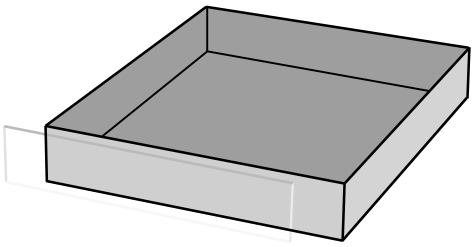
square nets

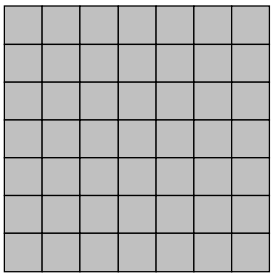
intro

This investigation is about open-top boxes and their nets; starting with a given square of card, the idea is to produce a number of different boxes and then to see which of them has the largest volume. It's a good investigation because at first it looks easy to see what's going on – but then further research shows that it's not quite so straightforward after all . . .

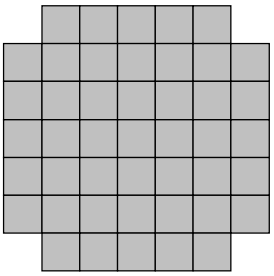


the investigation

For a given size of square card you can make different nets by cutting away larger or smaller square corners. For example, suppose you start with a 7cm x 7cm piece of card, like this :

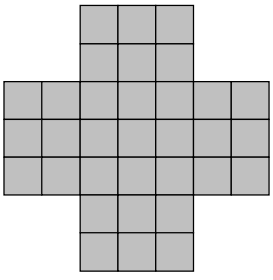


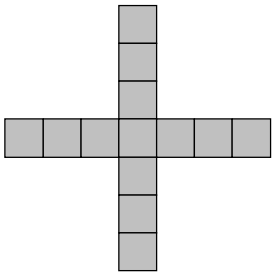
Now cut away a 1cm x 1cm square from each corner :



This net will give you a 5cm x 5cm x 1cm open-top box (rather like the one at the top of the page).

– and here are the other two possibilities :





The first of these boxes has a volume of $25cm^3$ and the other two have volumes of $18cm^3$ and $3cm^3$.

first steps

Start with a 3cm x 3cm piece of card; there's only one box you can make – and the same is true of the 4cm x 4cm version. The question of largest possible volume doesn't arise. But with a 5cm x 5cm piece of card you can cut away 1cm square corners or 2cm square corners, giving you two different shapes of box. A 6cm x 6cm card also produces two alternatives whilst the 7cm and 8cm versions give you three . . . With every one of these first four cases, it's the option involving cutting just a 1cm x 1cm square from each corner which generates the box with the largest volume. After investigating these first few cases, some children will think they've seen the pattern but . . . a 9cm x 9cm piece of card gives you four possibilities and suddenly you see that it's not the first option which gives the maximum volume!

*nb in this investigation we're sticking to whole numbers throughout.

practical

Decide (according to the group involved) whether you want your pupils to make up some nets from card or whether they can investigate by drawing nets on squared paper and then just sketching the boxes. It's not essential to make the boxes but it might be a good lead-in to make the odd one or two (larger size perhaps) to demonstrate at the beginning.

How to record results? Encourage them to find a sensible way of setting down their findings.

further questions

**for pupils who have not seen a table of results*

- In every case, one of the results is a square number. Why?
- Which sizes of card would you use to make a box with volume = $100cm^2$? What about boxes with volumes = $200cm^2$, $300cm^2$, $400cm^2$, or $500cm^2$
- Which sizes of cards allow you to make the nets of cubes?
- Could you ever get two boxes, one with double the volume of the other, starting from the same size square?

notes

Explanation / calculation of max volume for a given square of card is beyond scope of this age-group – which makes it a bit unsatisfying for them in one way ie in the end there isn't a clear pattern which they can identify. However, the investigation is still a valuable one as this is just the sort of thing which often happens to 'real' mathematicians: you can find something of a pattern but can't really tie it down or explain it (yet) . . . Usefully, though, you can once again drive home the need to look at enough cases before forming a judgement about what's going on in any mathematical situation . . .

results

Here's a complete set of results for square nets up to18cm x 18cm:

square edge	corner edge	volume		square edge	corner edge	volume	
4	1	4	⇦	14	1	144	
5	1	9	⇦	14	2	200	⇦
5	2	9		14	3	192	
6	1	16	⇦	14	4	144	
6	2	8		14	5	80	
7	1	25	⇦	14	6	24	
7	2	18		15	1	169	
7	3	3		15	2	242	
8	1	36	⇦	15	3	243	⇦
8	2	32		15	4	196	
8	3	12		15	5	125	
9	1	49		15	6	54	
9	2	50	⇦	16	1	196	
9	3	27		16	2	288	
9	4	4		16	3	300	⇦
10	1	64		16	4	256	
10	2	72	⇦	16	5	180	
10	3	48		16	6	96	
10	4	16		16	7	28	
11	1	81		17	1	25	
11	2	98	⇦	17	2	338	
11	3	75		17	3	363	⇦
11	4	36		17	4	324	
11	5	5		17	5	245	
12	1	100		17	6	150	
12	2	128	⇦	17	7	63	
12	3	108		17	8	8	
12	4	64		18	1	256	
12	5	20		18	2	392	
13	1	121		18	3	432	⇦
13	2	162	⇦	18	4	400	
13	3	147		18	5	320	
13	4	100		18	6	216	
13	5	45		18	7	112	
13	6	6		18	8	32	