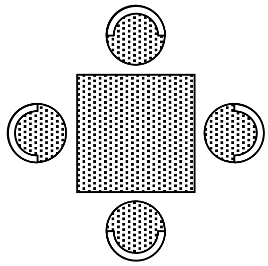
intro

At the Maths Café the tables are all square and each table seats 4 people. That's simple enough but then customers don't always come along in groups of 4 and what's more, larger groups often say that they want to sit together . . .

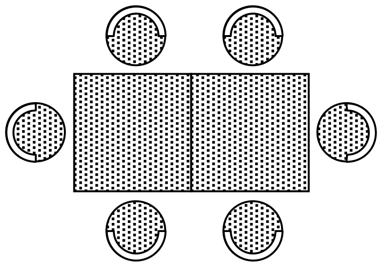


This investigation is all about arranging tables to accommodate different numbers of diners.

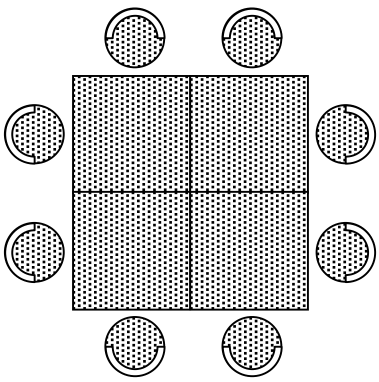
first steps

A good way of starting the thing off is to look at some specific seating problems; by working on one or two easy examples, the children will start to get a feel for how different arrangements work – and it should then be easy enough to move them on to a wider investigation . . .

Suppose 6 people come into the café . . . if they're happy to be in two groups, you can sit them 3 to a table at two different tables . . . but if they want to stay as one group, you'll have to put two tables together like this:



Of course, it's not always this easy . . . Say one evening 8 people arrive at the café; your first idea might be to take the above arrangement for 6 and just add an extra table – but suppose it's towards the end of the evening and you'd like to use the four empty tables you have left (to make sure the café looks totally full) . . . how could you arrange four tables together to seat 8 diners?



One large square seems to do the trick!

The rules, by the way, are that you must always have :

1. whole sides together when you join tables (no half-and-half)
2. only 1 person per place – and no empty places allowed!

Keeping to the these rules, try to find seating arrangements which will work for these seating problems :

- **a** use 3 tables to seat 8 diners who wish to be in one group
- **b** use 5 tables to seat 12 diners who wish to be in one group
- **c** use 5 tables to seat 14 diners who don't mind being split up
- **d** use 6 tables to seat 12 diners who wish to be in one group
- **e** use 6 tables to seat 18 diners who don't mind being split up

* see separate sheet for answers.

the investigation

The aim is to find and record ways of accommodating different numbers of diners using a given number of tables. For example, if you've got 3 tables, could you seat 6 diners with no seats left empty? Could you seat 12 diners? Could you seat 18 diners? What different arrangements are possible with 3 tables? Remind the children that they must stick to the rules ie just one person per seat / no empty seats / tables joined only by whole sides.

The first real challenge for the pupils is in the actual gathering of results. They will need to be quite thorough in their approach to be sure they've covered all possibilities for a given number of tables. After they've been working for a while it's probably worth stopping them and getting everyone to discuss how they're going about achieving this objective. Obviously a systematic approach is more likely to be successful. Say, for example, you've got 5 tables. You start off by looking at different ways of putting all 5 tables together. Next, you could look at 4 tables together and 1 on its own – and ask see how many different arrangements there are for this combination. After this, you could look at 3 tables together alongside 2 tables together – and then 3 tables together alongside two single tables – and so on, each time looking at different ways in which the particular combination can be achieved.

practical

This investigation can be carried out as a pencil-and-paper exercise and some children will naturally work in this way. Others, though, will find it livelier and more engaging to have squares of card or polydron to represent the tables and counters to stand for the chairs. They can use squared paper to record their results and work in pairs or small groups.

