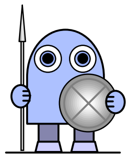


maths investigations

MATHS INVESTIGATIONS

What is investigative maths?

First of all, investigative maths is not just practical work; many investigations do involve practical activity but that's usually only the starting-point. Secondly, investigative maths is not the same as problem-solving. In both practical work and problem-solving children are given some initial information or instructions and they know what sort of answer they're looking for – even if they don't know straight away what it is or exactly how to get it. In investigative maths children are given a starting-point and some clear procedures to try out but they have no idea at the outset what sort of results they're going to get – although clearly good investigations at this stage generate patterns of different sorts and it's these patterns we want children to look for and identify.

What's the point of investigative maths for children?

Even now some people have the idea that maths is a fixed body of knowledge with a rigid set of rules and procedures; they see it as a subject where you get right answers by just learning how to apply the rules correctly. On this view of maths there's just one correct way of doing each kind of thing and there's just one right answer. It's important for children to learn that real maths isn't all like this – there are plenty of topics still to be investigated and there really is room for being imaginative and creative. In real maths not all problems can be solved by one approach and not all problems have just one straightforward answer. Real maths involves asking questions, investigating and using your imagination.

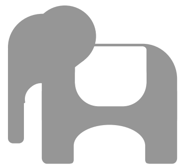
Our investigations

Our investigations come from a variety of sources; some you might have seen before, whilst others are completely original – but all have been tried and tested in the classroom and found to be practicable, worthwhile and engaging. They're presented here in a way which makes them easy to grasp and straightforward to carry out in the classroom. Please look at the list below to see which investigations might well suit which of your age-groups – but remember these are only suggestions, so feel free to use them as you judge best. Some of the earlier ones can be adapted to provide interesting investigations for older pupils.

elephant parade

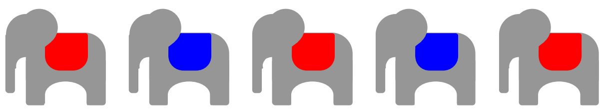
intro

There are lots of investigations about putting things into different orders. This one is about elephants. Although it's quite straightforward, it is well worth doing – and you can use it to get across a few important maths ideas . . .



first steps

Every spring animals come from all parts of the jungle to watch the most exciting event of the year, the jumbo gymnastics. If you ever go there, remember not to stand too close – an elephant on a trampoline is not quite as safe as you might think. At each competition the judges choose the five best jumbo gymnasts and afterwards these five winners parade through the jungle – to great applause from the other animals! The boy elephants always wear blue and the girl elephants wear red. This year there were 2 boy winners and 3 girl winners and they paraded through the jungle in this order :



As you can see, the order here is red, blue, red, blue, red – but of course, they could chosen a different order . . . can anyone here think of a different order for these five elephants? (Don't let them give you more than one or two!)

the investigation

The challenge now for the children is to find as many different orders as they can for the five elephants, remembering that there are 2 male elephants and 3 female elephants in each victory parade . . .

practical

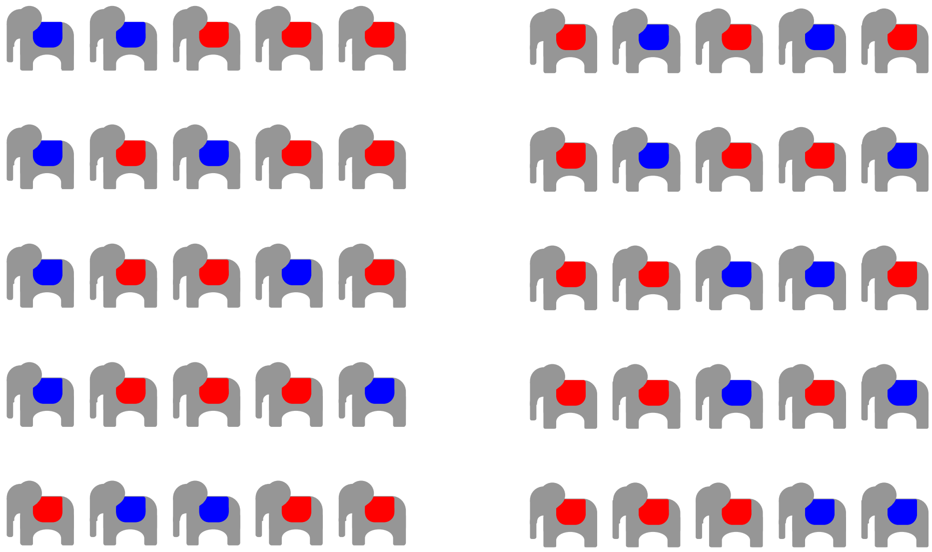
Children can work on their own or in twos or threes. They can use red and blue counters (or anything else they can shuffle about easily) to stand for the elephants.

You might want to get them to think about how to record their results (some children like drawing elephants) or alternatively you could provide them with results sheets like the one given here (blank parades printed 12 to a sheet ie no need to tell them in advance how many different answers there are). If you have the energy, a different idea is to print this answer sheet on card and then cut into strips – which makes it easier when children want to compare one answer with another.

elephant parade

results

The children should eventually find 10 different orders for the elephant parade. Here are all of them :



notes

Once you've got all the results, you can get the children to answer one or two questions, such as :

- How many of these parades have 2 blues next to each other?
- How many parades are there with the first and last elephants in the same colour?
- Is it the boy elephants or the girl elephants who are more often the leaders?
- How many of the parades are 'palindromes' (same order from front to back as from back to front)?

extension

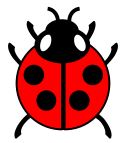
For those who finish early or as something for the whole class to go on to, you can vary the initial terms of the investigation and ask pupils to see how many different orders they can find for eg :

- A parade with 1 blue and 4 red elephants
- A parade with 2 red and 2 blue elephants
- A parade with 3 elephants and 3 different colours eg red, blue and yellow

Be careful, though, as interest fades quickly after more than one or two of these variations . . .

intro

There are quite a few investigations like this one. Although the initial enquiry doesn't go much beyond collecting results, there is still plenty to interest and involve younger pupils.



first steps

The investigation is easy to explain . . . In Scotland there are two kinds of ladybird, the *highland ladybird* and the *island ladybird* – one kind has 4 spots and the other kind has 7 spots (though no-one can ever remember which is which). In some parts of Scotland you can see both kinds of ladybird and that's how this investigation comes about . . .

Fiona saw two ladybirds on a leaf and said she could count 11 spots altogether; can you work out what were the two ladybirds on her leaf? Rob had to go one better – he found a leaf with three ladybirds on it and he counted 12 spots; what were the three ladybirds on his leaf? After this, Martin said he'd also seen a leaf with some ladybirds on it – and that on his leaf the spots added up to 13 . . .

the investigation

The aim of the investigation is to look at different ways of combining these two sorts of ladybird on one leaf – and to find what different numbers of spots you can make. To start things off, you might need to get the children thinking about one or two specific examples eg, 'How could you make up 8 spots? How could you make11 spots?'

practical

You can make simple 'ladybirds' from red card (and you can buy stick-on spots from a stationer); for leaves, use green paper stuck onto white card (and then laminated to use again and again). Working in twos or threes, children can use these to investigate different combinations and count the spots. They'll have to think about how to record their results – some might like to draw pictures to show all the combinations they've found whilst others will prefer to make lists . . .

results

When the children have completed their investigations, you can get them to tell you what they've found and you can summarise their results on the board. Ask them :

- Has anyone found a number that's impossible to arrange?
- What other numbers are impossible to arrange?
- Are there any numbers you can reach in more ways than one?

Here's a list of combinations giving up to 30 spots :

4-spot ladybirds	7-spot ladybirds	total no of spots
0	1	4
1	0	7
2	0	8
1	1	11
3	0	12
0	2	14
2	1	15
4	0	16
1	2	18
3	1	19
5	0	20

4-spot ladybirds	7-spot ladybirds	total no of spots
0	3	21
2	2	22
4	1	23
6	0	24
1	3	25
3	1	26
5	1	27
7	0	28
0	4	28
2	3	29
4	2	30

impossible : 1, 2, 3, 5, 6, 9, 10, 13, 17 . . .

notes

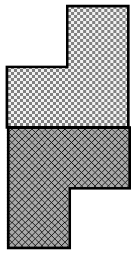
- If you need a challenge for the more able, ask them to work out how many 4-spot and 7-spot ladybirds they would need to produce eg 40 spots, 70 spots, 33 spots, 50 spots, 99 spots . . . (they should be looking at combining results already obtained)
- There's plenty of scope here for displaying findings!

extension

A simple way of varying this investigation is of course to try some different spot pairings (ie mutant ladybirds) and see what the results are eg 3-spot with 8-spot or 2-spot with 5-spot or even perhaps 4-spot with 6-spot . . .

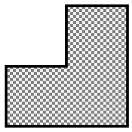
intro

This investigation is about putting simple shapes together to produce new shapes. Although the actual investigtion is relatively straightforward, it does give pupils an opportunity to think about shape and to use concepts such as symmetry, reflection and rotation – and it teaches them a useful lesson about the need to be systematic and thorough in work of this sort.



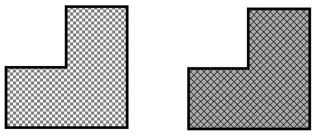
first steps

Let's start off with this simple shape :

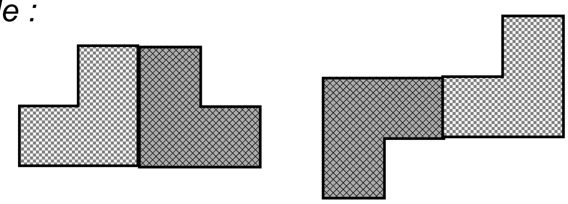


How would you describe this shape? Imagine you're speaking on the telephone to someone who can't see it . . . *it's like three squares put together to make an L-shape (or . . . if you think of four small squares put together to make a larger square and then you take away one of the small squares, you're left with a kind of L-shape, and that's it!)*

Suppose we have two of these shapes :



By putting two of these shapes together, we can make new shapes, like these, for example :

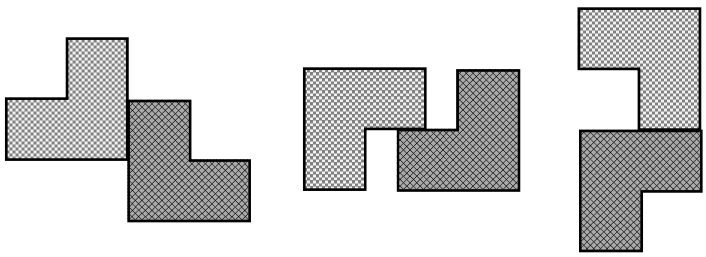


the investigation

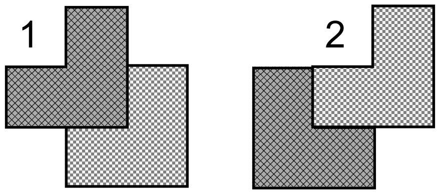
Ask the children to find as many different shapes as they can using these two simple pieces – but tell them that, for this investigation, they must follow this rule :

Each piece must have at least one whole side attached to one whole side of the other piece.

– which means that shapes like the two just above are fine, but shapes like these are definitely not :



There might need to be some discussion, either before children get started or after they've begun to produce results, about whether (or why) these two shapes are or are not allowed :



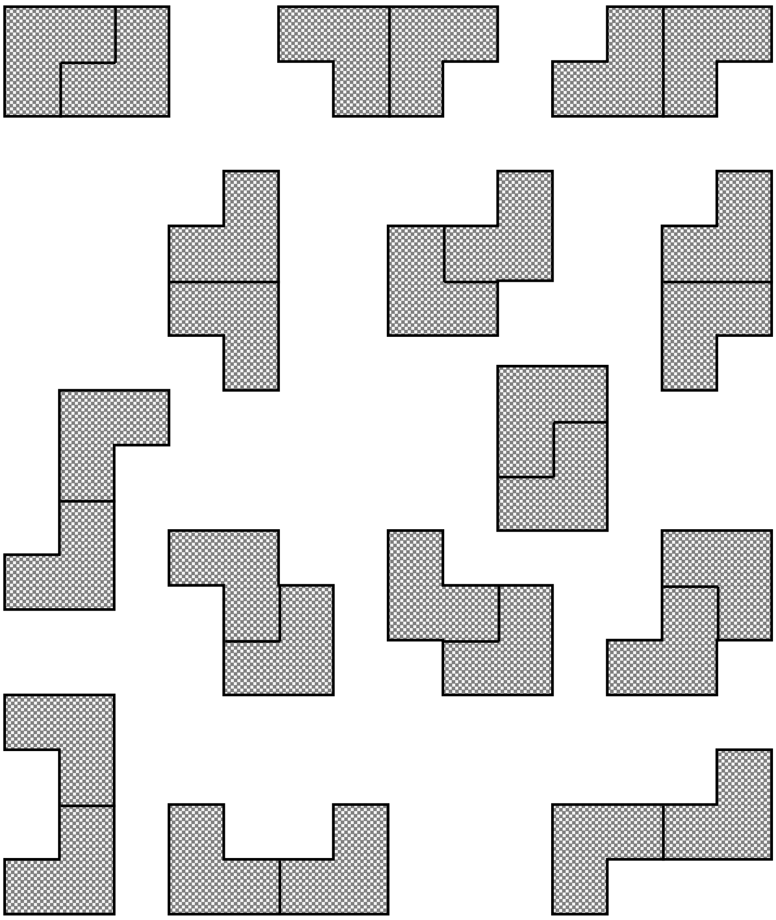
** Answer : Shape 1 is not allowed, because one of the L-shapes doesn't have a single whole edge next to the other L-shape. But shape 2 is allowed, because both L-shapes have at least one edge attached.*

practical

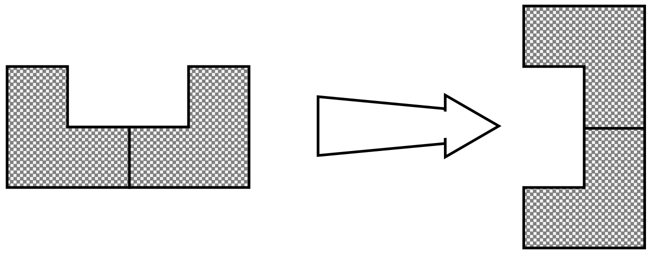
Children can work on their own or in pairs. The investigation can be done as a pencil-and-paper exercise but L-shapes made from coloured card are more cheerful – and they make it easier for the children to try out different arrangements. They can use squared paper to record their results – and at the end the original pieces can be taken in and used for a poster to display findings.

results

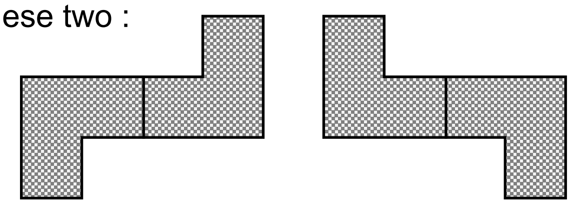
All these results are possible :



But clearly, they're not all different. We need to be sure what counts as different and what counts as the same. Pupils will have their own ideas and there's no 'right' answer – although generally investigations like this we call two shapes 'the same' if one can be rotated to get the other, for example :

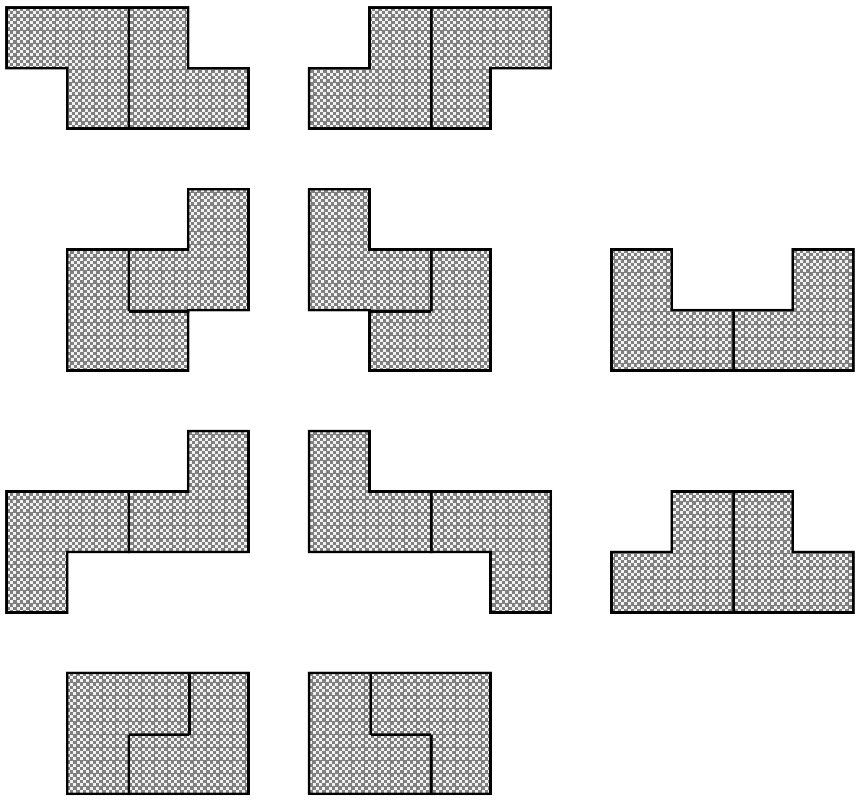


But not these two :



– which are certainly 'alike', since each one is a reflection or mirror-image of the other, but neither of which can be rotated to give the other.

