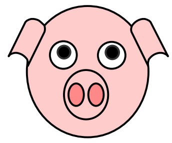
intro

This is an investigation which is relatively straightforward to explain and enjoyable to carry out. But it's not so easy to find a pattern in the results . . .

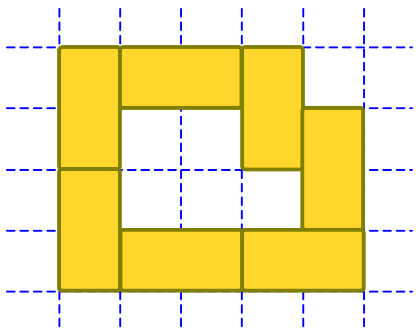


first steps

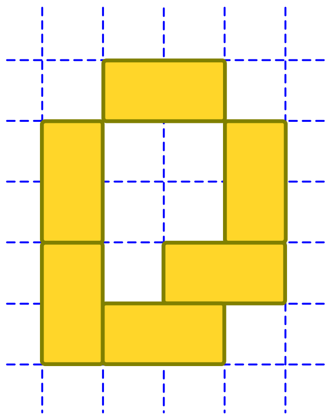
The pigs at Croft Farm have some of the most comfortable sleeping quarters in the world. On hot summer nights, though, some of them prefer to sleep outdoors. The farmer is happy enough to let them do this but he always makes up a pig pen from bales of straw, just to discourage them from roaming.

These bales of straw – looking at them from above – are 2m by 1m rectangles and they have to be put together with at least 1m adjoining (pigs can eat their way out of corner-to-corner arrangements). The other thing to remember is that these pigs are sociable animals, so they always sleep together in one pen. However, every pig does like its own square metre of field to sleep on – so if for example you've got 5 pigs sleeping out, you need a pig pen enclosing 5m^2 (5 square metres) . . .

like this perhaps :



but definitely not like this :



As you can see, the arrangement on the left uses 7 bales of straw. Over the years, the farmer at Croft Farm has become quite an expert at arranging bales of straw for his pigs.

the investigation

The challenge here is to find – for each number of pigs – the smallest number of bales needed to make a pig pen. Remember the rules :

- The pigs should be in one single pen – with 1m^2 for each pig.
- Corner-to-corner arrangements for the bales are not allowed.
- Wherever one bale is next to another, the join must be at least 1m long.

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practical

This investigation can be done as a pencil and paper exercise but it's easier and more fun to use something (face-down dominoes are ideal) to stand for the bales – and to have a suitable grid to work on (see the photocopy masters). The exercise works well if children work in pairs or in small groups. 1cm square grid paper is fine for recording the results. Some children like to draw a pig's face in each square of the pig-pen . . .

results

Children will obviously be aware from the start that the more pigs you want to enclose, the more bales you'll need. And they'll soon discover that for any particular number of pigs, they can arrange the enclosure in different ways. Even when they agree on the smallest number of bales needed in a particular case, they might still find they've got different ways of arranging the bales. In the end, though, there should be no ambiguity about the actual minimum number of bales needed in each case. Here's our table of results for up to 20 pigs in an enclosure (and for interest we show our set of pig-pens on the last page) :

number of pigs	smallest number of bales		number of pigs	smallest number of bales
1	4		11	9
2	5		12	9
3	6		13	10
4	6		14	10
5	7		15	10
6	7		16	10
7	8		17	11
8	8		18	11
9	8		19	11
10	9		20	11

The interesting question is : can we find a pattern in here ?

There's nothing immediately obvious – so let's try arranging the results in a different way. We can see that 4 bales is the minimum number of bales for a 1-pig enclosure, 5 bales is the minimum number for a 2-pig enclosure, 6 bales is the minimum number for both a 3-pig enclosure and a 4-pig enclosure . . .

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We can summarise all this in a table :

number of bales	= minimum number for
4	1
5	2
6	3, 4
7	5, 6
8	7, 8, 9
9	10, 11, 12
10	13, 14, 15, 16
11	17, 18, 19, 20

So 4 is the minimum number of bales for just 1 size of pig-pen. And 5 is the minimum number for just 1 size of pig-pen. But 6 is the minimum number for two different sizes of pig-pen . . . and so is 7. Let's follow this idea through :

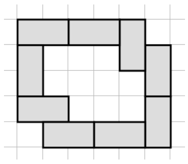
number of bales	= minimum for how many sizes
4	1
5	1
6	2
7	2
8	3
9	3
10	4
11	4

It rather looks as though we've found a pattern . . .

