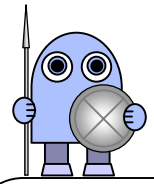
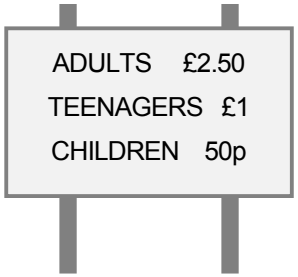
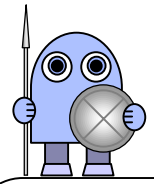


ferry fares

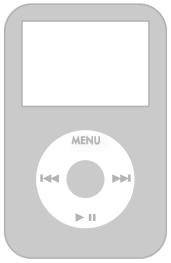


Felix is a ferryman. He has a small boat and takes people back and forth across the River Dee. You can see his charges on the poster above. Some days he has lots of passengers and some days just a few. Last Tuesday morning, Felix noticed something a bit special – he had ferried 20 passengers (some adults, some teenagers and some children) and he had taken exactly £20.

Work out how many of each type of passenger there must have been on Tuesday morning to make this possible.



mean downloads



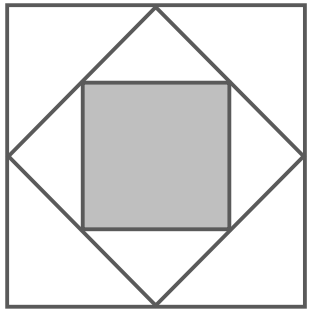
On Monday morning, Daniel downloaded five tracks to put on his ipod. Here are the prices he paid for these tracks :

89p / £1.18 / 89p / 89p / £1.15

a What's the (mean) average price of these tracks ?

b Later in the day Daniel downloaded another track and this brought his average payment down to 96p per track. How much did Daniel pay for the extra track ?

a shady square . . .



The small shaded square has an area of 9 cm^2.

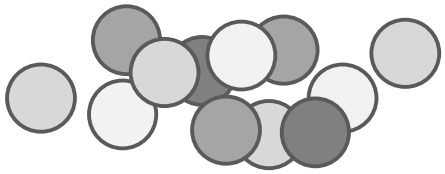
a What's the area of the middle square ?

b How long are the diagonals of the middle square ?

c How long are the sides of the outside square ?

MATHS CHALLENGE CARDS SET D

counter-attack

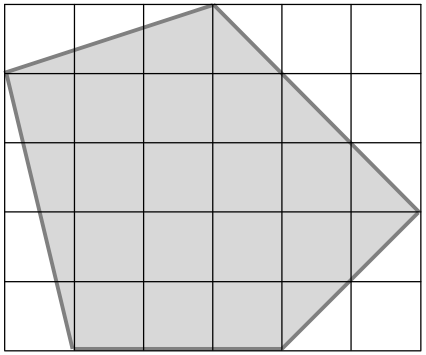


Mr Pascal, the maths teacher, has a pile of counters on the desk in front of him. He asks the class to guess how many there are. Here's the information he gives them :

- the number is under 60
- divide the number by 2, you get a remainder of 1
- divide the number by 3, you get a remainder of 2
- divide the number by 4, you get a remainder of 3
- divide the number by 5, you get a remainder of 4

How many counters are there on Mr Pascal's desk ?

pentagon area



a Use any method you like to work out the exact area (in squares) of the shaded pentagon.

b Now draw a fresh 6 x 5 rectangle and draw a pentagon inside it, different in shape from the one above but with the same area. Your pentagon must touch all four sides of the rectangle.

powers of 2 . . .