results

It's pretty obvious that the more tables you have, the more flexibility there will be in how many diners you can accommodate. This table summarises the possibilities for up to 5 tables (see later for illustration of various arrangements for these numbers) :

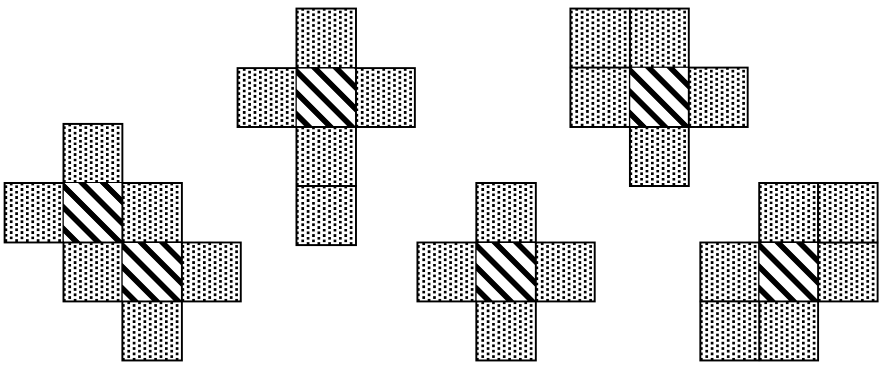
	possible diner numbers
1 table	4
2 tables	6 / 8
3 tables	8 / 10 / 12
4 tables	8 / 10 / 12 / 14 / 16
5 tables	10 / 12 / 14 / 16 / 18 / 20

Of course, you don't have to stop at 5 tables . . .

notes

There isn't one simple pattern to be found in this table of results – but there's plenty to draw attention to / question / discuss :

- What's the largest number of diners you can accommodate for any given number of tables? (ans : 4 x the number of tables!)
- Pupils might spot something special about 9 dining tables : if they're arranged in a large square, you end up with one table 'trapped' inside ie totally out of use. How many ways can pupils find to 'trap' tables using fewer than 9 tables? There are a few (there's even one way of trapping 2 tables using just 6 tables) :

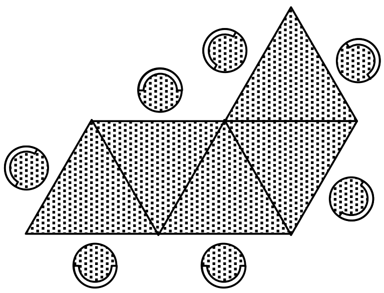


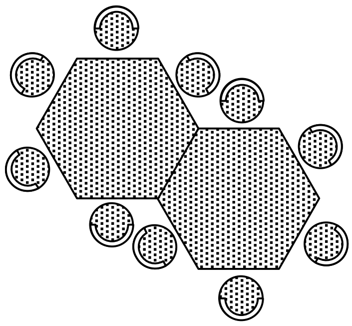
- Can any pupil explain why the results all seem to feature even numbers of diners? Why can't you have an arrangement for an odd number of diners?

- Could you ever have an arrangement with the same number of diners as tables? (ans : yes, eg 16 tables arranged in one large square or 18 tables arranged in a 6 x 3 rectangle)
- Could you ever have an arrangement where the number of diners is lower than the number of tables they're sitting at? What's the smallest number of tables where this could happen? (ans : yes, eg 20 diners can sit at 24 tables if the tables are in a 6 x 4 rectangle; however, if you take 2 tables away from one corner of this rectangle, you'll still have 20 diners but now sitting at just 22 tables – which is also the smallest number of tables where this can happen)
- What's the largest number of tables you can 'use up' if you have a group of 24 diners? (ans : 36 tables arranged in one large square)
- Of course, the key to each arrangement of tables is how many 'joins' you've got . . .

extension

This investigation is all about square tables. But dining tables aren't always square – so one way of extending the project might be to get pupils to see what happens* with eg equilateral triangle dining tables or hexagonal dining tables . . .

