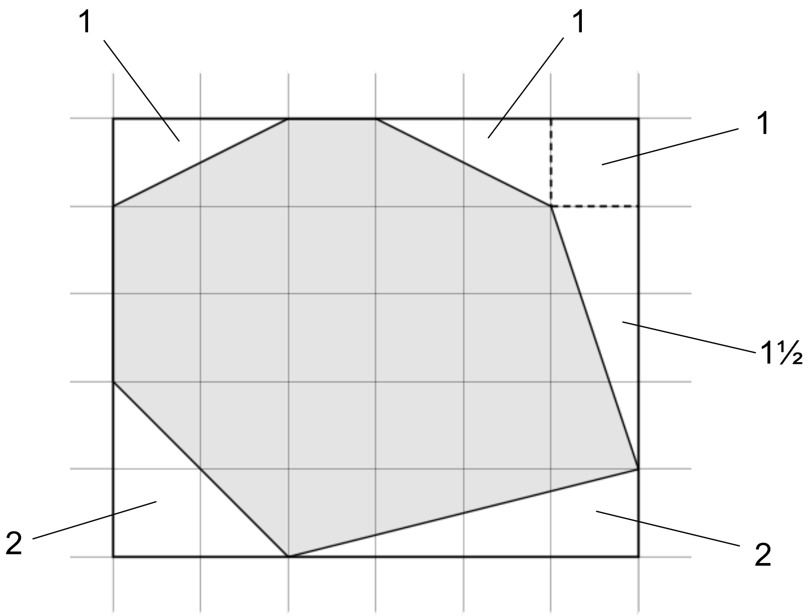


Each of these ways is equally likely. Two of the ways show a head / tail combination, so

probability of getting a head and a tail = 2/4 = 1/2

b If you take out 5 brown eggs, then you're leaving 1 white one in the fridge. So, the probability of selecting 5 brown eggs is exactly the same as the probability of leaving behind 1 white one, that's to say 1/5.

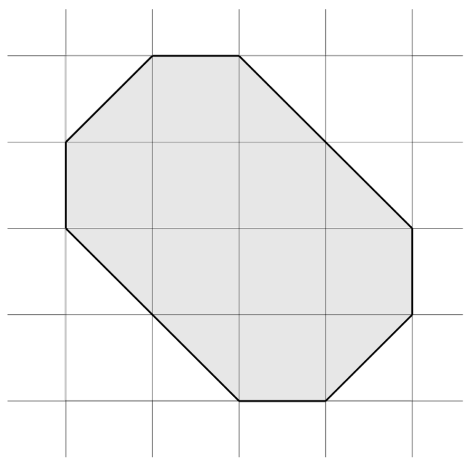
28 **a** An easy way to find the area of the heptagon is to draw a rectangle around it, like this :



The area of the rectangle is 30 squares and the various white bits around the heptagon add up to 8½ squares – so the area of the heptagon must be

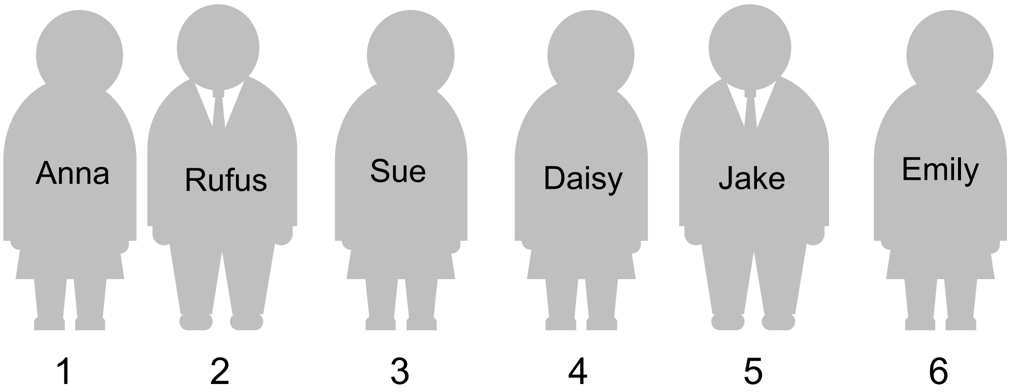
$$30 - 8\tfrac{1}{2} = 21\tfrac{1}{2} \text{ squares.}$$

b Here's one possible answer :



The area of this particular octagon is 11 squares.