With this in mind, there are just 10 different shapes to be found :



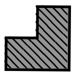
notes

With these different shapes before them, you can ask the children :

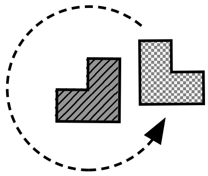
- Obviously there are three 'pairs' here – but which individual shapes have bilateral symmetry?
- Which shape has the longest perimeter? Which has the shortest perimeter?
- Can anyone think of a method for finding all the shapes? Is there any way we could make sure that we really have got all the shapes?

extension

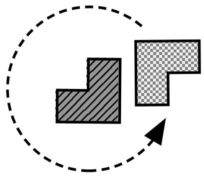
If you have the appetite for it, one useful extension of this investigation is to look for ways of checking that you've got **all** the new shapes which can be made by combining two simple L-tiles. Here's one approach (called 'Walking the Dog') :

1 Select one of the L-shapes : 

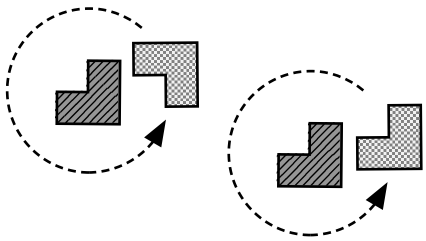
2 Keep this shape in exactly the same orientation, then take a second L-shape and (always keeping it in the same orientation) 'walk' it around the first one into every position the rules allow.



3 Next, without altering the first L-shape, rotate the second L-shape clockwise by 90˚ and repeat the 'walk-around' process to find every position allowed by the rules. Now you have two sets of new shapes.

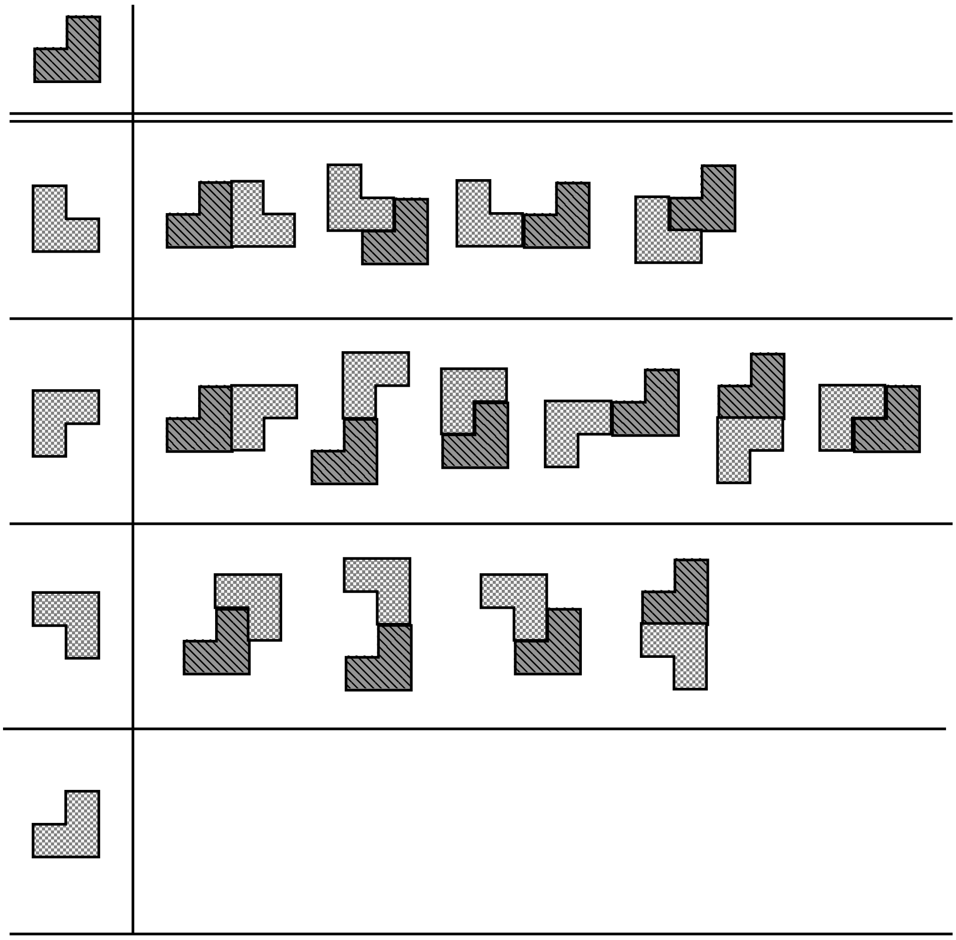


4 Rotating the second L-shape clockwise by a further two right-angles and repeating the 'walk-around' process will give you your third and fourth sets of new shapes.

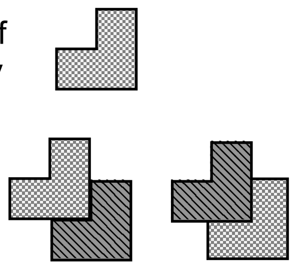


Now you've covered every possibility! See the next page for our table of results. Of course, the 13 shapes we you see there really amount to only 10 different ones (the same 10 shapes we've already met).

Walking the Dog : table of results



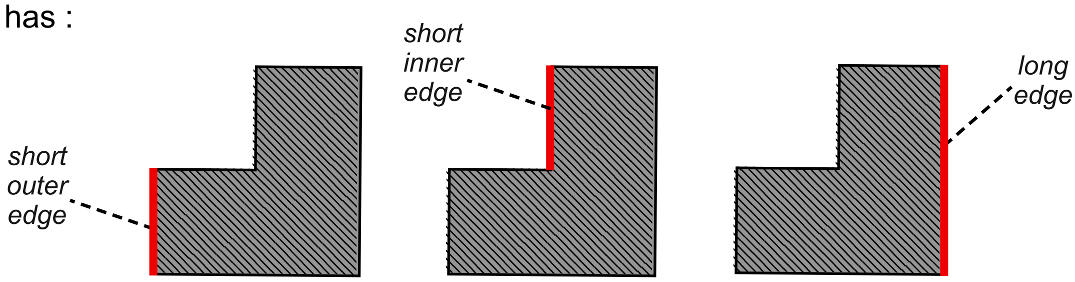
As you can see, the space for our fourth set of results is empty. Why haven't we included any results for 'walking' this orientation of shape 2 around shape 1?
The answer is that the only new shapes this will give you are these two . . .



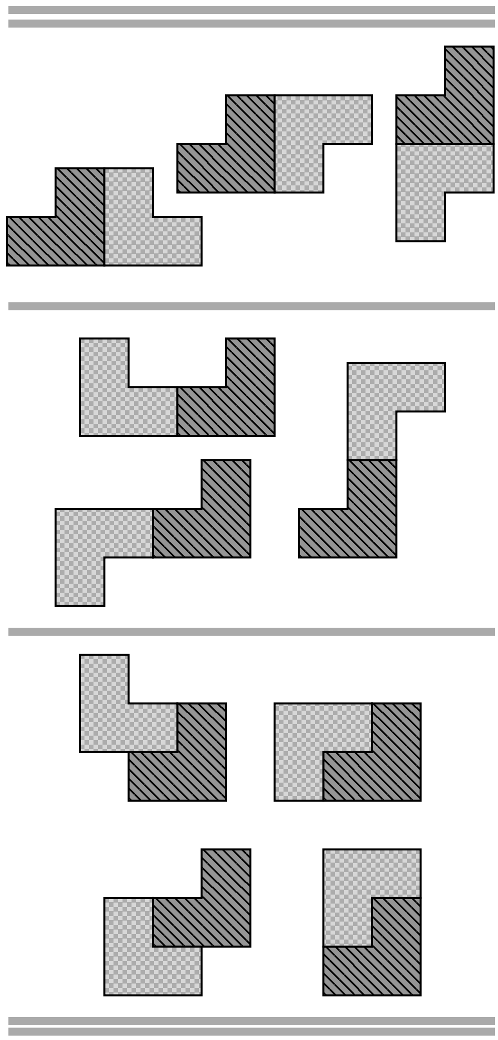
. . . and these two don't qualify. Why not? Because although the dark L-shape has two complete edges involved, the lighter L-shape doesn't have any complete edges involved.

Another way of making sure we've got **all** possible results is based on what you can do starting with different edges. See the next page for an account of this approach . . .

Walking the Dog is not the only way of checking for completeness. Another way is to take a first L-shape and identify the three types of edge which it has :



Then select one type of edge eg the long edge – and see how many ways you can find of attaching L-shape 2 to this edge. Repeat this process for the other two types of edge. You're left with three sets of new shapes, making 10 new shapes in all (same result as before!) :



If an investigation you're doing involves finding a set of shapes, numbers or whatever, checking for completeness is always a good follow-up, as long as you can find a fairly straightforward way of doing it. The completeness tests above perhaps stray a little beyond the normal year 3 limits of understanding (but they are optional).

How else could you extend this investigation? Here are two suggestions :

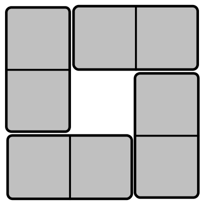
- Investigate how many shapes can be found using three of the L-shapes instead of two.
- Investigate how many shapes can be found using two (simple) shapes of a different kind as 'building blocks'.

No doubt you can think of your own variation on the theme. But – for each investigation you'll need to make sure that the children understand exactly what the rules are.

domino squares

intro

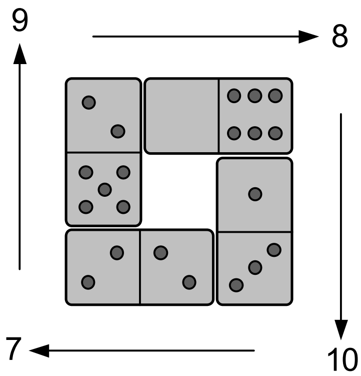
Here we begin with a problem – which has a number of solutions – and then move on to a wider investigation. The challenges involved are quite easy for pupils to understand and the answers are not too hard to find . . .



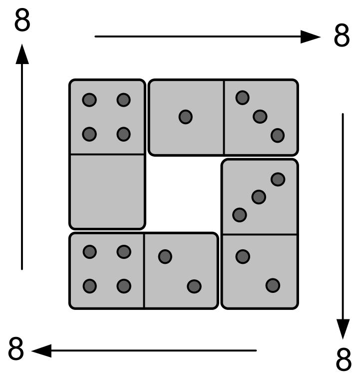
first steps

If you arrange 4 dominoes in a square, as above, and then add up the number of spots along each side of the square . . .

you might get four different totals :



or exactly the same total on all four sides :



The first challenge for pupils is to look for arrangements like the second one above ie with the same total on all four sides. There are quite a few ways of doing this . . .

the investigation

Once pupils have found a few arrangements which work, you can get them to investigate : What's the largest total you can get on all four sides? What's the smallest total you can get?

practical

Children can work on their own or in pairs. Obviously, they'll need dominoes and you can either let them choose how to record their results or you can provide them with gridsheets (see photocopy masters).

results

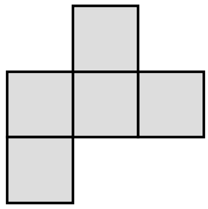
There are many possible arrangements which fit the bill. Results can be recorded on the board for all to see – and to identify the largest and smallest possible totals . . .

extension

A natural way of extending this investigation is to repeat with 10 dominoes arranged in a square instead of 4.

intro

Here we get pupils to investigate the different shapes they can make by joining squares together. Starting with two, then three and then four squares, pupils move on to generate the well-known set of pentominoes . . .

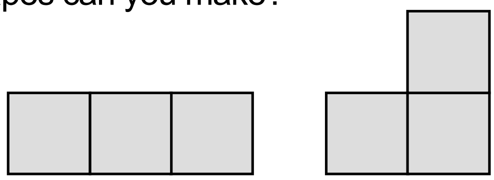


first steps

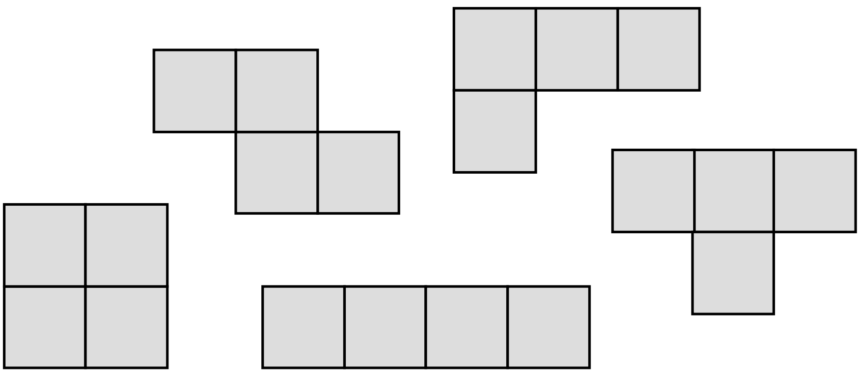
Everyone knows what a domino looks like :

We've drawn this one without spots because for now we're just interested in the actual shape of the domino . . . as you can see, it's made up of two squares joined together. How would you describe how these two squares are joined?

Suppose you have three squares and you join them edge-to-edge, how many different shapes can you make?