Look at this sequence of sums :

$$1 = 1$$

$$1 + 2 = 3$$

$$1 + 2 + 4 = 7$$

$$1 + 2 + 4 + 8 = 15$$

$$1 + 2 + 4 + 8 + 16 = 31$$

Can you see a pattern here ? If you can, write down the answer to this :

$$1 + 2 + 4 + 8 + 16 + \ldots + 1024 =$$

Please don't just add them all up !

at the pumps

Three cars pull in for petrol at the Mile-End filling station – a BMW, a Ford and a Land Rover. One of these cars is red, one is blue and one is white. The three drivers are Sue, Mike and Ed. Here are some interesting facts :

- the white car is a BMW
- Ed owns the Land Rover
- Sue doesn't drive a Ford
- Mike's car is red
- Ed doesn't drive a white car

Use this information to work out : What colour is the BMW ? – and who drives it ?

pulp fraction

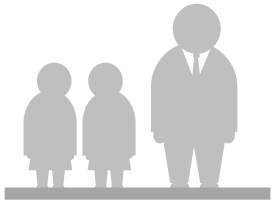
How do you turn 0.9 into a fraction ? The answer is simple – it already is one ! Because 0.9 just means 9/10. And in the same way, 0.27 just means 27/100.

But sometimes, to arrive at a fraction *in its lowest terms*, you have to do some cancelling. For example, 0.15 = 15/100, which (dividing top and bottom by 5) cancels down to 3/20. It's easy !

Now try turning each of these decimals into a fraction in its lowest terms :

a	0.3	**e**	0.35
b	0.8	**f**	0.85
c	0.5	**g**	0.125
d	0.75	**h**	0.375

double jeopardy



Millie and Mary are twins. Here's what they say about their ages and about their father's age :

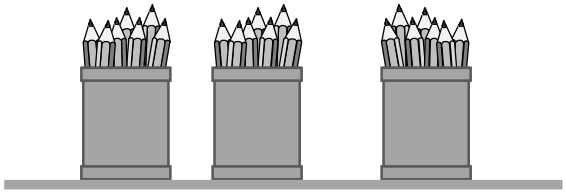
- Millie says, 'If you add our ages together and multiply by 3, you get dad's age.'
- Mary says, 'Just square my age to get dad's age.'

Now here are your questions :

a How old is the father of these two twins ?

b How many more years will it be before you can just double the sum of the twins' ages to get their father's age ?

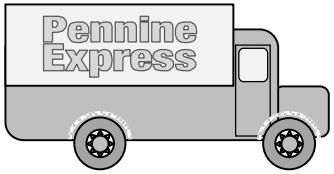
drawing the crowds



Miss Mondrian, the art teacher, wants to start an outdoor drawing club and wonders how many pupils will come along. Ben guesses 60 and Sally guesses 20. The actual number turns out to be between the two guesses. In fact, if you lower Ben's guess by a certain percentage, you get the right answer – and if you increase Sally's guess by the same percentage, you also get the right answer.

Using this information, work out how many pupils actually come along to the drawing club.

not so fast . . .



Andy drives a lorry for the Pennine Express delivery company, taking parcels to Yorkshire and the north-east. Last week he travelled from Manchester to Sunderland, a total distance of 120 km. He stopped half-way for a cup of coffee and a rest, so the journey was in two equal stages, each of 60 km. During the first stage Andy averaged just 20 km / hr (the roads were busy) – but during the second stage he averaged 30 km / hr.

What was Andy's average speed for the journey as a whole ?

five numbers . . .

Mr Bourbaki, the maths teacher, writes down five numbers on a sheet of paper. He tells his class that they must work out what the numbers are. Here's what he tells them :

- none of the numbers is bigger than 10
- the mean (average) of the numbers is 6
- the largest number is 8 more than the smallest
- the median (middle number) is 6
- one of the numbers is the square of another one
- one of the numbers is the product of two others

an odd place to live . . .

This question is all about people living in houses on the odd-numbered side of their street or road. In Ben's avenue, for example, there are just a few houses on the odd side : 1, 3, 5, 7 and 9. You can quickly see that there are five houses here and that the middle one is number 5.

a James lives on the odd side of Trafford Road and here the numbers go from 1 up to 77. How many houses are there here and which is the middle one ?

b Alex lives on the odd side of Wellington Road, where unusually the numbers go from 29 up to 125. (Numbers 1 to 27 were bombed in the war and were never rebuilt). How many houses are there here and which is the middle one ?