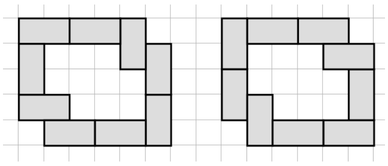
extension

Here are three ways of extending the investigation :

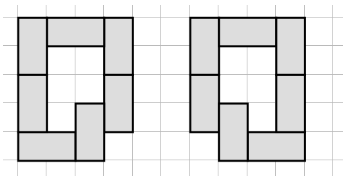
- In the first stage of the investigation we were looking for the minimum number of bales to enclose a given number of pigs. Pupils will soon have noticed that they often came to the same result – but with different arrangements of the bales. One obvious thing to investigate is : just how many different ways are there of getting the various results? But before embarking on this, it's definitely worth getting some agreement on what should count as 'different' arrangements. For example, you can make a pig-pen for 10 pigs with just 9 bales. Here's one way of doing it :



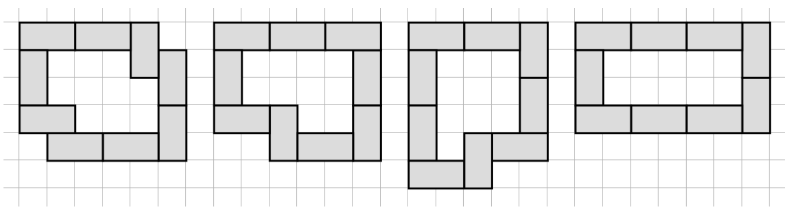
You obviously wouldn't want to count a simple re-ordering of the bales as a different arrangement :



Different *rotations* of one arrangement clearly shouldn't be counted as different arrangements – but you'll have to decide whether or not to allow *reflections* to count as different arrangements :



Most people probably wouldn't. What we're really looking for here is this : how many completely different *shapes* of enclosure can we find for each result in the table? There are plenty to find. For example, here are four different shapes for 9 bales enclosing 10 pigs :



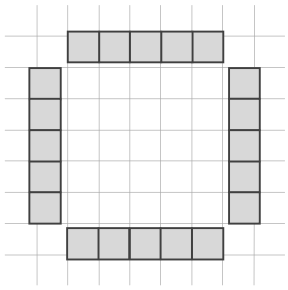
- Pupils who have the stamina can investigate larger enclosures (that's to say those holding more than 20 pigs). Here the interesting question is : does the pattern we found continue after 20?
- Square enclosures are obviously a special case and worth looking at on their own. There's a straightforward connection between the number of pigs / size of square and the number of bales needed – and it's a nice exercise for pupils to identify this pattern. To begin with, pupils could write down what happens in the first few cases. This is what they should find :

number of pigs		number of bales
1	→	4
4	→	6
9	→	8
16	→	10
25	→	12

Next, you could ask them to look at one particular case and use a diagram to show *why* the answer is what it is. Let's consider, for example, the 25-pig pen. We know $25 = 5^2$, so what we're looking at is how many bales we need to enclose a 5 x 5 square.

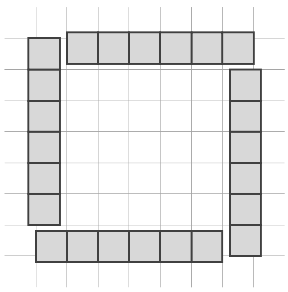
Let's imagine using just simple $1m^2$ bales to begin with :

We need 5 of these small squares to go along each side of the enclosure



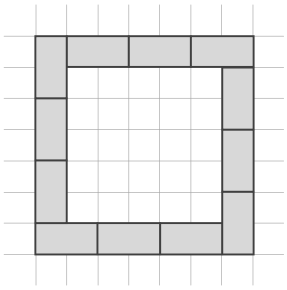
– so that's 4 lots of 5 small squares we need for the whole enclosure

But we also need a small square to cover each corner . . .



– so now that's 4 lots of 6 small squares we need for the whole enclosure

Finally, we have to remember that the bales we're actually using are not $1m^2$ but $2m^2$ . . .



– which means that the total number of bales needed for the enclosure = 2 lots of 6

Conclusion : for a 5^2 pen we need 2 x 6 bales.

– and if we looked at a 9^2 pen, we'd find we needed 2 x 10 bales; for a 12^2 pen we'd need 2 x 13 bales, and so on . . .

In algebra terms, the rule for how many bales you need for an n x n square enclosure is :

$$n \rightarrow 2(n + 1)$$

nb The photocopy masters are grids for use with standard 22mm x 44mm dominoes (these are the smaller dominoes sold in packs containing sets of different colours); the first grid can be printed as a straight A4 sheet whilst the second grid needs to be enlarged to A3 for pupils needing a larger working space.