29

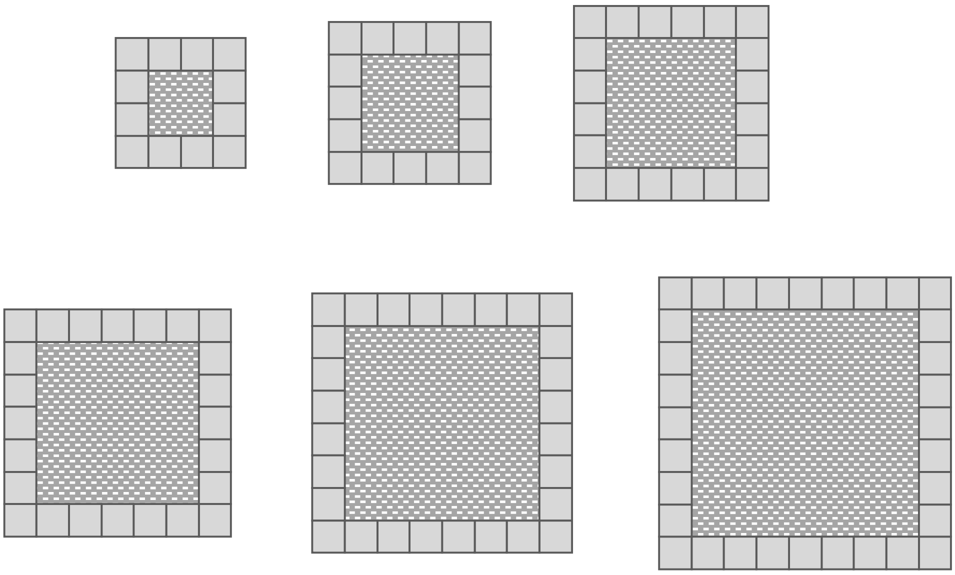


30 **a** 1, **3**, **5**, 9, 17, **31**, 57 . . .

b 1, 5, 7 is one possible answer.

c 1, 1, 2 is one possible answer; and 3, 1, 1 is another.

31 The answer to this question is no. If you're using a whole number of slabs, your square path will enclose an area that's either bigger or smaller than the area of the path itself. Here's what you get when your flower-bed is 2, 3, 4, 5, 6 or 7 metres square :

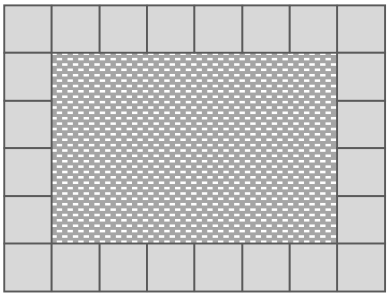


– which give you these results for the areas :

side of bed	area of bed	area of path
2	4	12
3	9	16
4	16	20
5	25	24
6	36	28
7	49	32

As you can see, when the side of the flower-bed is 2, 3 or 4 metres, the area of the path is larger than the area of the bed. But from 5 metres upwards, the area of the bed is larger than the area of the path. There's no whole number where areas are equal !

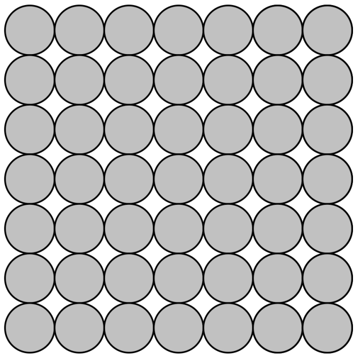
** Of course if Bob's wife had been happy to have a rectangle instead of a square, Bob could have done the trick :*



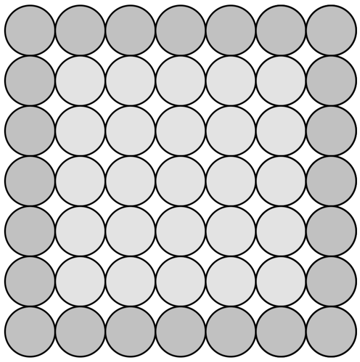
With this arrangement, area of path = 24 m^2 and area of flower-bed = 24 m^2.