two magic squares

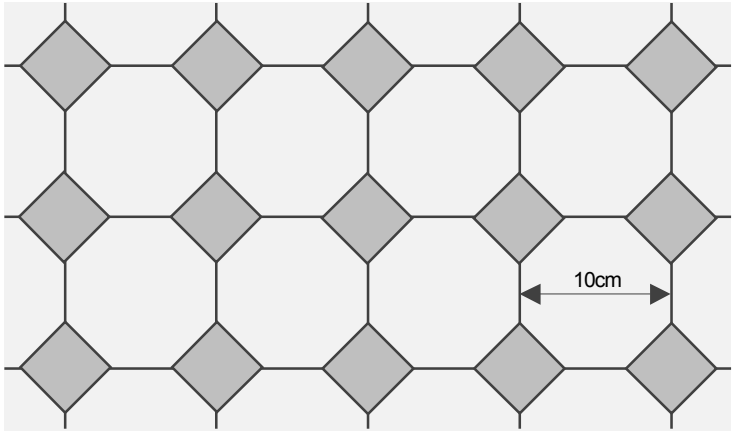
Find the right numbers to complete these two magic squares :

	24	
	13	
12		

		27
28	15	2

** remember, a magic square must have the same total for each row, column and diagonal.*

floor tiles



You've probably seen a floor somewhere tiled like this, with a mixture of octagons and squares. In this particular one, the octagons are 10 cm across (from side to side or from top to bottom) and the small squares all have a side length of 3 cm. Can you work out what percentage of the floor is covered by the octagons ?

square cubes

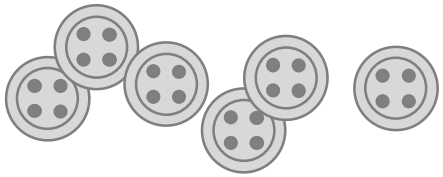
- 64 is a special number : it's both a square number ($64 = 8^2$) and a cube number ($64 = 4^3$).
- 729 is a special number in the same way : it's a square number ($729 = 27^2$) and it's a cube number at the same time ($729 = 9^3$).

a Try to explain exactly why these two numbers can be square numbers and cube numbers at the same time.

b Are there any numbers below 50 which are special in the same way ?

c What's the smallest power of 10 which is both a square number and a cube number ?

shirt alert !



Bob and Emma work in a shirt factory. When the shirts have been sewn together, they put the buttons on. Here's some information about how they work :

- Working together, Bob and Emma can finish 15 shirts per hour.
- On his own, Bob can finish one shirt every 10 minutes.

If Emma is working on her own, how many shirts per hour can she finish ?

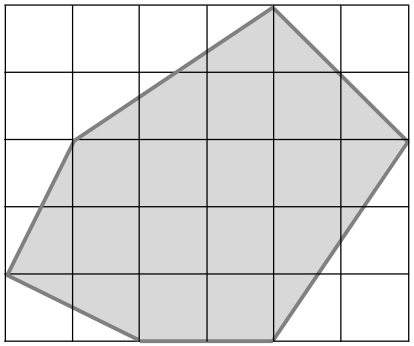
house-bound rat

Five rats live together in a cellar in Liverpool. They're called Annabelle, Boris, Charlie, Daisy and Eric. At any hour of the day or night, there will always be two of them out hunting for food and three of them at home keeping house. Here's how it works :

- Boris and Daisy hate each other, so they must never be out together or at home together.
- No two females must ever be out together – the outdoor hunting pair must always be either two males or a male and a female.
- Eric is happy to go out hunting with any of the rats but he can't be left at home with Charlie.

Which unlucky rat never gets to go out at all ?

hexagon area

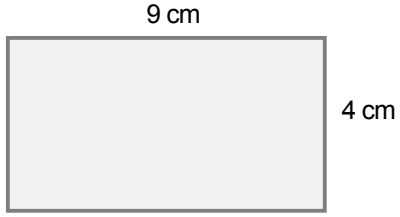


a Use any method you like to work out the exact area (in squares) of the shaded hexagon.

b Now draw a fresh 6 x 5 rectangle and draw a parallelogram inside it with the same area as the hexagon above. Your parallelogram must touch all four sides of the rectangle.

odd perimeter

Take a look at this rectangle :