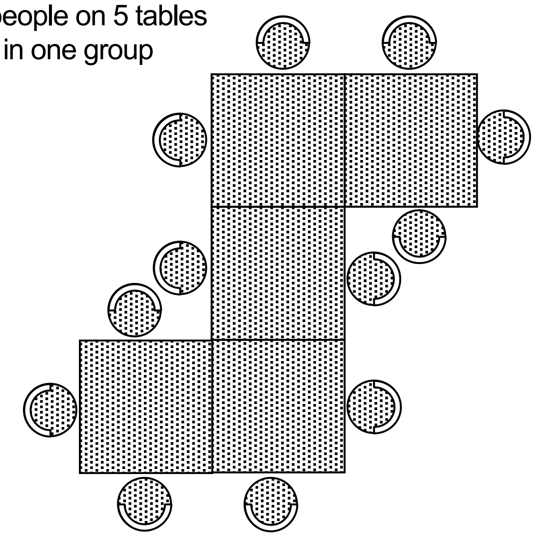




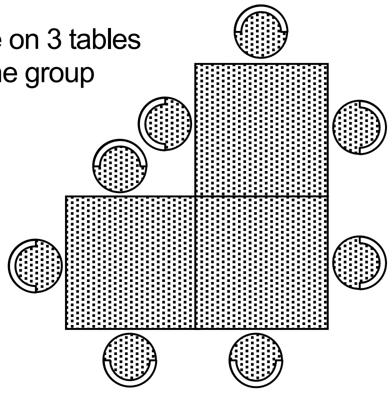
* using isometric grid paper to record results