32 **a** It’s easy to explain this in diagrams :

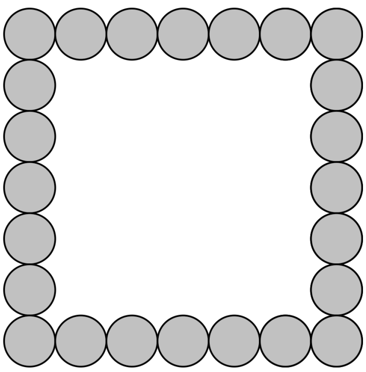
Let’s take $7^2 - 5^2$ as an example. You can show 7^2 like this :



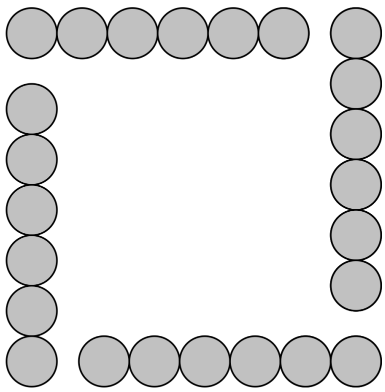
– or you can show it like this, with 5^2 on the inside :



If you take away the 5^2, what’s left is $7^2 - 5^2$:



– which you can show like this :



– a number which is very obviously a multiple of 4.

b The pattern is not hard to spot. You just need to look at the number between the two squares to see how many lots of 4 you'll get. For example, $3^2 – 1^2$ will give you 2 lots of 4 and $7^2 – 5^2$ will give you 6 lots of 4. So straight away we can say that $101^2 – 99^2$ must be the same as 100 lots of 4.

c Yes, the **a** and **b** results above work for even numbers too.

If we're using algebra, we can write the difference between any square number and the one 2 above it like this :

$$\text{difference} = (n + 2)^2 – n^2$$

$$= (n^2 + 4n + 4) – n^2$$

$$= 4n + 4 = 4(n + 1)$$

– which is of course a multiple of 4. You'll notice it doesn't matter whether n is odd or even, you still end up with a multiple of 4. (And either way this multiple will equal n + 1 lots of 4.)

---

33 **a** Glyn's field is $1\,{}^1/_4$ acres. There are different ways of getting to the answer but it's easy with algebra :

$$\text{if} \quad 5x = 5 + x\text{, then } 4x = 5 \text{ and so } x = {}^5/_4.$$

b John's field is $1\,{}^4/_5$ acres. Again, algebra gives you an easy way:

$$\text{if} \quad {}^9/_4\,y = {}^9/_4 + y\text{, then } {}^9/_4\,y = {}^9/_4 + {}^4/_4\,y = {}^1/_4\,(9 + 4y)$$

$$\text{from which we get : } 9y = 9 + 4y$$

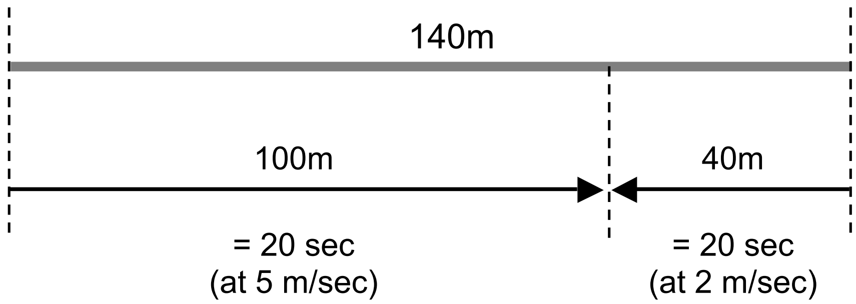
$$\text{so that } 5y = 9\text{, or in other words, } y = {}^9/_5.$$

---

34 **a** Obviously it's exactly the same for both wheels ! This means the small wheel also covers 405 metres.

b The circumference of the small wheel is about 3 times the diameter, that's to say 3 x 45 cm or 135 cm. So each time it goes round, the small wheel covers about 135 cm. This means that in 1000 rotations, the small wheel covers 1000 x 135 cm or 135,000 cm. This is the same as 1.35 km.

35 Of course it does ! The clue to this problem lies in the picture : Don’t think of *one* lift on Monday setting off at 12 noon and the same lift on Tuesday setting off at 12 noon. Instead, just think of *two lifts setting off at exactly the same time*, one leaving the ground floor and going up – and one leaving the 28th floor and going down. Obviously there *has* to be a point at which the lift going up and the lift going down pass each other. Exactly where and when doesn’t take long to work out :



As you can see, the crossing point is at height = 100 metres and the journey time = 20 seconds.