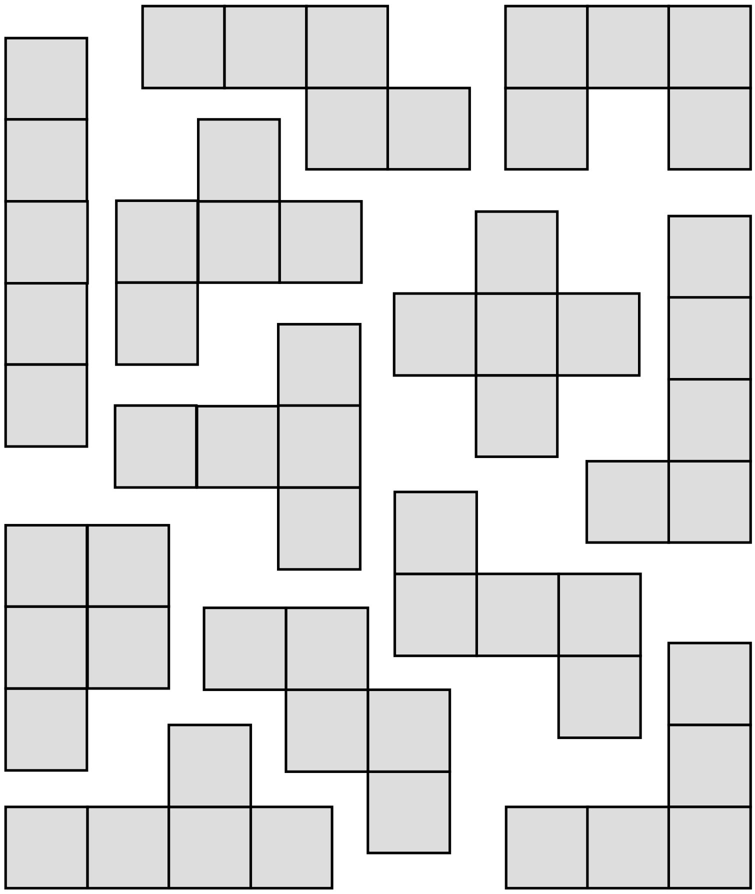
These shapes are called 'trominoes' (or sometimes 'triominoes') and there are just two* of them. The shapes you can make from four squares joined together are called 'tetrominoes'; try to find all of them if you can.



This time there are five* different shapes to be found.

* special note : In this investigation we're not counting mirror-images as different shapes.

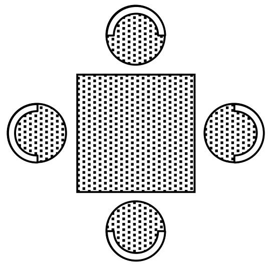
the investigation	The challenge now is to find as many different shapes as possible made from five squares, joined edge-to-edge as before . . .
practical	All pupils can do this on their own; square grid paper is all they need . . .
results	The children should eventually find the 12 different pentominoes :



notes	This investigation is simple and straightforward but well worth doing – if only as a lead-in to work with actual pentominoes.
extension	○ There are always the hexominoes to move on to . . . ○ Time spent investigating various properties of the pentominoes is time well spent – see separate investigation 'pentomino properties'.

intro

At the Maths Café the tables are all square and each table seats 4 people. That's simple enough but then customers don't always come along in groups of 4 and what's more, larger groups often say that they want to sit together . . .

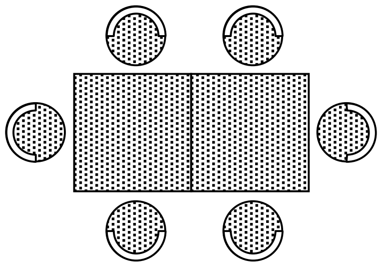


This investigation is all about arranging tables to accommodate different numbers of diners.

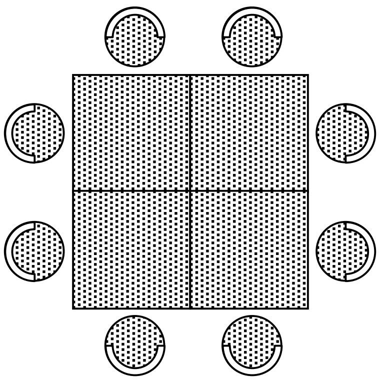
first steps

A good way of starting the thing off is to look at some specific seating problems; by working on one or two easy examples, the children will start to get a feel for how different arrangements work – and it should then be easy enough to move them on to a wider investigation . . .

Suppose 6 people come into the café . . . if they're happy to be in two groups, you can sit them 3 to a table at two different tables . . . but if they want to stay as one group, you'll have to put two tables together like this:



Of course, it's not always this easy . . . Say one evening 8 people arrive at the café; your first idea might be to take the above arrangement for 6 and just add an extra table – but suppose it's towards the end of the evening and you'd like to use the four empty tables you have left (to make sure the café looks totally full) . . . how could you arrange four tables together to seat 8 diners?



One large square seems to do the trick!

The rules, by the way, are that you must always have :

1 whole sides together when you join tables (no half-and-half)

2 only 1 person per place – and no empty places allowed!

Keeping to the these rules, try to find seating arrangements which will work for these seating problems :

a use 3 tables to seat 8 diners who wish to be in one group

b use 5 tables to seat 12 diners who wish to be in one group

c use 5 tables to seat 14 diners who don't mind being split up

d use 6 tables to seat 12 diners who wish to be in one group

e use 6 tables to seat 18 diners who don't mind being split up

* see separate sheet for answers.

the investigation

The aim is to find and record ways of accommodating different numbers of diners using a given number of tables. For example, if you've got 3 tables, could you seat 6 diners with no seats left empty? Could you seat 12 diners? Could you seat 18 diners? What different arrangements are possible with 3 tables? Remind the children that they must stick to the rules ie just one person per seat / no empty seats / tables joined only by whole sides.

The first real challenge for the pupils is in the actual gathering of results. They will need to be quite thorough in their approach to be sure they've covered all possibilities for a given number of tables. After they've been working for a while it's probably worth stopping them and getting everyone to discuss how they're going about achieving this objective. Obviously a systematic approach is more likely to be successful. Say, for example, you've got 5 tables. You start off by looking at different ways of putting all 5 tables together. Next, you could look at 4 tables together and 1 on its own – and ask see how many different arrangements there are for this combination. After this, you could look at 3 tables together alongside 2 tables together – and then 3 tables together alongside two single tables – and so on, each time looking at different ways in which the particular combination can be achieved.

practical

This investigation can be carried out as a pencil-and-paper exercise and some children will naturally work in this way. Others, though, will find it livelier and more engaging to have squares of card or polydron to represent the tables and counters to stand for the chairs. They can use squared paper to record their results and work in pairs or small groups.

results

It's pretty obvious that the more tables you have, the more flexibility there will be in how many diners you can accommodate. This table summarises the possibilities for up to 5 tables (see later for illustration of various arrangements for these numbers) :

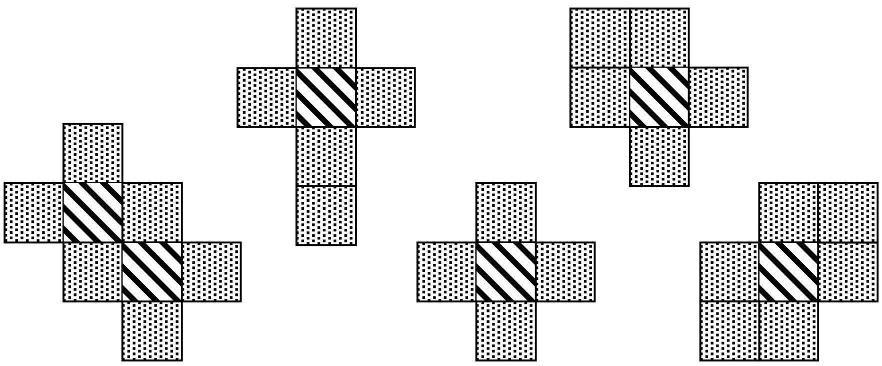
	possible diner numbers
1 table	4
2 tables	6 / 8
3 tables	8 / 10 / 12
4 tables	8 / 10 / 12 / 14 / 16
5 tables	10 / 12 / 14 / 16 / 18 / 20

Of course, you don't have to stop at 5 tables . . .

notes

There isn't one simple pattern to be found in this table of results – but there's plenty to draw attention to / question / discuss :

- What's the largest number of diners you can accommodate for any given number of tables? (ans : 4 x the number of tables!)
- Pupils might spot something special about 9 dining tables : if they're arranged in a large square, you end up with one table 'trapped' inside ie totally out of use. How many ways can pupils find to 'trap' tables using fewer than 9 tables? There are a few (there's even one way of trapping 2 tables using just 6 tables) :

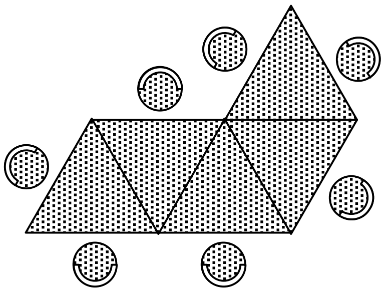


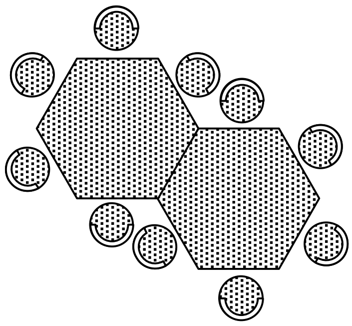
- Can any pupil explain why the results all seem to feature even numbers of diners? Why can't you have an arrangement for an odd number of diners?

- Could you ever have an arrangement with the same number of diners as tables? (ans : yes, eg 16 tables arranged in one large square or 18 tables arranged in a 6 x 3 rectangle)
- Could you ever have an arrangement where the number of diners is lower than the number of tables they're sitting at? What's the smallest number of tables where this could happen? (ans : yes, eg 20 diners can sit at 24 tables if the tables are in a 6 x 4 rectangle; however, if you take 2 tables away from one corner of this rectangle, you'll still have 20 diners but now sitting at just 22 tables – which is also the smallest number of tables where this can happen)
- What's the largest number of tables you can 'use up' if you have a group of 24 diners? (ans : 36 tables arranged in one large square)
- Of course, the key to each arrangement of tables is how many 'joins' you've got . . .

extension

This investigation is all about square tables. But dining tables aren't always square – so one way of extending the project might be to get pupils to see what happens* with eg equilateral triangle dining tables or hexagonal dining tables . . .

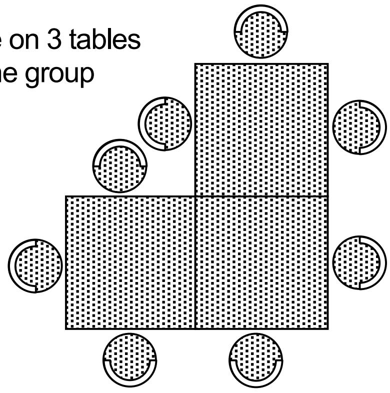




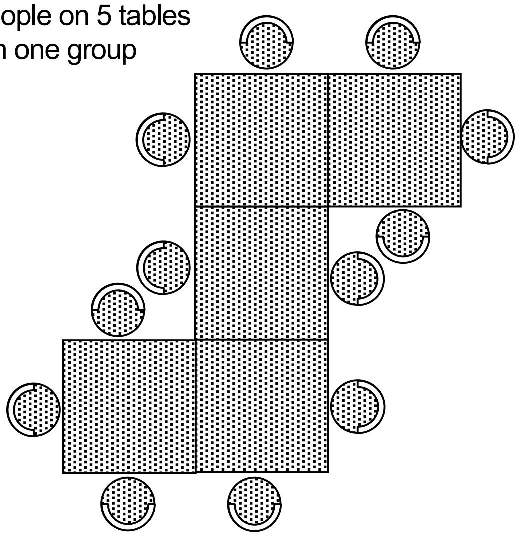
* using isometric grid paper to record results