answers to seating problems

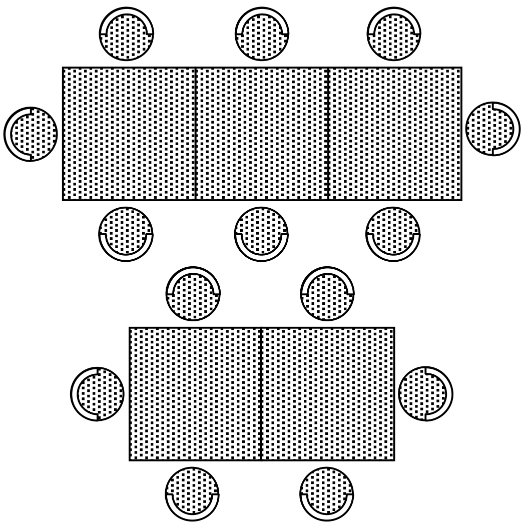
12 people on 5 tables
in one group



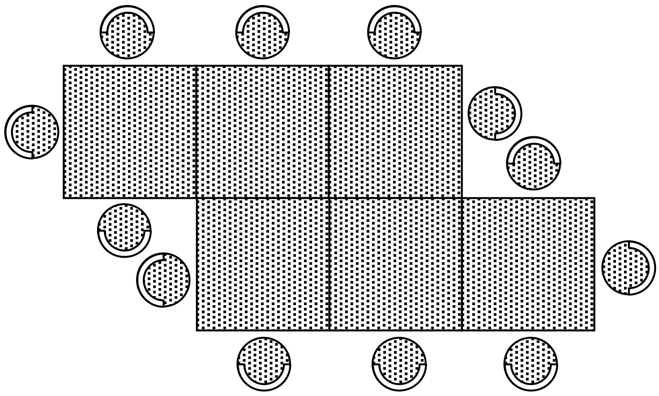
8 people on 3 tables
in one group



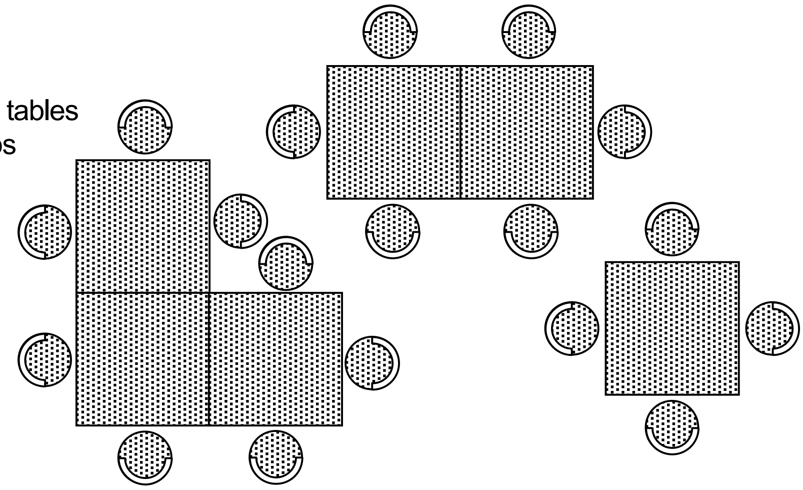
14 people on 5 tables
in two groups



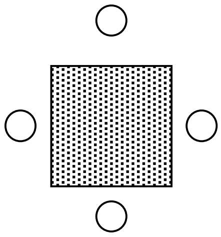
12 people on 6 tables
in one group