---

36 Here’s one example of each :

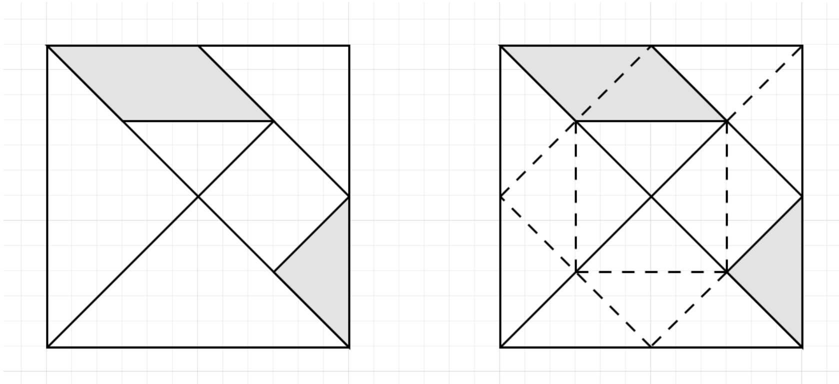
magic square

0	1	3
7	2	5
6	4	8

anti-magic square

3	8	1
2	4	6
7	0	5

37 **a** If we add one or two dotted lines, we can picture the whole square as made up of 16 right-angled triangles, all of the same size :



Clearly the area of the shaded parallelogram (2 triangles) is exactly one-eighth of the area of the square (16 triangles), that's to say $12^2 \div 8 = 18$ cm^2.

b The shaded small triangle has an area of one-sixteenth of the area of the square, that's to say $12^2 \div 16 = 9$ cm^2

c As you can easily work out, if you divide the square's side by 2, you divide the various areas by 4. So the answers to questions **a** and **b** above become 4.5cm^2 and 2.25cm^2.

---

38 **a** = 47 / **b** = 36 / **c** = 145 / **d** = 84 / **e** = 3

---

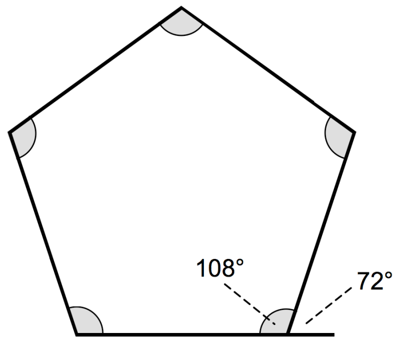
39 Three different combinations will give you a total of 100 kg :

10 kg	13 kg	17 kg	total no. of pumps	total mass
7	1	1	9	100 kg
4	2	2	8	100 kg
1	3	3	7	100 kg

Only the last one gives you fewer small pumps than any other size. So the answer is : 1 small, 3 medium, 3 large.

40 **a** Obviously, six exterior angles of 60° each add up to 360°. (This is not surprising! Imagine an ant starting in the middle of one side and walking all the way round the hexagon until he's back where he started. He'll have changed direction six times along the way but as he ends up where he started – and facing exactly the same way – he must have turned through 360° in all. So, with six changes of direction, that's 60° at each corner.)

b We can use the same sort of thinking for the regular pentagon. An ant who goes all the way round the pentagon will turn through 360° in all (so that's what the pentagon's exterior angles must add up to). That means we must have 360 ÷ 5 = 72° at each corner.



And, with an exterior angle of 72°, each interior angle must be 108°.

c Total of exterior angles = 360°, so each exterior angle = 45° (360 ÷ 8). Therefore, each interior angle = 135°.

---

41 Sides of 2cm, 3cm and 7cm will do the trick !

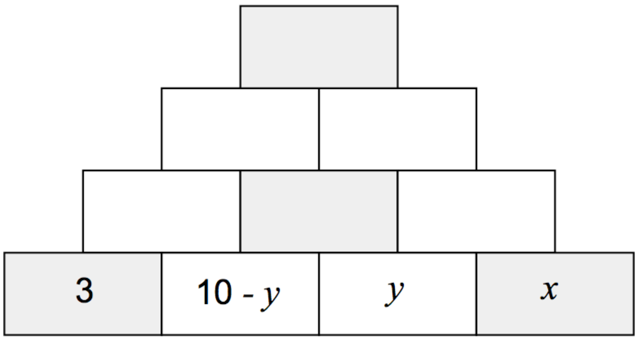
---

42 **a** x must be 15.