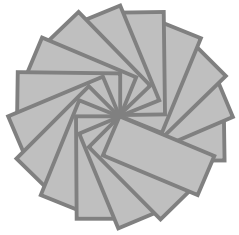


As you can see, the rectangle has an area of 36 cm^2 and a perimeter of 26 cm. A 3cm x 12cm rectange or a 6cm x 6cm square would have the same area. Notice that for all these rectangles, the perimeter is an even number.

a Can you think of a rectangle with an area of 36 cm^2 but whose perimeter is an odd number ?

b Can you think of a rectangle with an area of 24 cm^2 whose perimeter is an odd number ?

frog romance



Romeo and Juliet are two frogs who live by a pond near an old church. The frogs often climb to the top of the church tower, which has 200 steps from top to bottom. At 10:45 one Sunday morning, Juliet is on step number 197 and Romeo is on step number 22. The church bell starts ringing, inviting people to the morning service. On every chime, Romeo jumps 2 steps upwards and Juliet jumps 3 steps downwards.

Will the two frogs eventually land on the same step ?

take your pick !

Dan has a bag with 20 M&Ms in it. There are 8 red ones, 6 blue ones and 6 green ones.

a Rebecca puts her hand into the bag and, without looking, takes out an M&M and eats it. What's the probability that she's had a red one ?

b Suppose Rebecca's M&M was a red one. If Dan now puts his hand into the bag and, without looking, takes out an M&M, what's the probability that his M&M is *not* a red one ?

special operation

The operation a ○ b combines two numbers in this way :

a ○ b = 100 - ab

In other words, to find a ○ b for two numbers a and b, you just find the product of a and b and take it from 100.

example : 2 ○ 5 = 100 – 10 = 90

a What's 5 ○ 7 ?

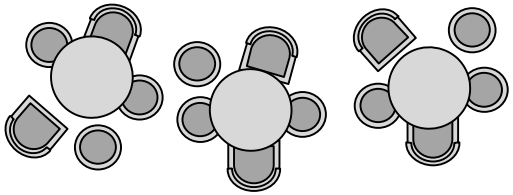
b What's 10 ○ 10 ?

c What's 5 ○ (9 ○ 10) ?

d What does n equal if 2 ○ (n ○ n) = 62 ?

note : Where there are brackets, you should work them out first.

100 wooden legs

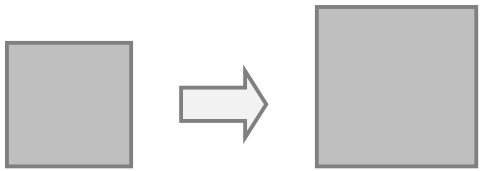


At the Lighthouse Inn in Barmouth, all of the seats are wooden – but some of them are chairs (4 legs) and some of them are stools (3 legs).

a In the main room there are 28 seats – and the barman counts that these seats have 100 legs altogether. How many of these seats are chairs and how many are stools ?

b In the small room there are fewer seats. The barman counts 42 wooden legs altogether. What's the smallest number of seats there could be to make this many legs ? What's the largest number ?

growing squares . . .



Ruben has a square flower-bed in the middle of his lawn; this flower-bed measures 5m x 5m. Ruben decides he'd like to make the flower-bed a little larger, so he increases the length of each side by 20%.

a What's the new area of the flower-bed ?

b By what percentage has the flower-bed increased in area ?

MATHS CHALLENGE CARDS SET D

boy racer

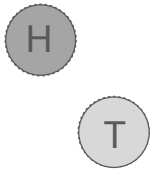
The cycle path from the main road to the library is exactly 72 metres long. Ben and his dad turn onto this path and Ben straight away starts to cycle twice as fast as his dad. Ben is soon far ahead and when he reaches the end he immediately turns around and heads back towards his dad.

a How far has Ben cycled when he meets his dad again ? How far has the dad cycled ?

