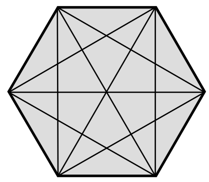
polygon diagonals

intro

This is a straightforward investigation with a well-defined pattern in the results. In fact, the pattern is there for all to see in every corner of every polygon . . .



first steps

This investigation is about polygons and diagonals. *Polygons* are the straight-sided 2-dimensional shapes we call triangle, quadrilateral, pentagon, hexagon, heptagon etc. In any polygon (diagonals are not just for rectangles!) a *diagonal* is a line which joins one vertex (corner) to any other non-neighbouring vertex (a line joining a vertex to its neighbour is simply a *side* of the polygon).

the investigation

Having established the definitions, you can now ask pupils to work through a range of polygons (3-gon, 4-gon, 5-gon etc), drawing and counting the diagonals for each. Explain to them that the aim is to find a pattern of some sort in the results.

practical

Pupils can draw their own polygons (which in itself can be a worthwhile exercise) but you might prefer to give them sheets already printed with the various polygons to be investigated. This is an individual project ie all pupils should draw and count the diagonals for themselves.

results

There should be little disagreement about the results :

no of sides	no of diagonals
3	0
4	2
5	5
6	9
7	14
8	20

– nor about the fact that 27 comes next, then 35, then 44 . . .

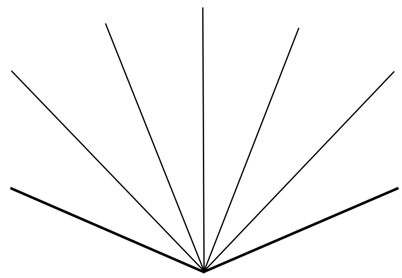
notes

Pupils will readily spot that as the number of sides goes up, the number of diagonals increases by 2, by 3, by 4 and so on . . . but can they work out eg how many diagonals a 10-gon or 20-gon will have *without* going through all the intermediate results? In other words, can we find a rule which will tell us for any polygon how many diagonals there are? The rule is in fact :

for a polygon with n sides, number of diagonals = $\frac{1}{2}n(n-3)$

So a 10-sided polygon will have $\frac{1}{2}(10 \times 7)$ ie 35 diagonals and a 20-sided polygon will have $\frac{1}{2}(20 \times 17)$ ie 170 diagonals.

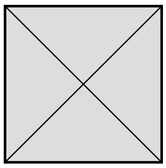
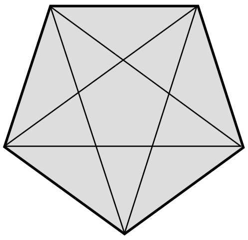
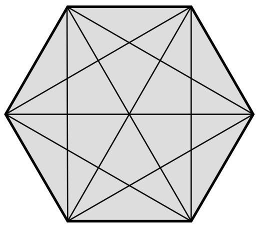
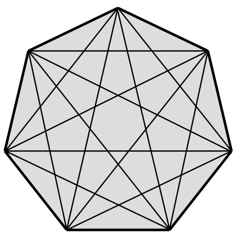
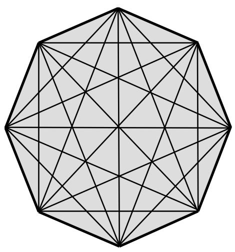
In the abstract this rule is obviously rather hard to identify for pupils of this age . . . but by looking at what actually happens at each vertex when the diagonals are drawn in, you can make things much clearer. Look at one vertex of an octagon, for example :



Apart from this vertex itself, there are 7 others in the shape. 2 of these are neighbours, though, which leaves just 5 for you to join up to when you're drawing diagonals. So from this one vertex you can draw 5 diagonals – and the same applies to every other vertex . . . which means that altogether there'll be 8 lots of 5 ie 40 diagonal-ends in the final shape . . . and as every diagonal has 2 ends, that means 20 actual diagonals (as pupils will have found).

So that's it : however many sides your polygon has, there'll be that many vertices and that many minus 3 diagonal-ends at each vertex; multiply these two numbers together, halve the result (every diagonal has 2 ends) and that's the total number of diagonals.

polygon diagonals

	polygon	number of sides	number of diagonals
	square	4	2
	pentagon	5	5
	hexagon	6	9
	heptagon	7	14
	octagon	8	20