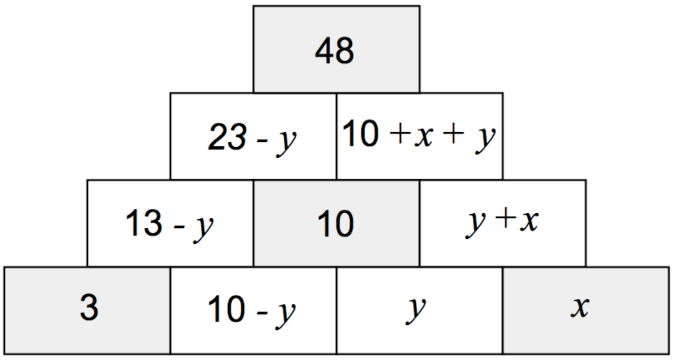
b As long as the two middle numbers on the bottom row add up to 10, you'll always get 48 at the top.

To get a picture of what's going on here, let's start with the bottom row. We know the two end numbers are 3 and x. If we call one of the two middle numbers y, the other one will obviously be $10 - y$. (This is because these two numbers must add up to 10.)

So now the bottom row numbers are: 3,10 – y, y, and x :



Adding upwards to complete the number-wall gives us this :



To get the top number-brick (48), we know we have to add together the two bricks it's resting on. So :

$$(23 - y) \ + \ (10 + y + x) \ = \ 48$$

$$23 - y + 10 + y + x \ = \ 48$$

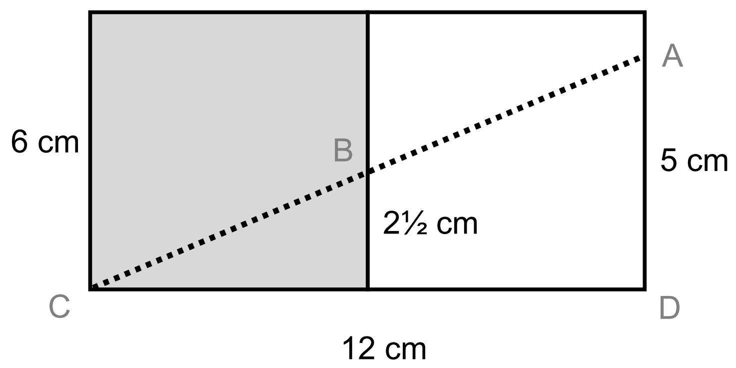
$$33 + x \ = \ 48$$

$$33 + x \ = \ 48$$

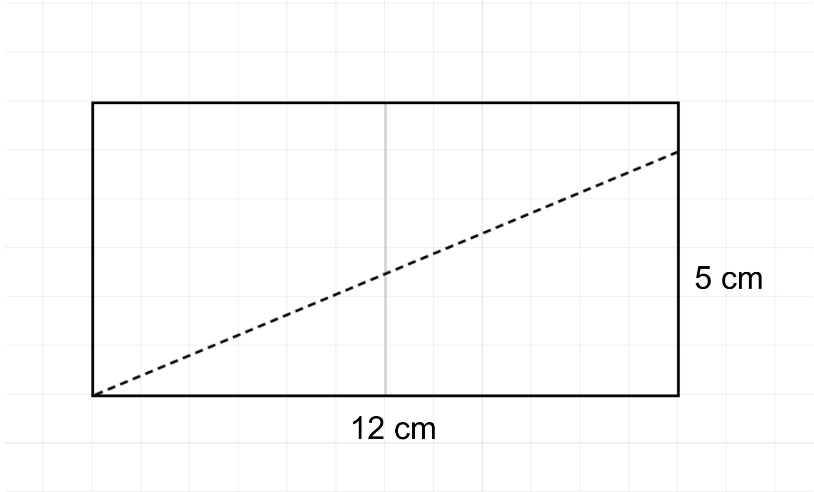
$$x \ = \ 15$$

– which tells us clearly that x has to be 15 but that y can be anything (or anything from 0 to 10 if you want to keep it all in the realm of positive numbers).