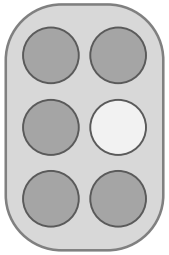
b If, when they meet, Ben straight away turns around, cycles to the end and then turns round, heading back towards his dad, how far are they both from the end when they meet again?

eggs and coins

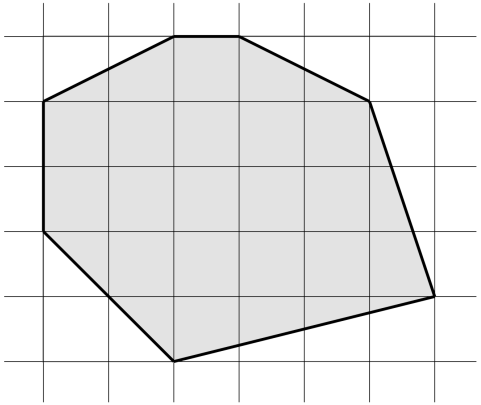
a John has two coins. He throws them up in the air together, catches them and then slaps them down on the table. What's the probability that he's got a head and a tail ?



b John's mum has an egg-box in the fridge, containing five brown eggs and one white one. Without looking, she takes out five eggs to make an omelette. What's the probability that all five eggs are brown ones ?



tricky areas



a Use any method you like to work out the exact area (in squares) of the shaded heptagon.

b Now draw a 4 x 4 square and inside it draw an octagon – an octagon which touches all four sides of the square and which has bilateral (mirror-image) symmetry. What's the area of your octagon ?

know your place !

Use the following information to work out who is standing where in this line of six children :

- Rufus is between two girls.
- Daisy is further forward than Emily.
- Jake is two places behind Sue.
- Sue and Daisy are next to each other.
- Emily is last in the queue.
- Anna is three places ahead of Daisy.

Tribonacci

You know how a Fibonacci sequence works : you just add two consecutive terms together to get the next one.

A *Tribonacci* sequence is similar – but here you add three consecutive terms together to get the next one, eg starting with 2, 3, 4, you get : 2, 3, 4, 9, 16, 29, 54, 99 . . .

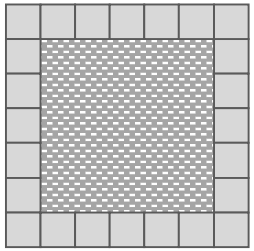
a Fill in the gaps in the Tribonacci sequence

1, __ , __ , 9, 17, __ , 57 . . .

b Find three starting numbers which would give you 13 as the sixth term of a Tribonacci sequence.

c Find three starting numbers which would give you 45 as the sixth term of a Tribonacci sequence.

Bob the gardener



Bob and his wife have just bought a new house. There's a huge lawn in front and Bob's wife says it would look better with a flower-bed in the middle. There are lots of square paving slabs in the garage, so Bob thinks he'll dig the square flower-bed and put a path around it using some of the slabs. Each slab measures 1 metre x 1 metre.

Bob thinks it would be good if the area of the slabs came to exactly the same as the area of the flower-bed. Can you find the right number of slabs to make this possible ?

odd squares

Look at this sequence of numbers :

$1^2,\ 3^2,\ 5^2,\ 7^2,\ 9^2\ \ldots$

As you can see, these are just the odd square numbers.

a. If you subtract any one of these from the next one, you always get a multiple of 4. Find a way of explaining why this happens. **A diagram might help.*

b. There's a pattern to how many lots of 4 you end up with when you subtract one odd square from the next one. Find this pattern and then, without any working out, write down the answer to the question 'How many lots of 4 will I get if I subtract 99^2 from 101^2 ?'

c. Does the same sort of thing happen with even square numbers ?

bigger fields

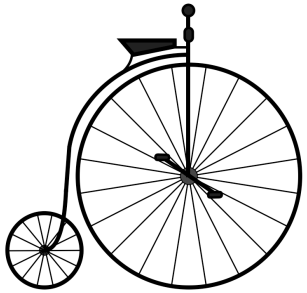
Glyn and John are two friends. Each of them owns a field and each one has a horse. Glyn's field is somewhere between 1 and 2 acres in size and so is John's.

a Glyn is thinking of buying some more land to make his field bigger. One evening he's is in the Plough Inn and he says to his friend John, 'If you multiply the size of my field by 5, you get just the same result as if you add 5 acres to the size !' What exactly is the size of Glyn's field ?

b * John says, 'Well, if you multiply the size of my field by 2¼ , you get the same result as if you add 2¼ acres to the size !' What is the size of John's field ?