answers to seating problems

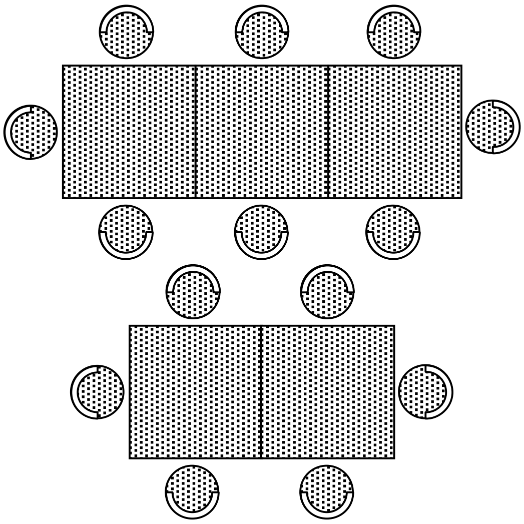
8 people on 3 tables
in one group



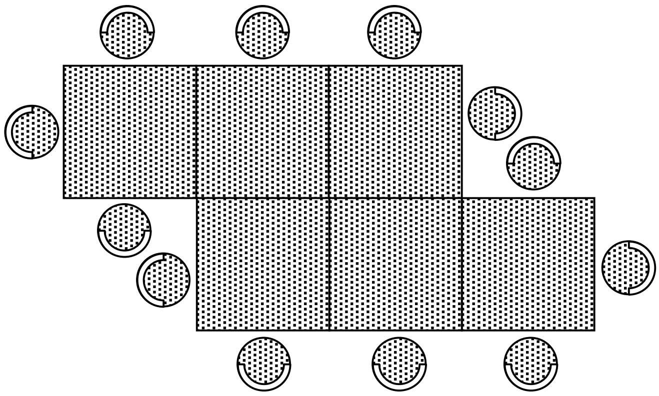
12 people on 5 tables
in one group



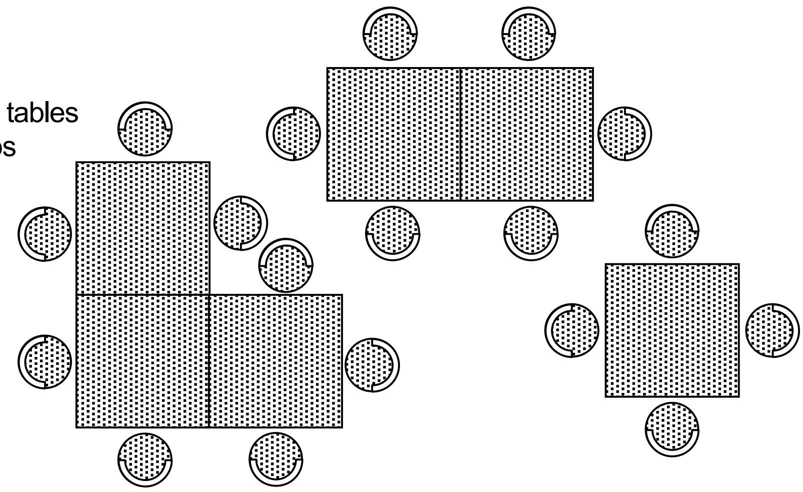
14 people on 5 tables
in two groups



12 people on 6 tables
in one group

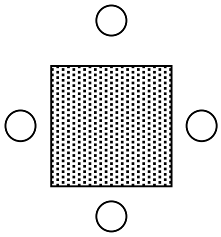


18 people on 6 tables
in 3 groups

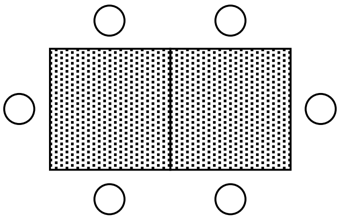


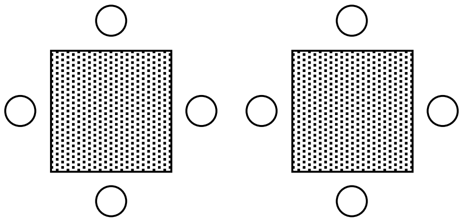
seating plans

1 table

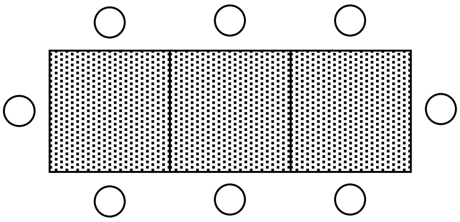


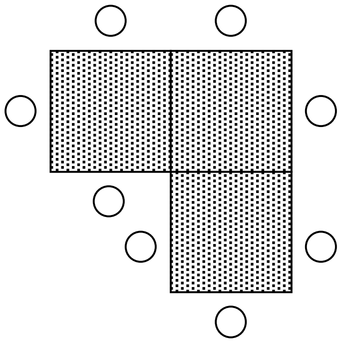
2 tables

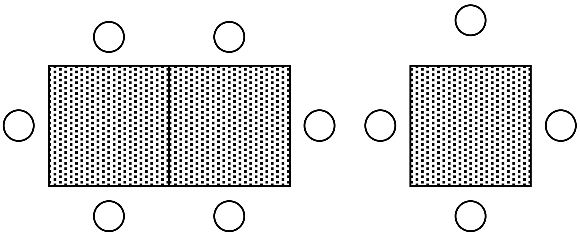


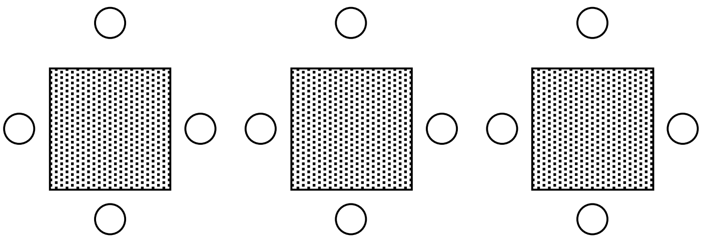


3 tables

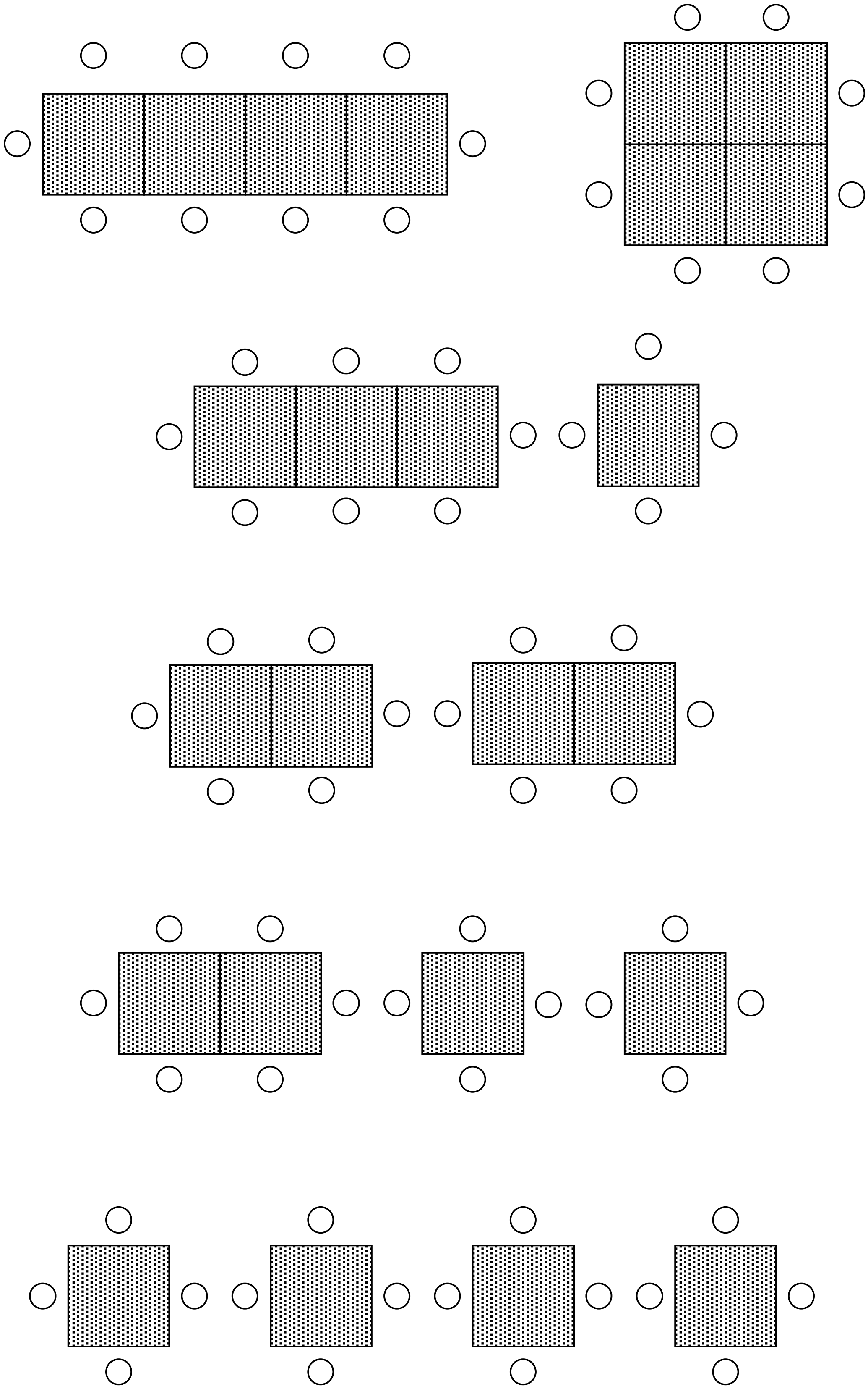




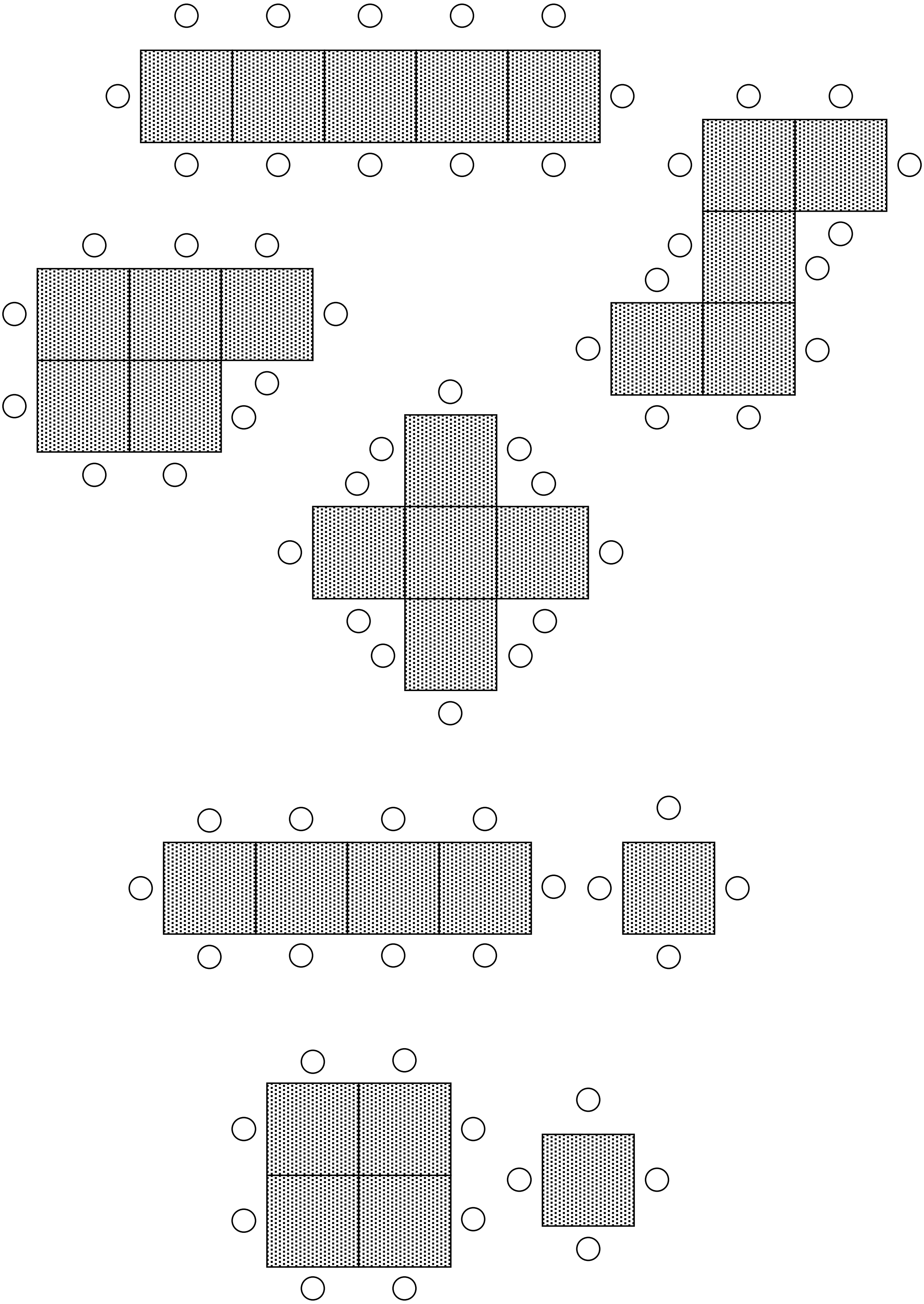




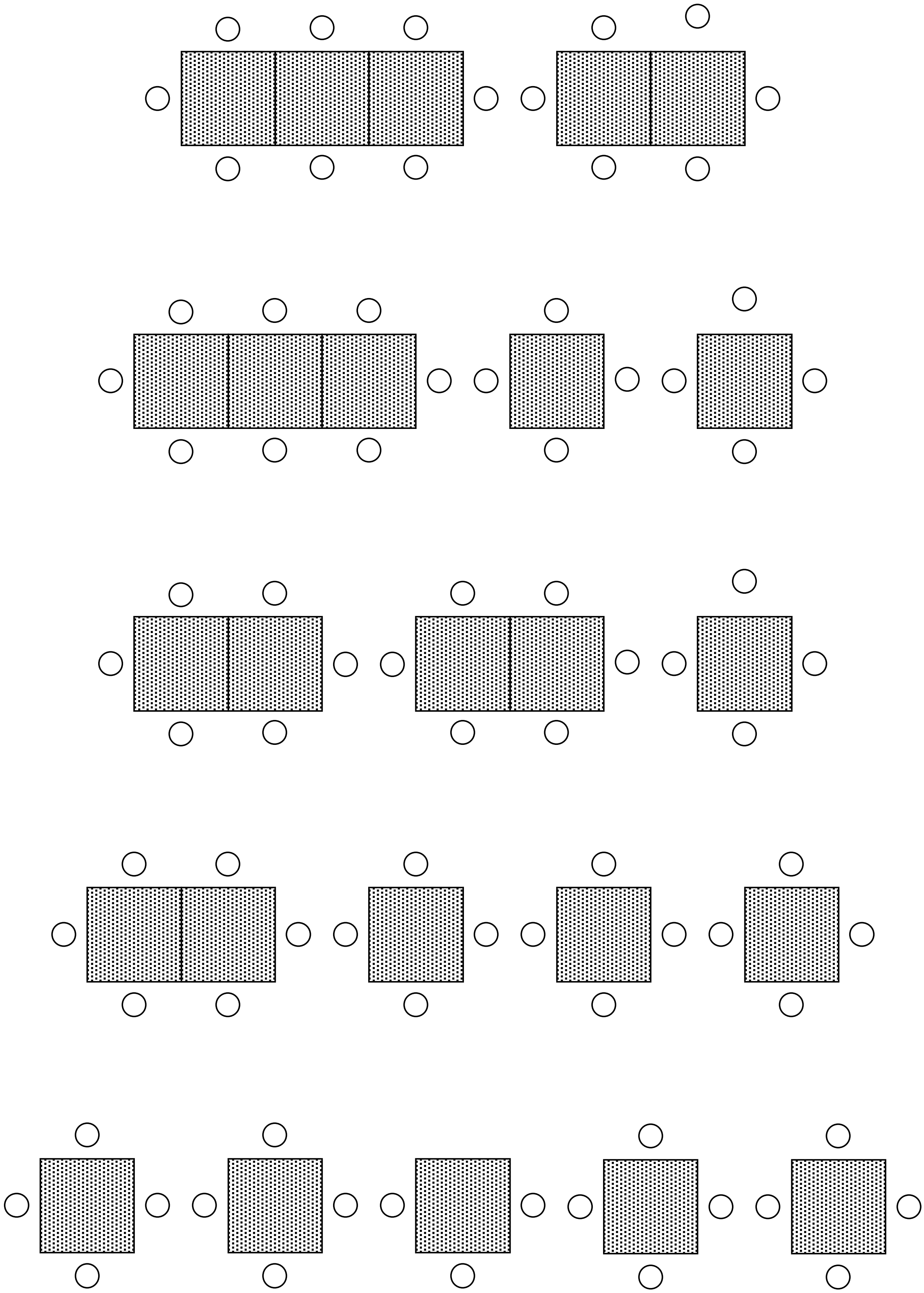
4 tables



5 tables



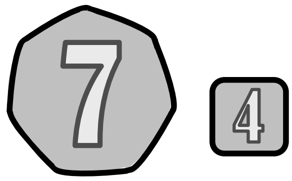
5 tables



all change

intro

There are plenty of problems and investigations which ask children to look at which totals can or can't be made up from combinations of certain given numbers. This investigation is similar but it takes things further – and it requires a bit more thinking . . .



first steps

Long, long before the Romans came to Britain, there were some people using coins. These early traders were actually 'buying' and 'selling' rather than just trading one thing for another . . .
A group of tribes living in one valley, for example, had a currency system using just two kinds of coin, the 4p coin and the 7p coin. The 'p' stands for potato, by the way, the idea being that 1p was about the value of 1 large potato.

the investigation

With this system of currency, it's pretty obvious how you're going to pay for something which costs 4p or 7p or 11p – but what about something costing 3p? Or, even harder, 5p? The answer is that you'll have to pay some coins over and then the person selling will have to give you some coins back – your 'change' in other words. As you can see, these people knew all about change . . .

Now for our investigation: If we look at any of the transactions above we can add up how many coins have changed hands; for example, to buy something costing 5p, you'd have to give three 4p coins and get back one 7p coin, so altogether four coins will have changed hands. The aim of our investigation is to look at a range of prices – say from 1p to 30p – and to work out for each price what's the 'neatest' way of using the coins ie what's the smallest number of coins which can be used to make the payment.

practical

Children can do this using just pencil and paper but if you – or they – have time, it's worth cutting out some 7p and 4p coins (see photocopy masters). This is definitely an exercise to do in pairs or in small groups! You can let pupils decide for themselves how they want to record their findings or you can give them copies of the results tables (see photocopy provided) and show them how to use it.

results

It’s worth going through the results with the whole class to make sure that everyone really has found the most efficient ways of paying for the different amounts.

notes

With an agreed set of results established, you can ask :

- What’s the largest number of coins you needed for amounts up to 30p in value? That’s to say, which were the amounts which needed most coins?
- To pay 9p you could give four 4p coins and get one 7p coin change, which uses five coins altogether, or you could give three 7p coins and get four 4p coins change, which obviously uses seven coins altogether. What other amounts did you come across which can be done in different ways using different numbers of coins?
- Are there any amounts where you could have two different ways of paying, each using the same number of coins?
- How would you describe the amounts which you can make up exactly (the ones where change isn’t needed)?

extension

You can leave things here but the full version of this investigation is really in two parts. If you – and the class – have the stamina it’s well worth going on to the second part (though probably not on the same day).

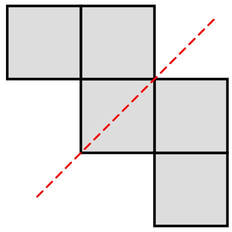
In part 2 we carry out essentially the same exercise but with a different currency system. In a valley not too far away from the one we’ve already mentioned, the tribes who live there have developed their own system. The unit of currency is still based on the potato – but instead of the 7p and 4p coins used by the others, we find these tribes using 2p and 5p coins. Apparently these people felt they had made a big improvement on the original idea.