43 To get from A to C, the ant must cross two faces of the cube. We can picture the ant's path by flattening out these two faces to form a 6cm x 12cm rectangle, like this :



Doing this doesn't make any difference to the basic problem – it's just an easier way to think of things. If you draw the diagram onto 1 cm squared paper, you'll get this :



The dotted line is the ant's path and if you measure it carefully with a ruler, you'll find that it's exactly 13 cm long. Point B is half-way along the path and of course this point is exactly 2½ cm from the nearest corner. So our final answers are :

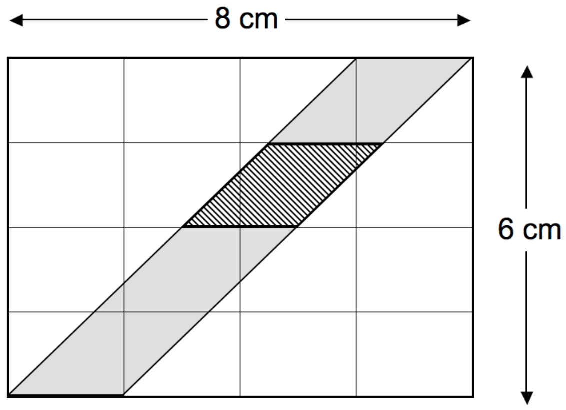
(a) 2½ cm **(b)** 13 cm

* *Of course, if you know Pythagoras' Theorem, you can find the length of the path easily enough by just writing :*

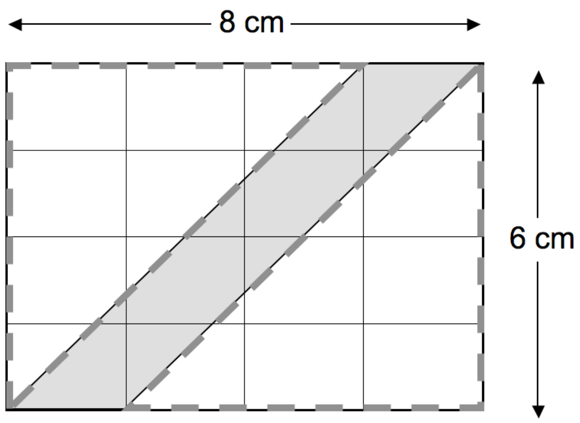
$$5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

44 There are different ways of doing this problem but here's one easy way :

For the moment, forget the horizontal band across the rectangle :



Now just think of the rectangle as made up of the sloping band and two right-angled triangles :

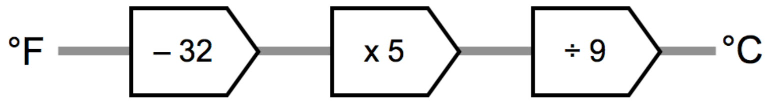


Each of these triangles has an area of 18 cm^2, so together their area is 36 cm^2. If we subtract this from the overall area of the rectangle (48 cm^2), we get 12 cm^2 as the area of the sloping band.

The parallelogram we're interested in is obviously one-quarter of this, so its area must be 3 cm^2.

** If you already know that area of a parallelogram = base x height, then it's even easier : 2 x 1.5 = 3*

45 To convert °F into °C, we can use this mapping diagram :



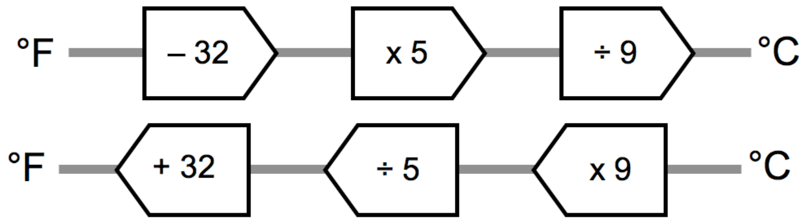
– which easily gives us the following results :

a 50°F = 10°C

b 140°F = 60°C

c 32°F = 0°C

To convert °C into °F, we just reverse the mapping diagram :