** this part is harder* !

penny farthing



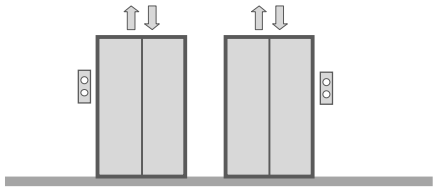
The year is 1875 and Lewis has just bought his first penny-farthing bicycle. The diameter of the large wheel is 135 cm and the diameter of the small wheel is 45 cm.

a. On Lewis's first journey, the front wheel covers a distance of 405 metres. What distance does the small wheel cover ?

b. On a later journey, the small wheel rotates exactly 1000 times. Roughly how far is this journey ?

* *you'll need to know that the circumference of a wheel (the distance all round it) is about 3 times the diameter.*

lift-off



- Bruce, on a trip to the USA, arrives in New York on a Monday and heads for his hotel. It's exactly 12 noon when he steps into the lift and starts his ascent to the 28^{th} floor (a height of 140 metres). The lift climbs at a speed of 2 metres per second.
- Bruce sleeps well and next day it's exactly 12 noon when he steps into the lift to go down to the ground floor. The lift goes down at a speed of 5 metres per second.

Does the lift pass any point at exactly the same time on the Tuesday as it did on the Monday ?

magic and anti-magic

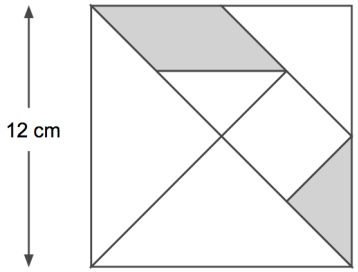
3	4	1
0	2	6
7	5	8

In a 3 x 3 *magic square*, adding up the numbers along the rows, columns and diagonals will give you exactly the same total over and over again. In a 3 x 3 *anti-magic square*, adding up the numbers in the rows, columns and diagonals will give you eight different totals.

a Using each of the numbers from 0 to 8, create your own magic square. (There are different ways of doing this but you'll find they all have the same 'magic total'.)

b Using each of the numbers from 0 to 8, create your own anti-magic square. You'll need to produce eight different totals from the rows, columns and diagonals !

it takes two to tangram . . .



You've probably come across the ancient Chinese *Tangram* puzzle. This has seven pieces (called *tans*) which you have to arrange into various different shapes. The diagram above shows how you can make a square. As you can see, this particular square has a side of 12 cm.

a What's the area of the shaded parallelogram ?

b What's the area of the shaded small triangle ?

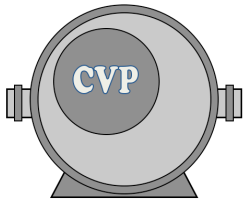
c What would your two answers be if you reduced the side of the square to just 6 cm ?

five numbers . . .

Use any method you like to work out what these five whole numbers must be :

- **a** is a whole number less than 100; it' s also a prime number and it's 2 less than a nearby square number. As well as that, if you square **a**, you get the 4-digit number 2 * * 9.
- **b** is a whole number. If you add together one third of **b** and one quarter of **b**, you get 21.
- **c** is exactly half-way between 11^2 and 13^2.
- **d** is the unknown number in the following sequence : 7, 18, 40, **d**, 172, 348 . . .
- **e** is half of the square-root of 60% of 60.

pump action

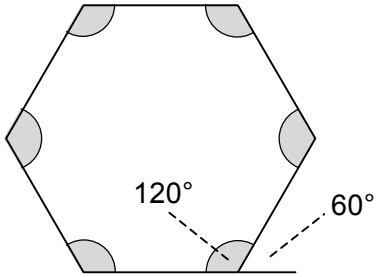


The Chelsea Village Pump factory is famous for central heating pumps. These sturdy pumps come in three different sizes : small (weighing 10 kg), medium (weighing 13 kg) and large (weighing 17 kg).