Once again, pupils can look at amounts from 1p to 30p and work out the ‘neatest’ ways of making the payments, this time using just 2p and 5p coins. Once the results are in, the interesting thing is to compare the two systems . . .

Look first at the amounts from 1p to 10p. If you add up the ‘number of coins needed’ column, you get a total of 30 coins used for the 4p / 7p system but just 22 coins used for the 2p / 5p system – which suggests the men’s system might indeed be better. However, if you look at the amounts from 21p to 30p, you might form a different impression . . .

intro

In this simple investigation, pupils are encouraged to look carefully at the twelve individual shapes which make up the standard set of pentominoes. By investigating their properties, pupils get to know the shapes better – and also to revise / reinforce their knowledge of some key spatial concepts.



the investigation

There are three parts to this investigation:

- **symmetry** – Pupils look at which pentominoes have bilateral or rotational symmetry (or both).
- **tessellation** – Pupils try to find which pentominoes can be used for tessellating and whether the tessellations they discover are unique.
- **nets of a cube** – Some of the pentominoes form the nets of open-top cubes . . . and some don't!

Obviously, it might well be worth going over the above concepts before pupils begin their investigations.

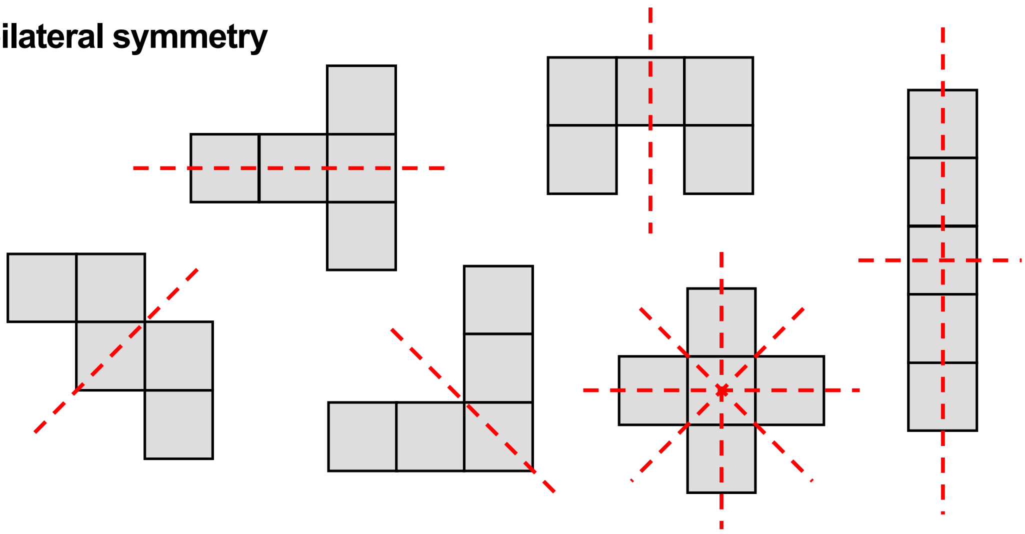
practical

Pupils can work on their own or in pairs / small groups. They should have sets of pentominoes to work with – and squared paper to record their results. You might like pupils to cover all three investigations or you might prefer to have different pupils working on the three different areas and perhaps reporting their findings back to the others.

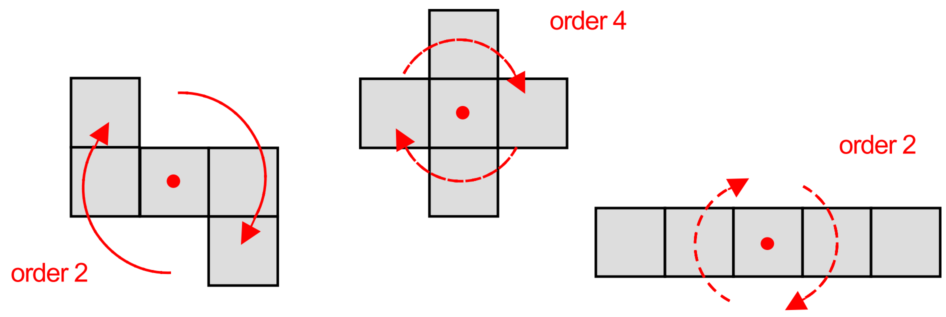
results

Pupils should have little difficulty in getting results. For reference :

bilateral symmetry

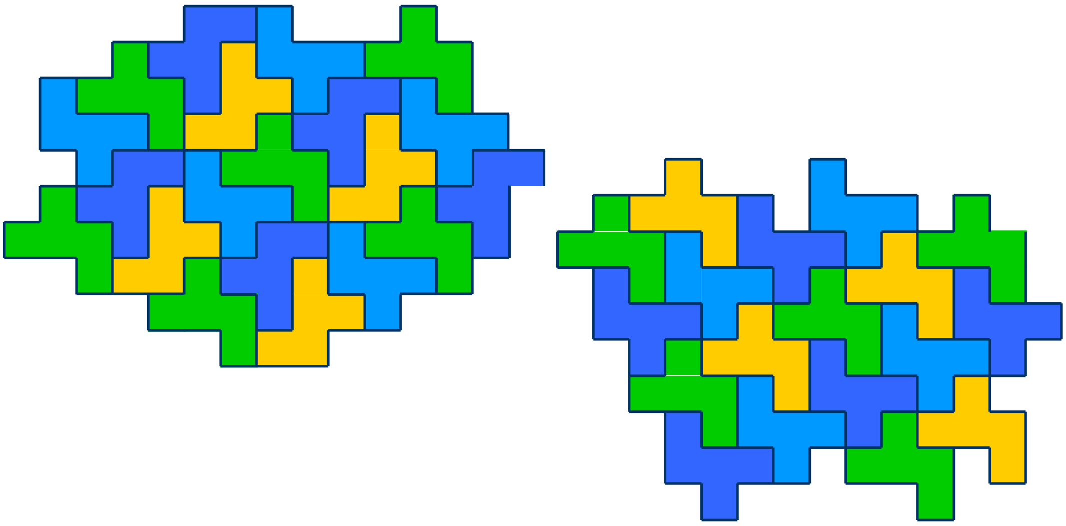


rotational symmetry



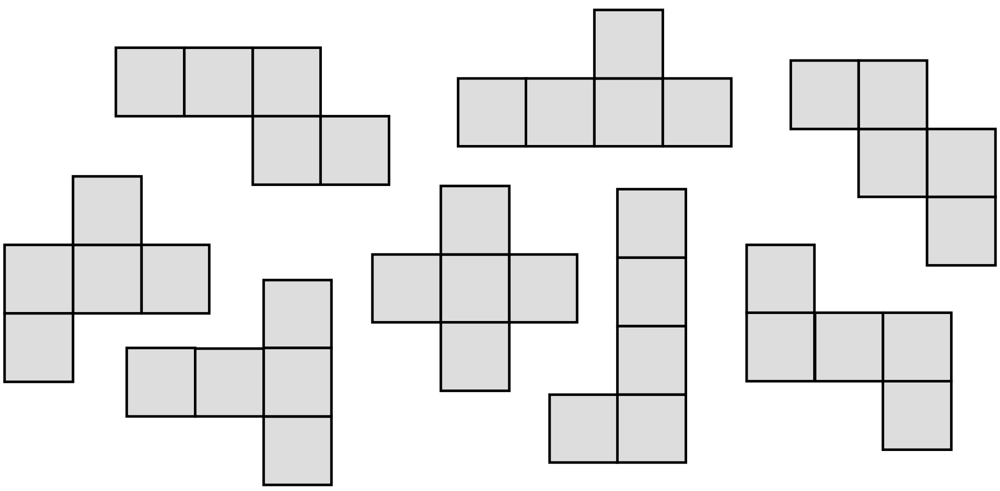
tessellation

Each of the pentominoes can be used for tessellating – and there are numerous alternatives to be found eg



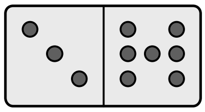
nets of a cube

any of these can be used as the net of a cube:



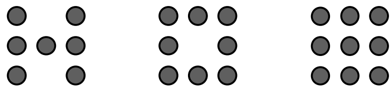
intro

This is a straightforward investigation which begins with a question : how many different dominoes are there in a set of 9-spot dominoes?



first steps

Pupils will be familiar with ordinary 6-spot dominoes. Begin by asking whether anyone knows how many different dominoes there are in a set like this . . . Usually one of the pupils will know that there are 28 (but if not, get out a set of dominoes and get someone to count them). Next, you can tell them that you can also buy 9-spot dominoes (and that there are various games you can play with them). Show them that the spots are arranged like this for the higher numbers :



Ask whether pupils think that a set of 9-spot dominoes would have more dominoes than an ordinary 6-spot set, or fewer . . . The answer is of course more dominoes – but how many more? Ask pupils to guess how many dominoes you would need to make up a 9-spot set. Write a few of these guesses on the board. Now for the investigation . . .

the investigation

Ask pupils whether 6-spot and 9-spot sets of dominoes are the only possibilities . . . and when one of them suggests 2-spot dominoes (or you have to), ask what dominoes you would need to make up a set. Write / draw their answers on the board until you've got the complete set (6 different dominoes for a 2-spot set). Don't arrange them in any specific order as ordering / completeness are things for pupils to think out for themselves later on.

Now they've got the idea, explain that you want them to work out the make-up of all the possible sets up to 6-spot ie a 0-spot set, a 1-spot set, a 2-spot set and so on . . . and that each time they should list all the dominoes in the set – and count them. Tell them they'll need to think carefully about recording their results and about how they can make sure they really have got all the dominoes for each set.

practical

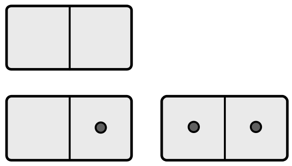
Children can work in twos or threes. The whole thing can be done as a pencil and paper exercise but it's probably better to give each group of pupils a set of ordinary (6-spot) dominoes; it won't take them long to see that the smaller sets they're investigating are just subsets of this set.

results

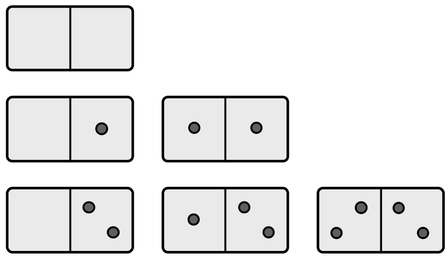
This is not a difficult investigation, so you can expect broad agreement when it comes to results :

- there is only one 0-spot domino

- there are three dominoes in the 1-spot set



- there are six dominoes in the 2-spot set



and so on . . . so that finally we have this table of results :

domino set	no of dominoes in set
0-spot	1
1-spot	3
2-spot	6
3-spot	10
4-spot	15
5-spot	21
6-spot	28

Pupils might recognise the numbers in the right-hand column as the *triangle numbers.* Using differences, they can now work out from the table how many dominoes there will be in a 7-spot set of dominoes (ans 36), in an 8-spot set (ans 45) and finally in a 9-spot set (ans 55). Were any of the original guesses near?

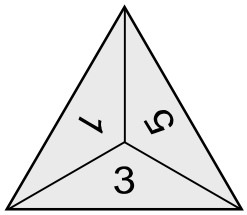
notes

At some stage in the proceedings it's well worth getting the children to stop and think about

- how they can be absolutely sure of finding all the members of any particular set
- how best to show their results

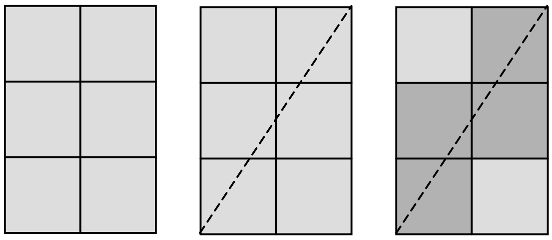
extension

Dominoes are not the only number tiles . . .



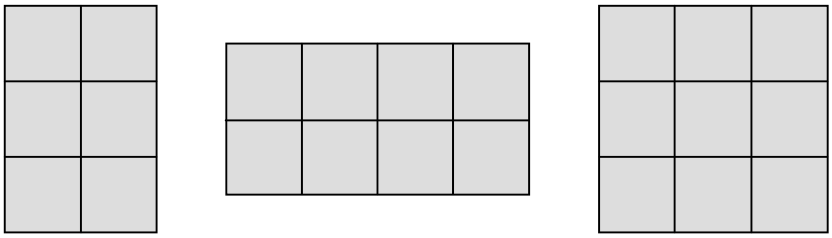
intro

This investigation – about diagonals and rectangles – is easy to explain and easy for children to carry out but there is a pattern to be found and that's a bit harder . . .



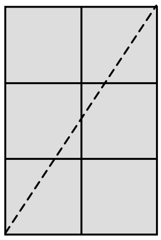
the investigation

In this investigation we're dealing with rectangles made up from unit squares, like these :

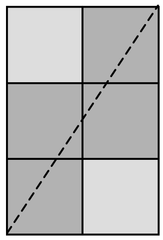


– here we have a 3 x 2 rectangle, a 4 x 2 rectangle and a 3 x 3 rectangle

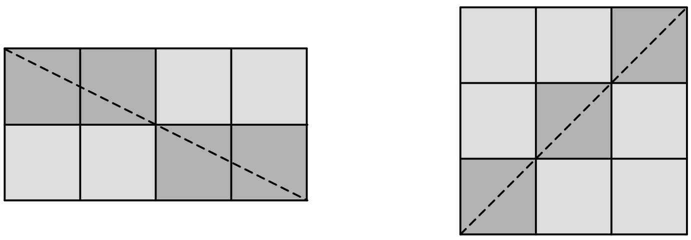
Let's take the first rectangle and draw in a diagonal :



As you can see, this diagonal cuts across 4 of the small squares :



Here are the results for the other two rectangles above :



first steps

Get your pupils – working on their own / in pairs / in small groups – to draw rectangles of various shapes and sizes and to record their results. If they find a systematic way of doing this, so much the better . . .

practical

Pupils can use ordinary 1 cm square grid paper for their drawings but they'll probably be happier with a slightly larger grid size.

next steps

Once they've drawn quite a few rectangles, pupils can start to look at their results to see whether they can identify a pattern (though this one is definitely not easy to see). You can assure them that there is a pattern here to be found and perhaps get the whole class to look at a selection of results on the board, inviting suggestions from anyone who thinks they have noticed something.

Ask them to think how they could make it easier to spot what's going on . . . with luck, someone will suggest looking at specific families of rectangles separately eg the set 4 x 1, 4 x 2, 4 x 3, 4 x 4, 4 x 5 etc, or all the squares (1 x 1, 2 x 2, 3 x 3 etc) . . . another suggestion might be to look at rectangles which are multiples of others (eg the 6 x 4 rectangle is made up of four 3 x 2 rectangles) . . .

Most classes will need some prompting to notice that :

- On average, the diagonals of larger rectangles cross more squares. This number does not go up quickly, however, suggesting that our answer is more likely to be something to do with the sum of the sides than with the product.
- There's a difference between rectangles where the two dimensions have a factor in common (such as 6 x 4 or 9 x 6) and rectangles where 1 is the only common factor (eg 7 x 4 or 11 x 6). So there's obviously something important about factors in this case.