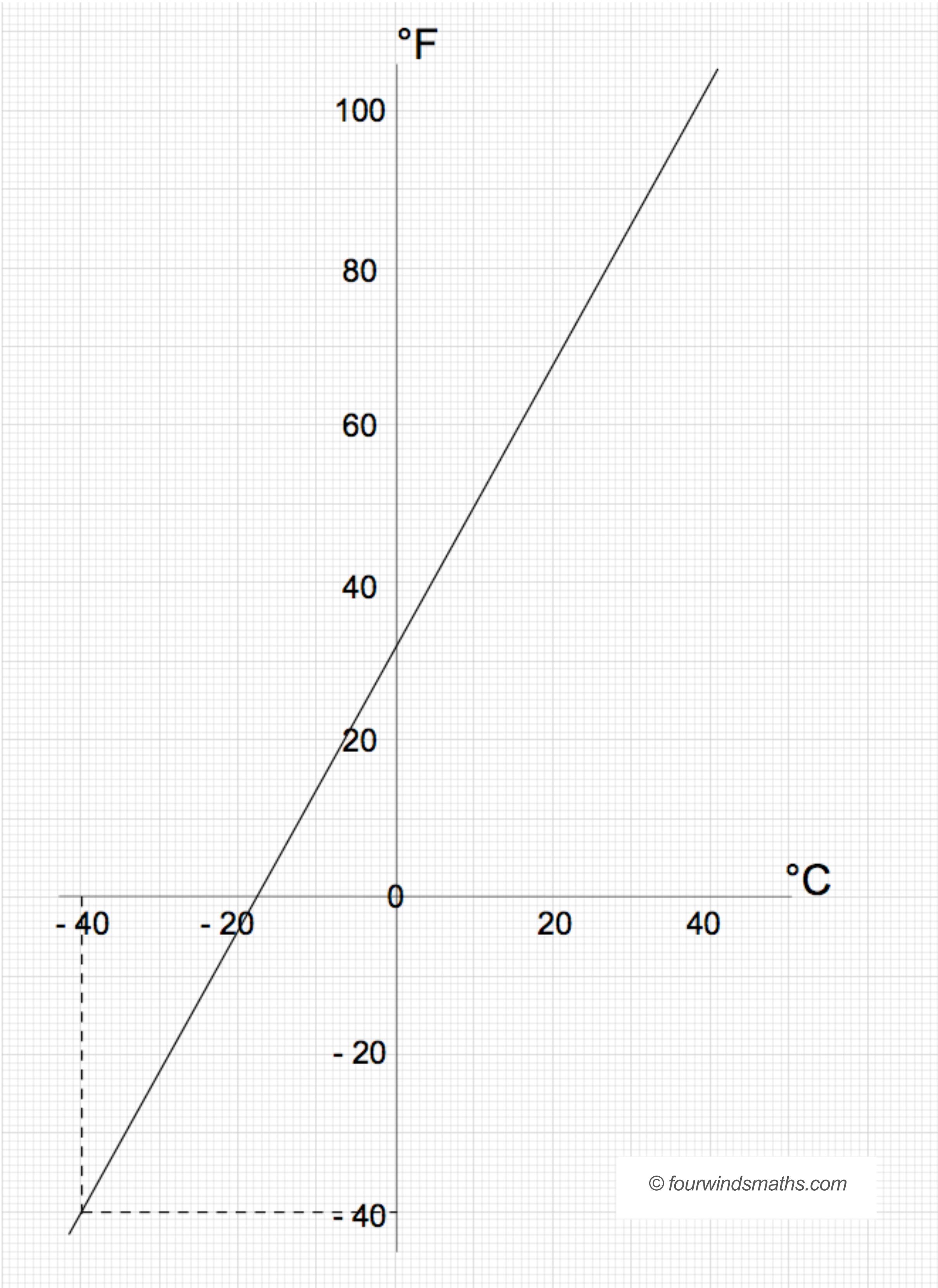


– from which we get :

d 85°C = 185°F

e Using trial-and-improvement or algebra (or any other method), you should soon find that – 40°C = – 40°F.

Another way of showing all these answers is by means of a graph :


°F
100
80
60
40
20
0
°C
- 40
- 20
20
40
- 20
- 40
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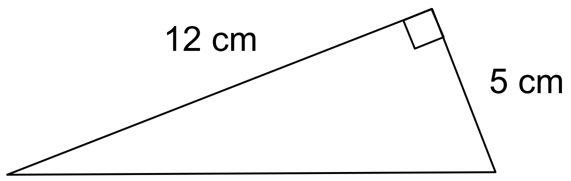
46 Here's one way to solve the problem :

The area of the outer square is 289 cm^2 and the area of the inner square is 169 cm^2. This means that the four right-angled triangles together have an area of 289 – 169 = 120 cm^2.

So, area of each triangle = 30 cm^2.

One short side of the triangle is 5cm, so the other short side of the triangle must be 12cm :



– and the longest side must be 13 cm long (since it's one side of a 169 cm^2 square).