One morning, Mark (a central heating engineer) takes his van to the CVP factory to get some pumps. The pumps he buys weigh 100 kg altogether, with fewer small pumps than any other size.

How many pumps of each size does Mark buy ?

equal angles



In a regular hexagon, each interior angle is 120° and each exterior angle is 60°.

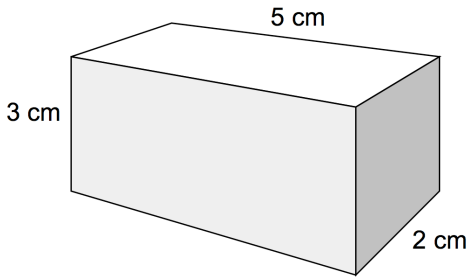
a What do the hexagon's exterior angles add up to ?

b What do a regular *pentagon's* exterior angles add up to ? Use your answer to work out what each interior angle of a regular pentagon must be.

c Now use the same method (or a different one if you prefer) to work out the interior angle of a regular *octagon*.

taking sides . . .

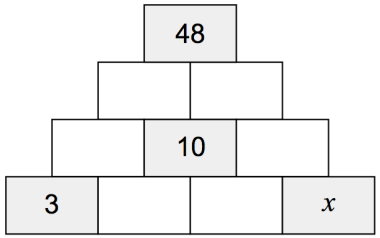
You know how to find the *surface area* of a cuboid : just add together the areas of the six faces ! For example, the cuboid below has sides of 2cm, 3cm and 5cm, so its surface area is $(2 \times 6) + (2 \times 5) + (3 \times 5) = 62$ cm^2.



Another cuboid has a total surface area of 82 cm^2. One of its sides is 2cm. What could the lengths of its other sides be ? (Stick to whole numbers for this one !)

you're my number-wall !

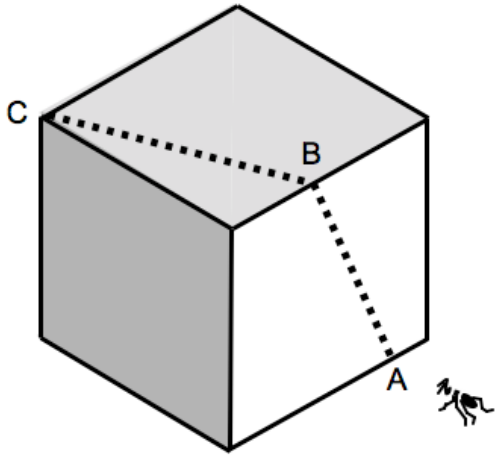
You know how a number-wall works : each number-brick is the sum of the two it's resting on. Often you're given a number-wall with a few gaps and you're asked to fill them all in. Here the problem is a bit different. First of all, take a look at this number-wall :



a What must x be ?

b What can you say about the two missing bottom-row numbers (the ones in the middle) ? Which pairs of numbers give the same total at the top ?

cubist ant

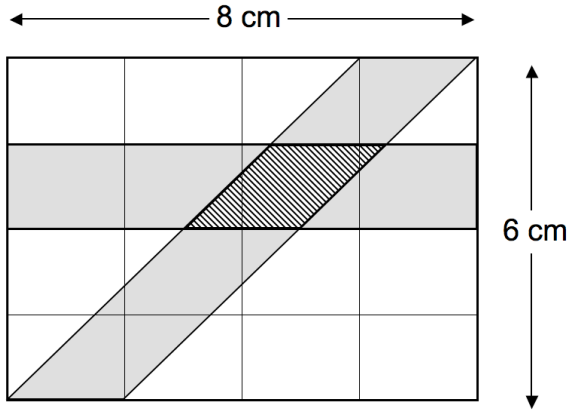


An ant comes across a 6cm wooden cube and thinks he'll climb on it. Starting at point A, just 1cm from the nearest corner, he decides to head for corner C by the shortest possible path, as shown above. Using a ruler and squared paper (or in any other way), work out :

a The distance from B to the nearest corner.

b The length of the shortest path from A to C.

parallel universe



This 8cm x 6cm rectangle has two bands drawn across it. You can see from the grid exactly where these bands lie on the rectangle.