In the end most classes will need to be given the definitive rule ie for any rectangle, to find the number of squares crossed by the diagonal, just add the two sides and then subtract the largest common factor.

notes

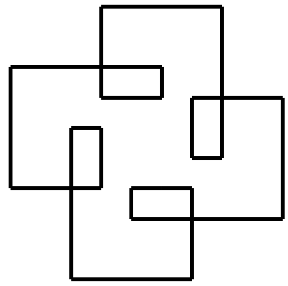
Finding the underlying pattern is a challenge even for the more able – some children will not really understand it even when it's explained to them – and therefore the investigation might not be suitable for all pupils. However :

- It's easy to explain and easy for pupils to carry out.
- There's plenty of opportunity for initiative – pupils can decide for themselves how to organise the tasks ie how work systematically, who should do what etc and how to record the results
- It combines spatial and number ideas – and it gives plenty of scope for displaying findings in different ways
- It does show pupils that not all investigations produce trivial or simple patterns ie in a 'real maths' investigation there isn't always a simple rule waiting at the end.

spirolaterals

intro

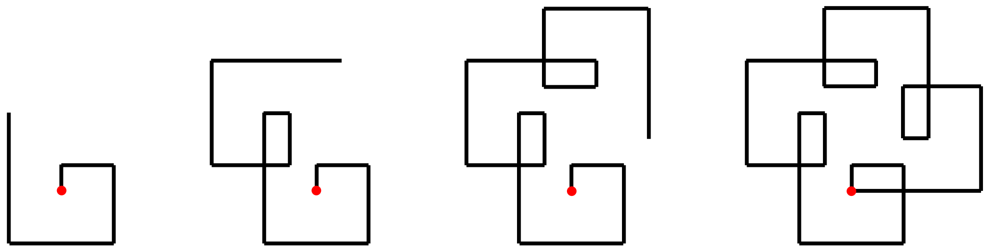
In this investigation children work on a rectangular grid and by following simple sequences of commands, trace out spiral patterns of increasing complexity. The basic idea of the spirolateral is attractively simple but the results can be surprising.



first steps

90° spirolaterals are the simplest since they involve only right-angle turns – and at this stage we'll stick to clockwise turns only. This is how the spirolateral of order 5 works :

Starting at a point on the grid, go along 1cm and turn 90° clockwise, then go 2cm and turn 90° clockwise, then go 3cm, then 4cm and then 5cm, each time turning 90° clockwise. Here you might think you're going to travel 6cm along but – this is an order 5 spirolateral and so at this point you begin the sequence again, travelling 1cm, 2cm, 3cm and so on, always remembering to turn 90° clockwise after each length, until . . .



the investigation

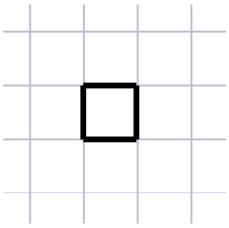
Once the children have understood how the thing works, you can get them to investigate spirolaterals of order 1, 2, 3, 4, 5, 6 and so on . . . obviously children will compare results etc but this is an activity which they should all do for themselves. Sadly, it's not as easy as it looks and some children will go wrong more readily than others . . .

practical

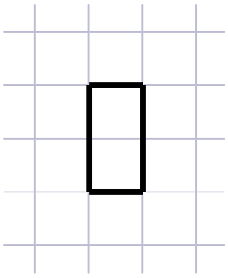
1cm square grid paper and coloured pens, pencils or markers are all the children will need to carry out the investigation. Grid paper with bigger squares might be good for display purposes.

results

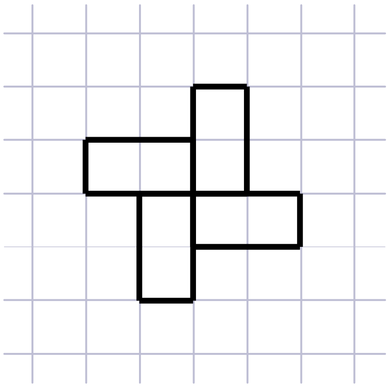
Results for spirolaterals up to order 7 are shown on the following pages. Ask the children to put into words what's different about the order 4 spirolateral; can they find any others like this one?



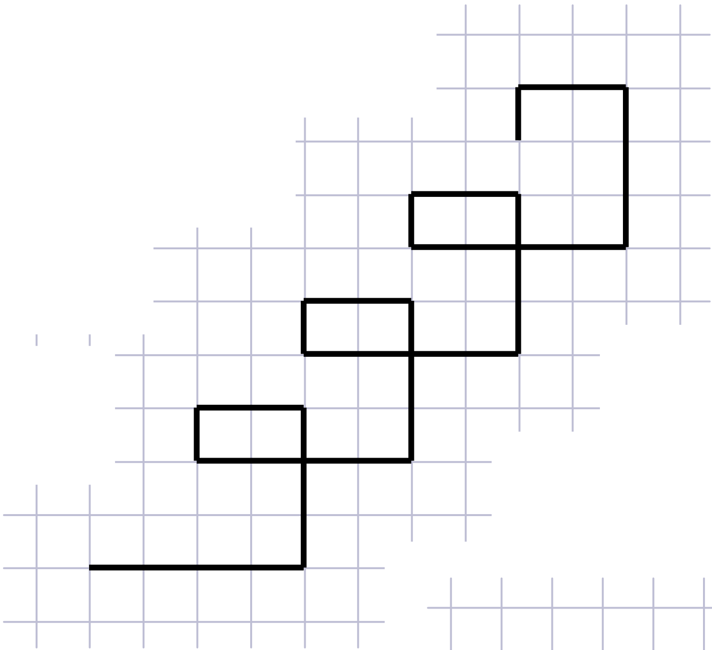
order 1



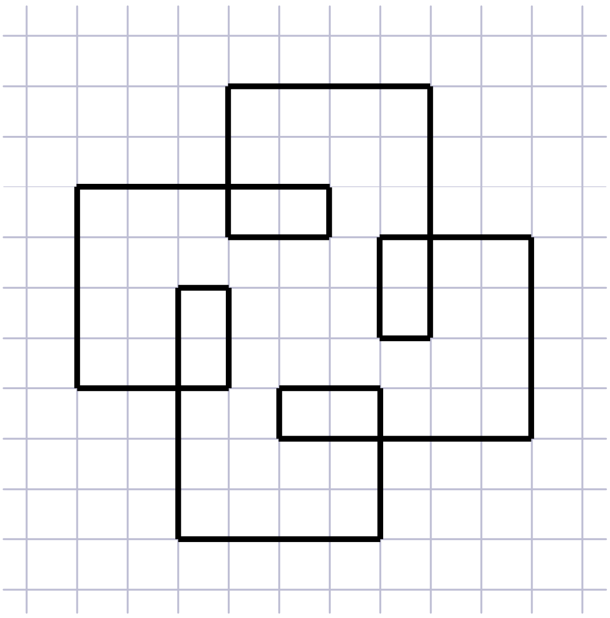
order 2



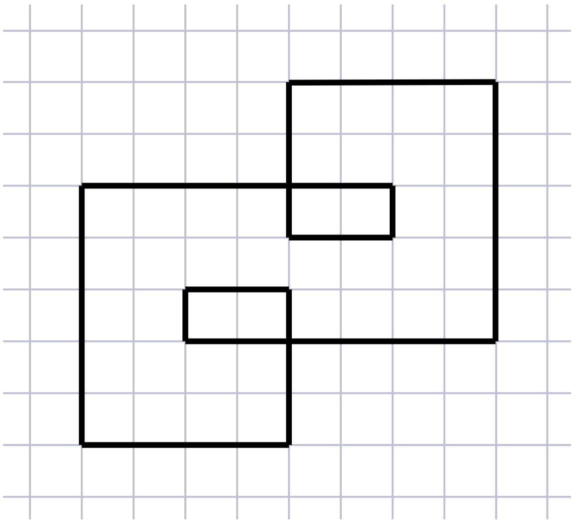
order 3



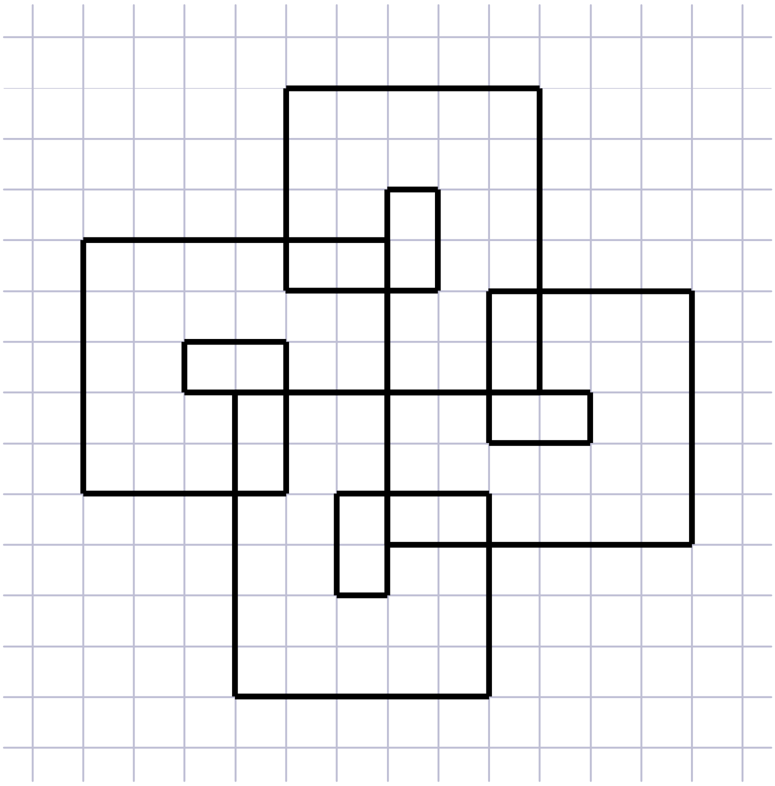
order 4



order 5



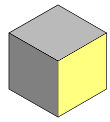
order 6



order 7

intro

In this investigation pupils look at the effect on surface area of breaking up a cube into smaller cubes.

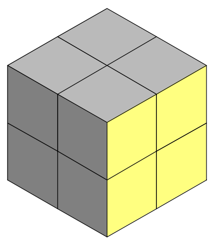


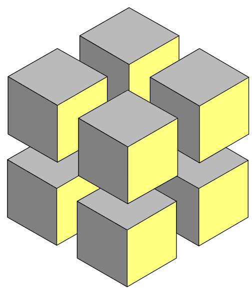
first steps

- You can find the surface area of a cuboid by adding together the separate areas of its faces. That's not too difficult! In the case of a cube, though, the surface area is very easy to calculate – since all the faces are the exactly the same, you just find the area of one face and multiply by 6.Suppose we have a cube and we cut it in half. Will the total surface area stay the same or will it increase? Pretty obviously it will increase. If you're not sure of this, imagine your cube is a wooden one and that you've painted it red all over; when you cut this cube in half, you'll still have the red surface you've already painted but now you'll have two unpainted faces as well (which means that to have all surfaces painted red, you'd have to do some more painting) . . . so the surface area has definitely increased!
- In this investigation we're going to look at what happens to the total surface area when we break up a cube into smaller cubes.

the investigation

Ask pupils to work out what happens to the total surface area when you take a cube and break it up into 8 smaller cubes.





If they're not sure how to get started, suggest that instead of trying to think of cubes in the abstract, they look at a cube with specific measurements eg 2cm x 2cm x 2cm and then investigate this.

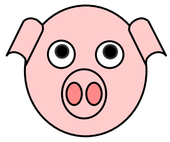
results

A 2cm cube has a surface area of 6 x 4 = 24 cm^2. Eight 1cm cubes will have a total surface area of 8 x 6 = 48 cm^2. In other words, the total surface area has doubled. Can any pupils explain graphically why this should be so? (Try to get them to think in terms of getting the 8 small cubes by successively cutting the original cube in half in each of the three directions).

next steps	Now get pupils to investigate what happens when a cube is broken into 27 or into 64 smaller cubes. Again, to make life easier they can give the original cube some suitable specific measurements eg 3cm x 3cm x 3cm or 4cm x 4cm x 4cm and take things from there.
notes	○ Pupils might already have encountered the problem of the cube which is painted red all over and then broken up into smaller cubes – and where they are asked to calculate the fraction / percentage of faces now painted red. ○ What would be the 2-D equivalent of this 3-D investigation? (*answer: Looking at how the total perimeter is changed when a square is split up into smaller squares.*) How do the results work out for the 2-D version?

intro

This is an investigation which is relatively straightforward to explain and enjoyable to carry out. But it's not so easy to find a pattern in the results . . .

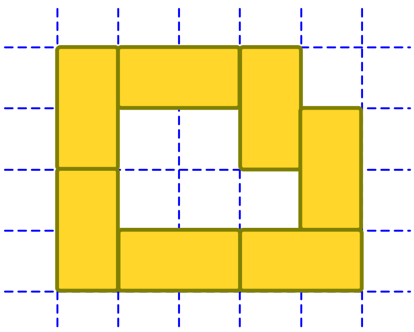


first steps

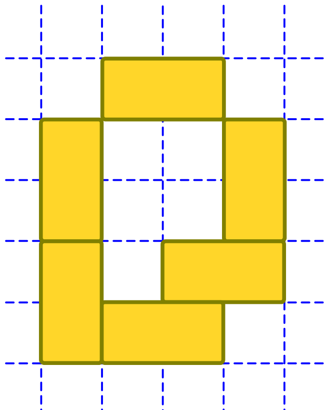
The pigs at Croft Farm have some of the most comfortable sleeping quarters in the world. On hot summer nights, though, some of them prefer to sleep outdoors. The farmer is happy enough to let them do this but he always makes up a pig pen from bales of straw, just to discourage them from roaming.

These bales of straw – looking at them from above – are 2m by 1m rectangles and they have to be put together with at least 1m adjoining (pigs can eat their way out of corner-to-corner arrangements). The other thing to remember is that these pigs are sociable animals, so they always sleep together in one pen. However, every pig does like its own square metre of field to sleep on – so if for example you've got 5 pigs sleeping out, you need a pig pen enclosing $5m^2$ (5 square metres) . . .

like this perhaps :



but definitely not like this :



As you can see, the arrangement on the left uses 7 bales of straw. Over the years, the farmer at Croft Farm has become quite an expert at arranging bales of straw for his pigs.

the investigation

The challenge here is to find – for each number of pigs – the smallest number of bales needed to make a pig pen. Remember the rules :

- The pigs should be in one single pen – with $1m^2$ for each pig.
- Corner-to-corner arrangements for the bales are not allowed.
- Wherever one bale is next to another, the join must be at least 1m long.

practical

This investigation can be done as a pencil and paper exercise but it's easier and more fun to use something (face-down dominoes are ideal) to stand for the bales – and to have a suitable grid to work on (see the photocopy masters). The exercise works well if children work in pairs or in small groups. 1cm square grid paper is fine for recording the results. Some children like to draw a pig's face in each square of the pig-pen . . .

results

Children will obviously be aware from the start that the more pigs you want to enclose, the more bales you'll need. And they'll soon discover that for any particular number of pigs, they can arrange the enclosure in different ways. Even when they agree on the smallest number of bales needed in a particular case, they might still find they've got different ways of arranging the bales. In the end, though, there should be no ambiguity about the actual minimum number of bales needed in each case. Here's our table of results for up to 20 pigs in an enclosure (and for interest we show our set of pig-pens on the last page) :

number of pigs	smallest number of bales		number of pigs	smallest number of bales
1	4		11	9
2	5		12	9
3	6		13	10
4	6		14	10
5	7		15	10
6	7		16	10
7	8		17	11
8	8		18	11
9	8		19	11
10	9		20	11

The interesting question is : can we find a pattern in here ?

There's nothing immediately obvious – so let's try arranging the results in a different way. We can see that 4 bales is the minimum number of bales for a 1-pig enclosure, 5 bales is the minimum number for a 2-pig enclosure, 6 bales is the minimum number for both a 3-pig enclosure and a 4-pig enclosure . . .