Of course, if you know Pythagoras' theorem, it's all quite easy :

Since the inner square has an area of 169 cm^2, its sides must be 13 cm long. So if we call the unknown side x, we can write :

$$5^2 + x^2 = 13^2$$

$$x^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$x = 12$$

– which again gives us 5 cm, 12 cm and 13 cm for the three sides.

47 **a** When you have a set of consecutive numbers, the easy way to find out how many numbers you've got is to subtract the smallest from the largest and add 1. So here the answer is (101 – 29) + 1 = 73

b The easy way to find what the middle number is in a set of consecutive numbers is to find the average of the first and last. So here the answer is ½ (101 + 29) = 65

c Using the pattern we've already seen, we can write : sum of series = 65 x middle number = 65 x 73. Multiplying this out gives you 4,745.

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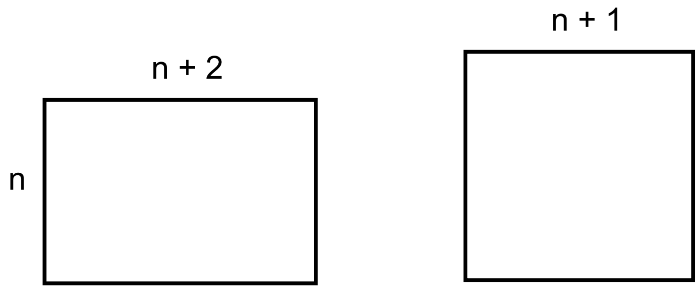
48 **a** Obviously this can't work for n and n + 1, because the square number would have to be between n and n + 1, and this can't happen with whole numbers !

b This one can't work either. The only square number which might work is n + 1 (because that's the only whole number between n and n + 1). But $(n + 1)^2$ is $n^2 + 2n + 1$, whereas n multiplied by n + 2 is only $n^2 + 2n$:



area = $n^2 + 2n$ area = $n^2 + 2n + 1$

Clearly these two can never be the same ! Another way of proving the case is this : Suppose n is even. Then (n + 2) will be even and so the product of n and (n + 2) will be even. This product can never be the square of an odd number and obviously (n + 1) would have to be an odd number. The same sort of thing applies if both n and (n + 2) are odd.

c Success at last ! There's just one example which works and that's n = 1. (A rectangle 1cm x 4cm has the same area as a square of side 2cm.)

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49 There were 4 people in the lift at the beginning.

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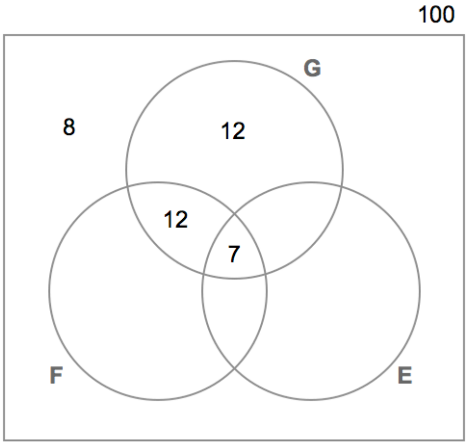
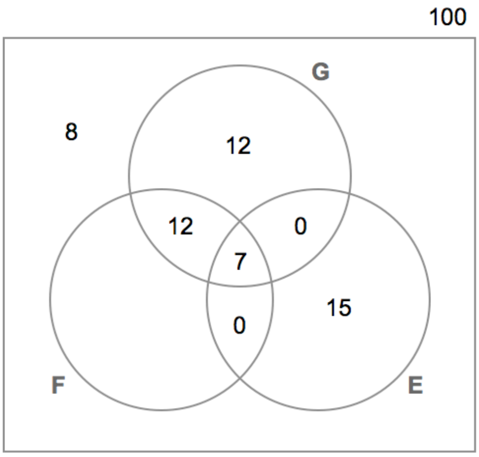
50 The easiest way to do this is probably to draw a Venn diagram :



As you can see, we've already entered the first four pieces of information we've been given.

The next thing we read is that all those who spoke English and French also spoke German. Another way of putting this is : percentage speaking English and French but not German = 0. So we can put this in :



Then we read that 31% spoke German. But the **G** set already has a total of 31 in it, so the remaining bit must contain 0. We also read that 22% spoke English, which means that (as we've already put in a 7 and two 0s) we must put 15 in the last part of the **E** set.

Finally, subtracting from 100 all the numbers we've entered, we get 54, the number we're after. So now we can say that 54% of these tourists spoke only French.