What's the area of the small parallelogram formed where the two bands intersect ?

keep cool !

You know that termperature is usually measured in either degrees Fahrenheit (°F) or degrees Celsius (°C). To convert a reading from °F to °C, you just subtract 32, divide by 9 and multiply by 5. For example, 95°F becomes 35°C.

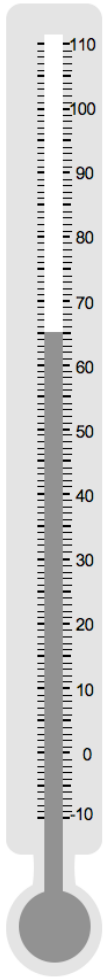
a What's 50°F in °C ?

b What's 140°F in °C ?

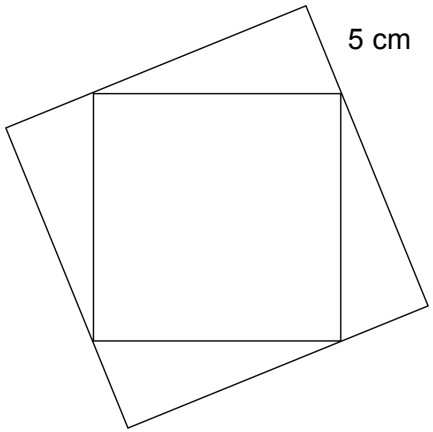
c What's 32°F in °C ?

d What's 85°C in °F ?

e Which temperature is the same in both °C and °F ?



square deal



The diagram shows one square drawn inside another.

- The area of the inner square is 169 cm^2.
- The area of the outer square is 289 cm^2.

As you can see, the shortest side of the right-angled triangle at the top is 5 cm. How long are the other two sides ?

it all adds up . . .

Take any *three* consecutive numbers and add them up. You will find that the total you get is 3 x the middle number.

example : 15 + 16 + 17 = 3 x 16

In the same way, if you add up any *five* consecutive numbers, the total is always 5 x the middle number.

example : 7 + 8 + 9 + 10 + 11 = 5 x 9

Now look at this set of consecutive numbers :

29 + 30 + 31 + . . . + 100 + 101

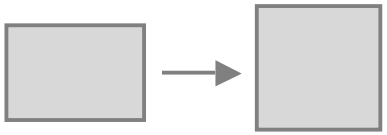
a How many numbers are there in this set ?

b What's the middle number in this set ?

c What do these numbers add up to ?

* *Don't try to answer these questions by writing the whole list out !*

squaring up



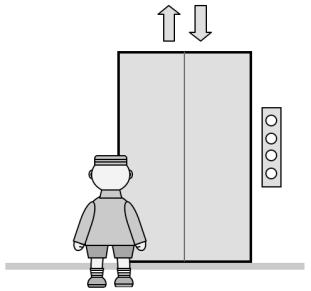
In this question we're just interested in whole numbers.

a Could n multiplied by (n + 1) ever be a square number ?

b Could n multiplied by (n + 2) ever be a square number ?

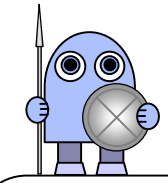
c What about n multiplied by (n + 3) ?

last man in . . .



Some people got into a lift; their mean (average) weight was 80 kg. On the way down, the lift stopped and a small boy got in, making the mean (average) weight of the people in the lift go down to just 70 kg.

If the boy himself weighed exactly 30 kg, can you work out how many people were in the lift in the first place ? Use any method you like.



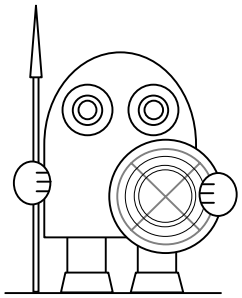
United Nations . . .

A survey of tourists at a Mediterranean holiday resort reported these facts about their knowledge of the English, French and German languages :

- 8% spoke none of these languages.
- 12% spoke only German.
- 7% spoke all three of these languages.
- 12% spoke French and German but not English.
- All those who spoke English and French also spoke German.
- 31% spoke German.
- 22% spoke English.

What percentage of these tourists spoke only French ?

four winds maths



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