We can summarise all this in a table :

number of bales	= minimum number for
4	1
5	2
6	3, 4
7	5, 6
8	7, 8, 9
9	10, 11, 12
10	13, 14, 15, 16
11	17, 18, 19, 20

So 4 is the minimum number of bales for just 1 size of pig-pen. And 5 is the minimum number for just 1 size of pig-pen. But 6 is the minimum number for two different sizes of pig-pen . . . and so is 7. Let's follow this idea through :

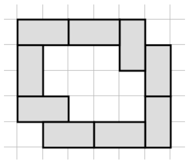
number of bales	= minimum for how many sizes
4	1
5	1
6	2
7	2
8	3
9	3
10	4
11	4

It rather looks as though we've found a pattern . . .

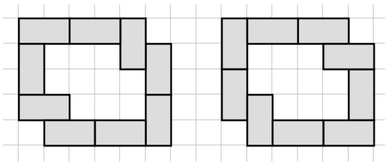
extension

Here are three ways of extending the investigation :

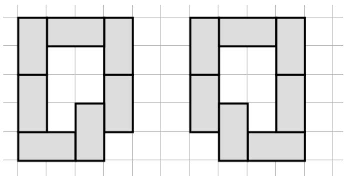
- In the first stage of the investigation we were looking for the minimum number of bales to enclose a given number of pigs. Pupils will soon have noticed that they often came to the same result – but with different arrangements of the bales. One obvious thing to investigate is : just how many different ways are there of getting the various results? But before embarking on this, it's definitely worth getting some agreement on what should count as 'different' arrangements. For example, you can make a pig-pen for 10 pigs with just 9 bales. Here's one way of doing it :



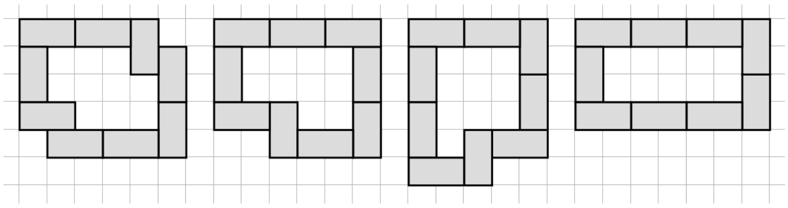
You obviously wouldn't want to count a simple re-ordering of the bales as a different arrangement :



Different *rotations* of one arrangement clearly shouldn't be counted as different arrangements – but you'll have to decide whether or not to allow *reflections* to count as different arrangements :



Most people probably wouldn't. What we're really looking for here is this : how many completely different *shapes* of enclosure can we find for each result in the table? There are plenty to find. For example, here are four different shapes for 9 bales enclosing 10 pigs :



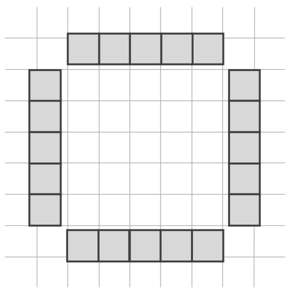
- Pupils who have the stamina can investigate larger enclosures (that's to say those holding more than 20 pigs). Here the interesting question is : does the pattern we found continue after 20?
- Square enclosures are obviously a special case and worth looking at on their own. There's a straightforward connection between the number of pigs / size of square and the number of bales needed – and it's a nice exercise for pupils to identify this pattern. To begin with, pupils could write down what happens in the first few cases. This is what they should find :

number of pigs		number of bales
1	→	4
4	→	6
9	→	8
16	→	10
25	→	12

Next, you could ask them to look at one particular case and use a diagram to show *why* the answer is what it is. Let's consider, for example, the 25-pig pen. We know $25 = 5^2$, so what we're looking at is how many bales we need to enclose a 5 x 5 square.

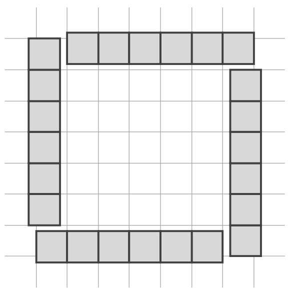
Let's imagine using just simple $1m^2$ bales to begin with :

We need 5 of these small squares to go along each side of the enclosure



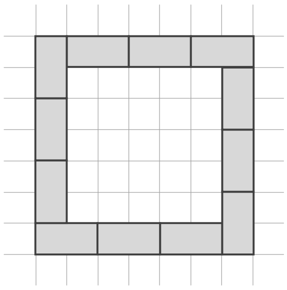
– so that's 4 lots of 5 small squares we need for the whole enclosure

But we also need a small square to cover each corner . . .



– so now that's 4 lots of 6 small squares we need for the whole enclosure

Finally, we have to remember that the bales we're actually using are not $1m^2$ but $2m^2$ . . .



– which means that the total number of bales needed for the enclosure = 2 lots of 6

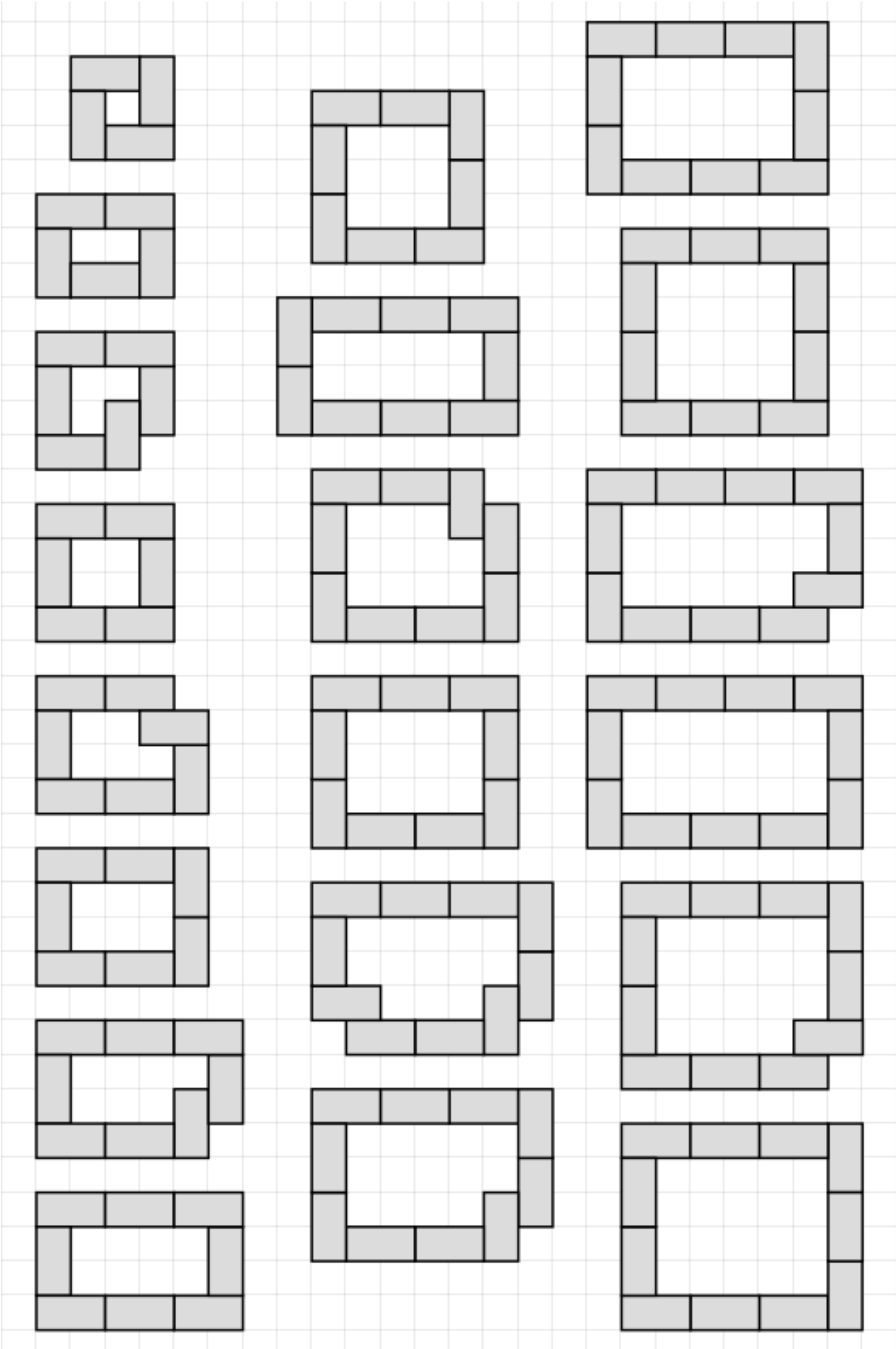
Conclusion : for a 5^2 pen we need 2 x 6 bales.

– and if we looked at a 9^2 pen, we'd find we needed 2 x 10 bales; for a 12^2 pen we'd need 2 x 13 bales, and so on . . .

In algebra terms, the rule for how many bales you need for an n x n square enclosure is :

$$n \rightarrow 2(n + 1)$$

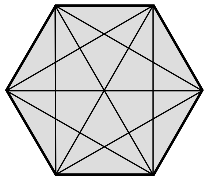
nb The photocopy masters are grids for use with standard 22mm x 44mm dominoes (these are the smaller dominoes sold in packs containing sets of different colours); the first grid can be printed as a straight A4 sheet whilst the second grid needs to be enlarged to A3 for pupils needing a larger working space.



polygon diagonals

intro

This is a straightforward investigation with a well-defined pattern in the results. In fact, the pattern is there for all to see in every corner of every polygon . . .



first steps

This investigation is about polygons and diagonals. *Polygons* are the straight-sided 2-dimensional shapes we call triangle, quadrilateral, pentagon, hexagon, heptagon etc. In any polygon (diagonals are not just for rectangles!) a *diagonal* is a line which joins one vertex (corner) to any other non-neighbouring vertex (a line joining a vertex to its neighbour is simply a *side* of the polygon).

the investigation

Having established the definitions, you can now ask pupils to work through a range of polygons (3-gon, 4-gon, 5-gon etc), drawing and counting the diagonals for each. Explain to them that the aim is to find a pattern of some sort in the results.

practical

Pupils can draw their own polygons (which in itself can be a worthwhile exercise) but you might prefer to give them sheets already printed with the various polygons to be investigated. This is an individual project ie all pupils should draw and count the diagonals for themselves.

results

There should be little disagreement about the results :

no of sides	no of diagonals
3	0
4	2
5	5
6	9
7	14
8	20

– nor about the fact that 27 comes next, then 35, then 44 . . .

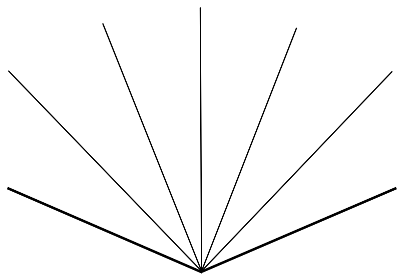
notes

Pupils will readily spot that as the number of sides goes up, the number of diagonals increases by 2, by 3, by 4 and so on . . . but can they work out eg how many diagonals a 10-gon or 20-gon will have *without* going through all the intermediate results? In other words, can we find a rule which will tell us for any polygon how many diagonals there are? The rule is in fact :

for a polygon with n sides, number of diagonals = $\frac{1}{2}n(n-3)$

So a 10-sided polygon will have $\frac{1}{2}(10 \times 7)$ ie 35 diagonals and a 20-sided polygon will have $\frac{1}{2}(20 \times 17)$ ie 170 diagonals.

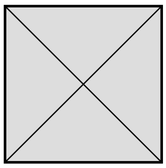
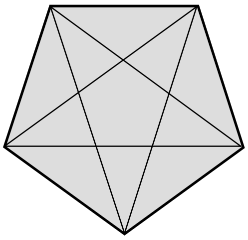
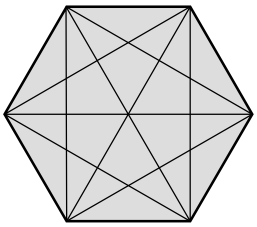
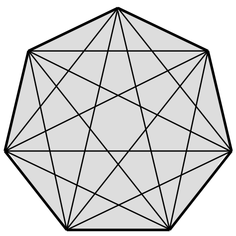
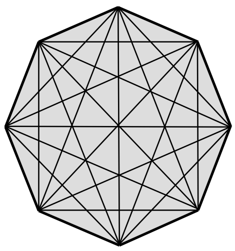
In the abstract this rule is obviously rather hard to identify for pupils of this age . . . but by looking at what actually happens at each vertex when the diagonals are drawn in, you can make things much clearer. Look at one vertex of an octagon, for example :



Apart from this vertex itself, there are 7 others in the shape. 2 of these are neighbours, though, which leaves just 5 for you to join up to when you're drawing diagonals. So from this one vertex you can draw 5 diagonals – and the same applies to every other vertex . . . which means that altogether there'll be 8 lots of 5 ie 40 diagonal-ends in the final shape . . . and as every diagonal has 2 ends, that means 20 actual diagonals (as pupils will have found).

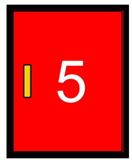
So that's it : however many sides your polygon has, there'll be that many vertices and that many minus 3 diagonal-ends at each vertex; multiply these two numbers together, halve the result (every diagonal has 2 ends) and that's the total number of diagonals.

polygon diagonals

	polygon	number of sides	number of diagonals
	square	4	2
	pentagon	5	5
	hexagon	6	9
	heptagon	7	14
	octagon	8	20

intro

The well-known ‘1000 lockers problem’ provides an interesting investigation for pupils in this age-group. The first challenge is to explain the problem; after this you can get your pupils to look at a simpler version and to investigate step-by-step how things develop. There is a simple pattern waiting to be found – which can then be used to solve the original problem.



The really hard question is: why do we get this particular pattern?

the problem

A college has 1000 students and just off the entrance hall there’s a bay with 1000 lockers, one for each student. On the first day of term, the lockers are all closed but student no 1 arrives early in the morning and straightaway opens the doors of all 1000 lockers. Student no 2 arrives and closes the doors of all the even-numbered lockers (ie 2, 4, 6, 8 and so on). Later, student no 3 turns up and he focuses on all the lockers numbered with a multiple of 3 (ie 3, 6, 9 and so on), opening those which are closed and closing those which are open. Student no 4 arrives and goes along lockers 4, 8, 12, 16 and so on (multiples of 4), again opening those which are closed and closing those which are open.

During the morning all the students arrive in turn (ie in number order) and they each do the same thing ie (*here you could ask your class ‘is there a neat way of describing what it is each student does when it’s his/her turn?’*) they change/reverse the ‘state’ of those lockers numbered with multiples of their own id number.

The problem is : How many lockers will remain open when all 1000 students have visited them, opening and closing in the way described above?

first steps

Obviously the first thing is to make sure that your pupils really do understand the problem. After this, remind them that when they have a difficult problem it’s often useful to look at a simpler version of it; here we can start by investigating what happens with say just 20 lockers and 20 students . . .

the investigation

Pupils should look at a bank of 20 lockers and systematically chart what happens as the students come along opening and closing the lockers. They will need to work carefully – but before too long they should begin to see a pattern . . .

practical

Pupils can work on their own or in twos or threes. It isn't easy to find a simple practical set-up here. A card strip with 20 'lockers' and card flaps works reasonably well – or alternatively, each group can mark 20 squares on a card strip and use two-sided counters to represent the open / closed states. If you have easy access to a computer lab, then pupils can use a spreadsheet, numbering a row of 20 cells, using two different cell colours to stand for the open / closed states and copying each completed line before making the changes needed for the next one . . .

