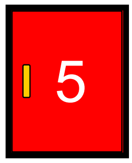
1000 lockers

intro

The well-known ‘1000 lockers problem’ provides an interesting investigation for pupils in this age-group. The first challenge is to explain the problem; after this you can get your pupils to look at a simpler version and to investigate step-by-step how things develop. There is a simple pattern waiting to be found – which can then be used to solve the original problem.



The really hard question is: why do we get this particular pattern?

the problem

A college has 1000 students and just off the entrance hall there’s a bay with 1000 lockers, one for each student. On the first day of term, the lockers are all closed but student no 1 arrives early in the morning and straightaway opens the doors of all 1000 lockers. Student no 2 arrives and closes the doors of all the even-numbered lockers (ie 2, 4, 6, 8 and so on). Later, student no 3 turns up and he focuses on all the lockers numbered with a multiple of 3 (ie 3, 6, 9 and so on), opening those which are closed and closing those which are open. Student no 4 arrives and goes along lockers 4, 8, 12, 16 and so on (multiples of 4), again opening those which are closed and closing those which are open.

During the morning all the students arrive in turn (ie in number order) and they each do the same thing ie (*here you could ask your class ‘is there a neat way of describing what it is each student does when it’s his/her turn?’*) they change/reverse the ‘state’ of those lockers numbered with multiples of their own id number.

The problem is : How many lockers will remain open when all 1000 students have visited them, opening and closing in the way described above?

first steps

Obviously the first thing is to make sure that your pupils really do understand the problem. After this, remind them that when they have a difficult problem it’s often useful to look at a simpler version of it; here we can start by investigating what happens with say just 20 lockers and 20 students . . .

the investigation

Pupils should look at a bank of 20 lockers and systematically chart what happens as the students come along opening and closing the lockers. They will need to work carefully – but before too long they should begin to see a pattern . . .

practical

Pupils can work on their own or in twos or threes. It isn't easy to find a simple practical set-up here. A card strip with 20 'lockers' and card flaps works reasonably well – or alternatively, each group can mark 20 squares on a card strip and use two-sided counters to represent the open / closed states. If you have easy access to a computer lab, then pupils can use a spreadsheet, numbering a row of 20 cells, using two different cell colours to stand for the open / closed states and copying each completed line before making the changes needed for the next one . . .

Squared paper and coloured pens are fine for recording the results line by line and should make it easy enough for pupils to see the pattern.

results

Before too long pupils will realise that it seems to be the lockers with square numbers on them which stay open. Once they've completed their investigations with 20 lockers they should find that the ones still open are those numbered 1, 4, 9 and 16 (four lockers in all).

answer / explanation

- To work out what happens with 1000 lockers, it seems that we just need to know how many square numbers there are up to 1000 . . . 30^2 is 900 and 31^2 is 961 but 32^2 is 1024, which is too large. So there are 31 square numbers in the first 1000 whole numbers – and the answer to our original problem is 31 lockers!
- The real challenge is to answer the question : why is it that in the end it's the lockers with square numbers on which are left open? It's worth getting pupils to think about this and to try to come up with an explanation. If they're completely stuck, get them to think about factors . . . and to focus not on what each student does but on what happens to *individual lockers* . . . ask them to look at some lockers which have square numbers on them and some which don't . . .
- 12 for example has six factors so that during the exercise, three students will find locker number 12 closed and will open it and three students will find the locker open and will close it – so that by the end, locker number 12 will be closed. Now think of eg locker 81; it has five factors ie during the whole exercise it will be closed, open, closed, open – and then closed! So in the end it's all down to the fact that every square number has an odd number of factors . . .

note

The last step is probably the most rewarding but pupils will only make it if they're familiar with the fact that a square number always has an odd number of factors . . . so, you might like to introduce or revise this fact in advance eg a couple of weeks before presenting the investigation (rather than just beforehand, which might make it all too easy).

1000 LOCKERS FIRST 20 ROWS