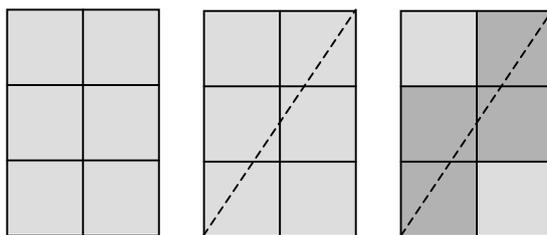


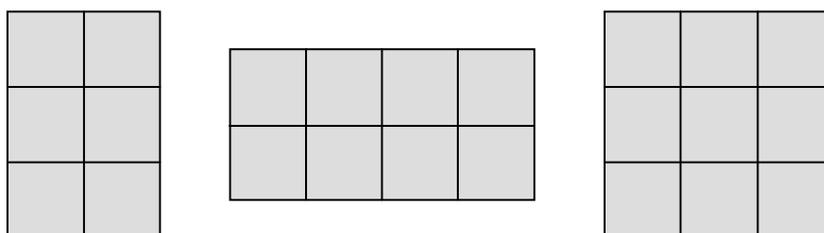
intro

This investigation – about diagonals and rectangles – is easy to explain and easy for children to carry out but there is a pattern to be found and that’s a bit harder . . .



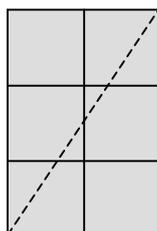
the investigation

In this investigation we’re dealing with rectangles made up from unit squares, like these :

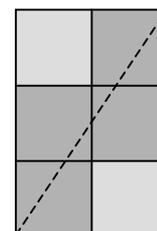


– here we have a 3 x 2 rectangle, a 4 x 2 rectangle and a 3 x 3 rectangle

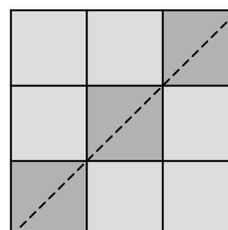
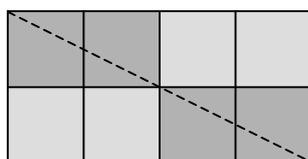
Let’s take the first rectangle and draw in a diagonal :



As you can see, this diagonal cuts across 4 of the small squares :



Here are the results for the other two rectangles above :



first steps

Get your pupils – working on their own / in pairs / in small groups – to draw rectangles of various shapes and sizes and to record their results. If they find a systematic way of doing this, so much the better . . .

practical

Pupils can use ordinary 1 cm square grid paper for their drawings but they’ll probably be happier with a slightly larger grid size.

next steps

Once they've drawn quite a few rectangles, pupils can start to look at their results to see whether they can identify a pattern (though this one is definitely not easy to see). You can assure them that there is a pattern here to be found and perhaps get the whole class to look at a selection of results on the board, inviting suggestions from anyone who thinks they have noticed something.

Ask them to think how they could make it easier to spot what's going on . . . with luck, someone will suggest looking at specific families of rectangles separately eg the set 4×1 , 4×2 , 4×3 , 4×4 , 4×5 etc, or all the squares (1×1 , 2×2 , 3×3 etc) . . . another suggestion might be to look at rectangles which are multiples of others (eg the 6×4 rectangle is made up of four 3×2 rectangles) . . .

Most classes will need some prompting to notice that :

- On average, the diagonals of larger rectangles cross more squares. This number does not go up quickly, however, suggesting that our answer is more likely to be something to do with the sum of the sides than with the product.
- There's a difference between rectangles where the two dimensions have a factor in common (such as 6×4 or 9×6) and rectangles where 1 is the only common factor (eg 7×4 or 11×6). So there's obviously something important about factors in this case.

In the end most classes will need to be given the definitive rule ie for any rectangle, to find the number of squares crossed by the diagonal, just add the two sides and then subtract the largest common factor.

notes

Finding the underlying pattern is a challenge even for the more able – some children will not really understand it even when it's explained to them – and therefore the investigation might not be suitable for all pupils. However :

- It's easy to explain and easy for pupils to carry out.
- There's plenty of opportunity for initiative – pupils can decide for themselves how to organise the tasks ie how work systematically, who should do what etc and how to record the results
- It combines spatial and number ideas – and it gives plenty of scope for displaying findings in different ways
- It does show pupils that not all investigations produce trivial or simple patterns ie in a 'real maths' investigation there isn't always a simple rule waiting at the end.