Squared paper and coloured pens are fine for recording the results line by line and should make it easy enough for pupils to see the pattern.

results

Before too long pupils will realise that it seems to be the lockers with square numbers on them which stay open. Once they've completed their investigations with 20 lockers they should find that the ones still open are those numbered 1, 4, 9 and 16 (four lockers in all).

answer / explanation

- To work out what happens with 1000 lockers, it seems that we just need to know how many square numbers there are up to 1000 . . . 30^2 is 900 and 31^2 is 961 but 32^2 is 1024, which is too large. So there are 31 square numbers in the first 1000 whole numbers – and the answer to our original problem is 31 lockers!
- The real challenge is to answer the question : why is it that in the end it's the lockers with square numbers on which are left open? It's worth getting pupils to think about this and to try to come up with an explanation. If they're completely stuck, get them to think about factors . . . and to focus not on what each student does but on what happens to *individual lockers* . . . ask them to look at some lockers which have square numbers on them and some which don't . . .
- 12 for example has six factors so that during the exercise, three students will find locker number 12 closed and will open it and three students will find the locker open and will close it – so that by the end, locker number 12 will be closed. Now think of eg locker 81; it has five factors ie during the whole exercise it will be closed, open, closed, open – and then closed! So in the end it's all down to the fact that every square number has an odd number of factors . . .

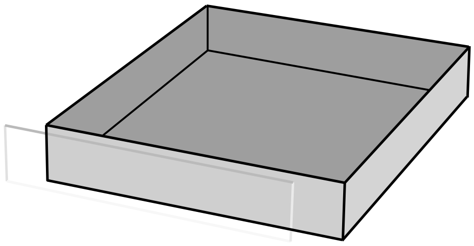
note

The last step is probably the most rewarding but pupils will only make it if they're familiar with the fact that a square number always has an odd number of factors . . . so, you might like to introduce or revise this fact in advance eg a couple of weeks before presenting the investigation (rather than just beforehand, which might make it all too easy).

1000 LOCKERS FIRST 20 ROWS

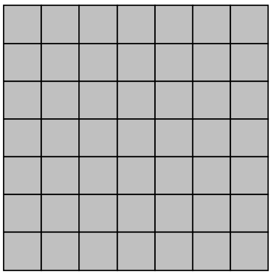
intro

This investigation is about open-top boxes and their nets; starting with a given square of card, the idea is to produce a number of different boxes and then to see which of them has the largest volume. It's a good investigation because at first it looks easy to see what's going on – but then further research shows that it's not quite so straightforward after all . . .

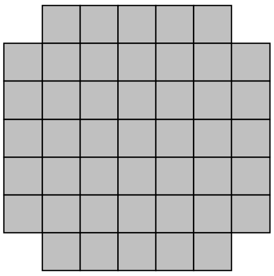


the investigation

For a given size of square card you can make different nets by cutting away larger or smaller square corners. For example, suppose you start with a 7cm x 7cm piece of card, like this :

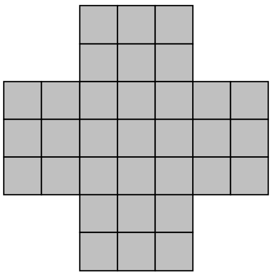


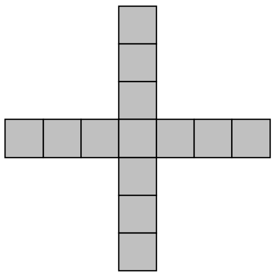
Now cut away a 1cm x 1cm square from each corner :



This net will give you a 5cm x 5cm x 1cm open-top box (rather like the one at the top of the page).

– and here are the other two possibilities :





The first of these boxes has a volume of 25cm^3 and the other two have volumes of 18cm^3 and 3cm^3.

first steps

Start with a 3cm x 3cm piece of card; there's only one box you can make – and the same is true of the 4cm x 4cm version. The question of largest possible volume doesn't arise. But with a 5cm x 5cm piece of card you can cut away 1cm square corners or 2cm square corners, giving you two different shapes of box. A 6cm x 6cm card also produces two alternatives whilst the 7cm and 8cm versions give you three . . . With every one of these first four cases, it's the option involving cutting just a 1cm x 1cm square from each corner which generates the box with the largest volume. After investigating these first few cases, some children will think they've seen the pattern but . . . a 9cm x 9cm piece of card gives you four possibilities and suddenly you see that it's not the first option which gives the maximum volume!

*nb in this investigation we're sticking to whole numbers throughout.

practical

Decide (according to the group involved) whether you want your pupils to make up some nets from card or whether they can investigate by drawing nets on squared paper and then just sketching the boxes. It's not essential to make the boxes but it might be a good lead-in to make the odd one or two (larger size perhaps) to demonstrate at the beginning.

How to record results? Encourage them to find a sensible way of setting down their findings.

further questions

**for pupils who have not seen a table of results*

- In every case, one of the results is a square number. Why?
- Which sizes of card would you use to make a box with volume = $100cm^2$? What about boxes with volumes = $200cm^2$, $300cm^2$, $400cm^2$, or $500cm^2$
- Which sizes of cards allow you to make the nets of cubes?
- Could you ever get two boxes, one with double the volume of the other, starting from the same size square?

notes

Explanation / calculation of max volume for a given square of card is beyond scope of this age-group – which makes it a bit unsatisfying for them in one way ie in the end there isn't a clear pattern which they can identify. However, the investigation is still a valuable one as this is just the sort of thing which often happens to 'real' mathematicians: you can find something of a pattern but can't really tie it down or explain it (yet) . . . Usefully, though, you can once again drive home the need to look at enough cases before forming a judgement about what's going on in any mathematical situation . . .

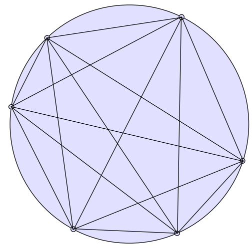
results

Here's a complete set of results for square nets up to18cm x 18cm:

square edge	corner edge	volume		square edge	corner edge	volume	
4	1	4	⇦	14	1	144	
5	1	9	⇦	14	2	200	⇦
5	2	9		14	3	192	
6	1	16	⇦	14	4	144	
6	2	8		14	5	80	
7	1	25	⇦	14	6	24	
7	2	18		15	1	169	
7	3	3		15	2	242	
8	1	36	⇦	15	3	243	⇦
8	2	32		15	4	196	
8	3	12		15	5	125	
9	1	49		15	6	54	
9	2	50	⇦	16	1	196	
9	3	27		16	2	288	
9	4	4		16	3	300	⇦
10	1	64		16	4	256	
10	2	72	⇦	16	5	180	
10	3	48		16	6	96	
10	4	16		16	7	28	
11	1	81		17	1	25	
11	2	98	⇦	17	2	338	
11	3	75		17	3	363	⇦
11	4	36		17	4	324	
11	5	5		17	5	245	
12	1	100		17	6	150	
12	2	128	⇦	17	7	63	
12	3	108		17	8	8	
12	4	64		18	1	256	
12	5	20		18	2	392	
13	1	121		18	3	432	⇦
13	2	162	⇦	18	4	400	
13	3	147		18	5	320	
13	4	100		18	6	216	
13	5	45		18	7	112	
13	6	6		18	8	32	

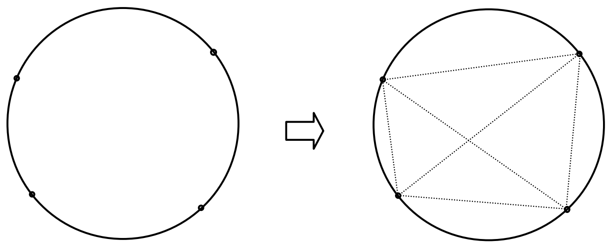
intro

This is an exercise which everyone should carry out at some time. Pupils know that an investigation often leads to a table of results – with of course a simple and straightforward pattern there to be found. Once they've seen a pattern, children are usually quite reluctant to test it out thoroughly. This investigation should be a warning to them . . .



first steps

Suppose you have a circle with a number of points marked around the circumference. By joining every point to every other point, you divide the circle up into a number of regions. A circle with 4 points marked, for example, will give you 8 regions :



the investigation

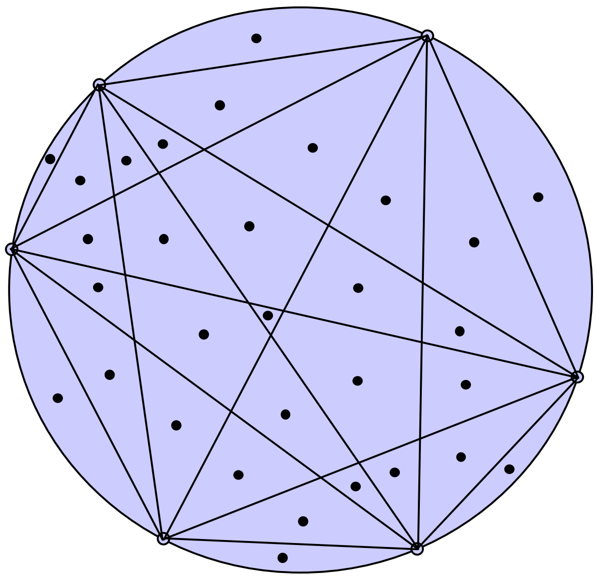
Get the pupils to join every point to every other point and then to count the number of regions for circles with 1, 2, 3, 4, 5 circumference points. It's absolutely essential to stop them at this stage – and then to ask them for their results. Put these up on the board for all to see and ask whether anyone can see a pattern and what the answer is going to be for the next circle. Most of the class will be keen to tell you that they've seen the pattern ie as the number of regions produced for these circles is 1, 2, 4, 8, 16, you're obviously just doubling as you go along – and so the next circle (the one with 6 points on the circumference) will obviously have 32 regions. Now let them draw in the lines for the last circle . . .

practical

Children will need to draw and count carefully. You can let them draw their own circles and put on the points but – the points must not be placed regularly around the circle or we begin to 'lose' regions (the idea being to look for the maximum number of regions we can obtain). A prepared gridsheet might be a better idea (see photocopy master).

results

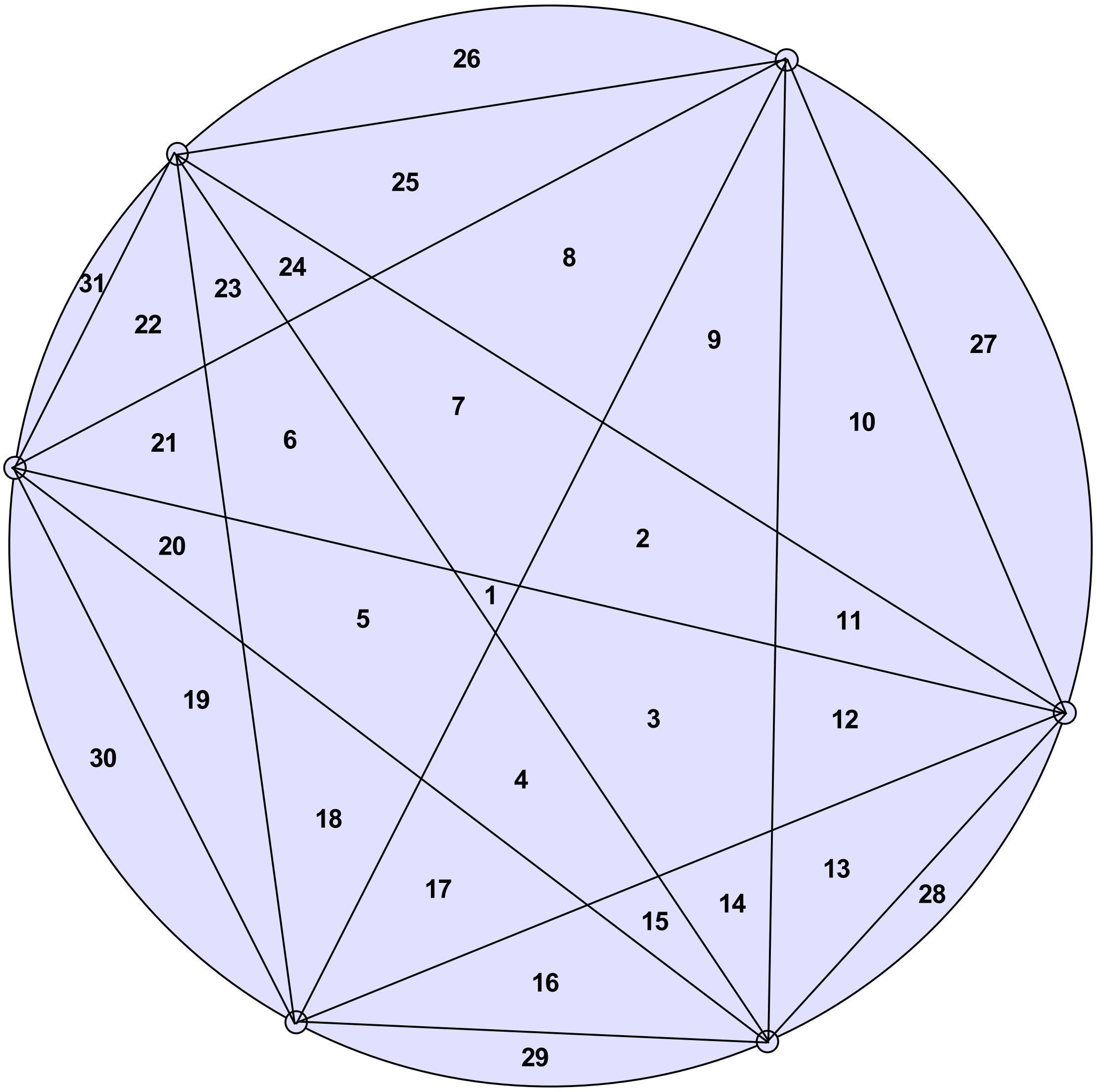
Yes, the maximum number of regions you can obtain with 6 points really is 31. Many children will find their own predictions so compelling that they'll refuse to believe the answer isn't 32; you can get them to count again . . . and you can show them a finished drawing (see later).



notes

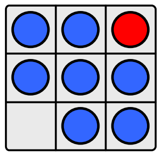
The moral of the story is clearly that one shouldn't jump to conclusions. A few examples might well suggest a pattern to you but you do need to check out your ideas on quite a few cases to be really sure . . .

regions in a circle



intro

Maths games and puzzles sometimes lead to interesting investigations. As a puzzle, ‘corner to corner’ is simple enough but it can be used as the starting point of a challenging investigation. In fact, it’s a double challenge – it takes some effort to obtain the results and then there’s a pattern which is not all that easy to see . . .



first steps

First of all, pupils should familiarise themselves with the ‘corner to corner’ puzzle : starting with a 3 x 3 grid and counters arranged as above, the challenge is to get the red counter from the top right-hand square to the bottom left-hand square in only 13 moves (where for any counter a ‘move’ means a slide into an adjacent empty square – which can be left / right or up / down ie no diagonal moves here and no jumping over!)

the investigation

Having got used to the puzzle on a 3 x 3 grid, pupils should now investigate 2 x 2, 4 x 4, 5 x 5 etc versions, each time aiming to establish the minimum number of moves needed to get the red counter home . . .

practical

This investigation is best carried out in pairs – one pupil can move the counters whilst the other pupil keeps count of the moves.

results

Here are the results which (we hope) will emerge :

square		minimum number of moves
2 x 2	⇨	5
3 x 3	⇨	13
4 x 4	⇨	21
5 x 5	⇨	29
6 x 6	⇨	37
. . .		. . .

notes

There is a pattern here which in one way is easy to spot : as you go up through 2 x 2, 3 x 3, 4 x 4 and so on, you just keep adding 8 to the minimum number of moves. But what about a general rule? What would the minimum number of moves be for a 20 x 20 square? Or for a 50 x 50 square? Or for a 100 x 100 square? Pupils might or might not be able to spot the general rule, which is :

for an n x n square, $n \Rightarrow 8n - 11$