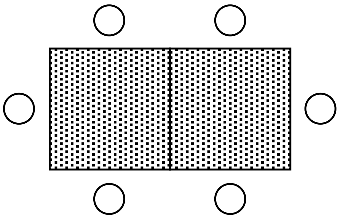
18 people on 6 tables
in 3 groups

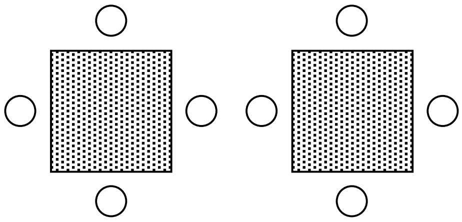


1 table

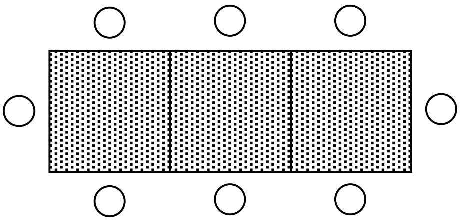


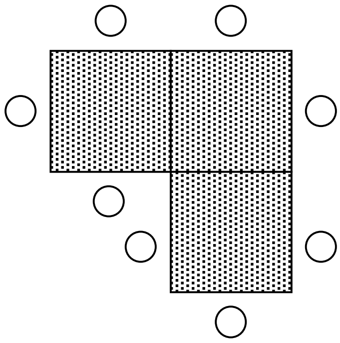
2 tables

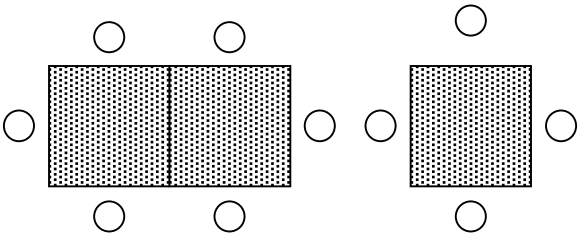


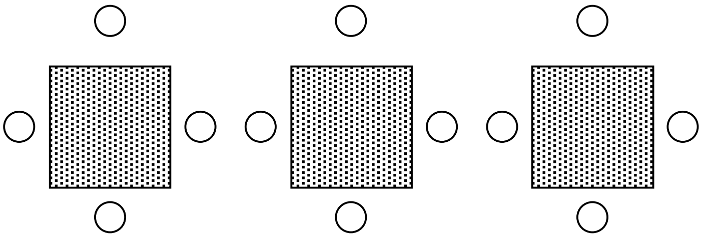


3 tables

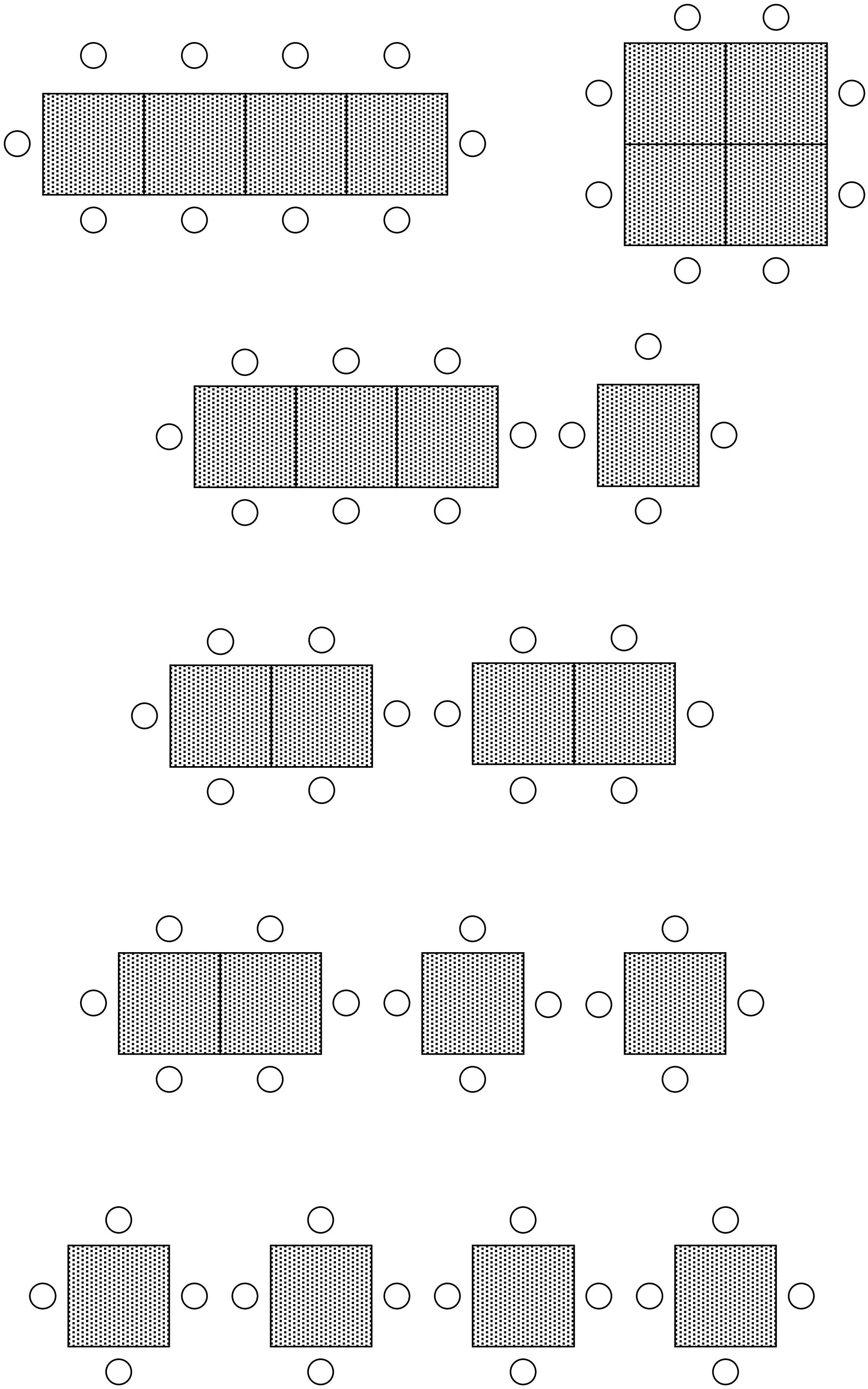




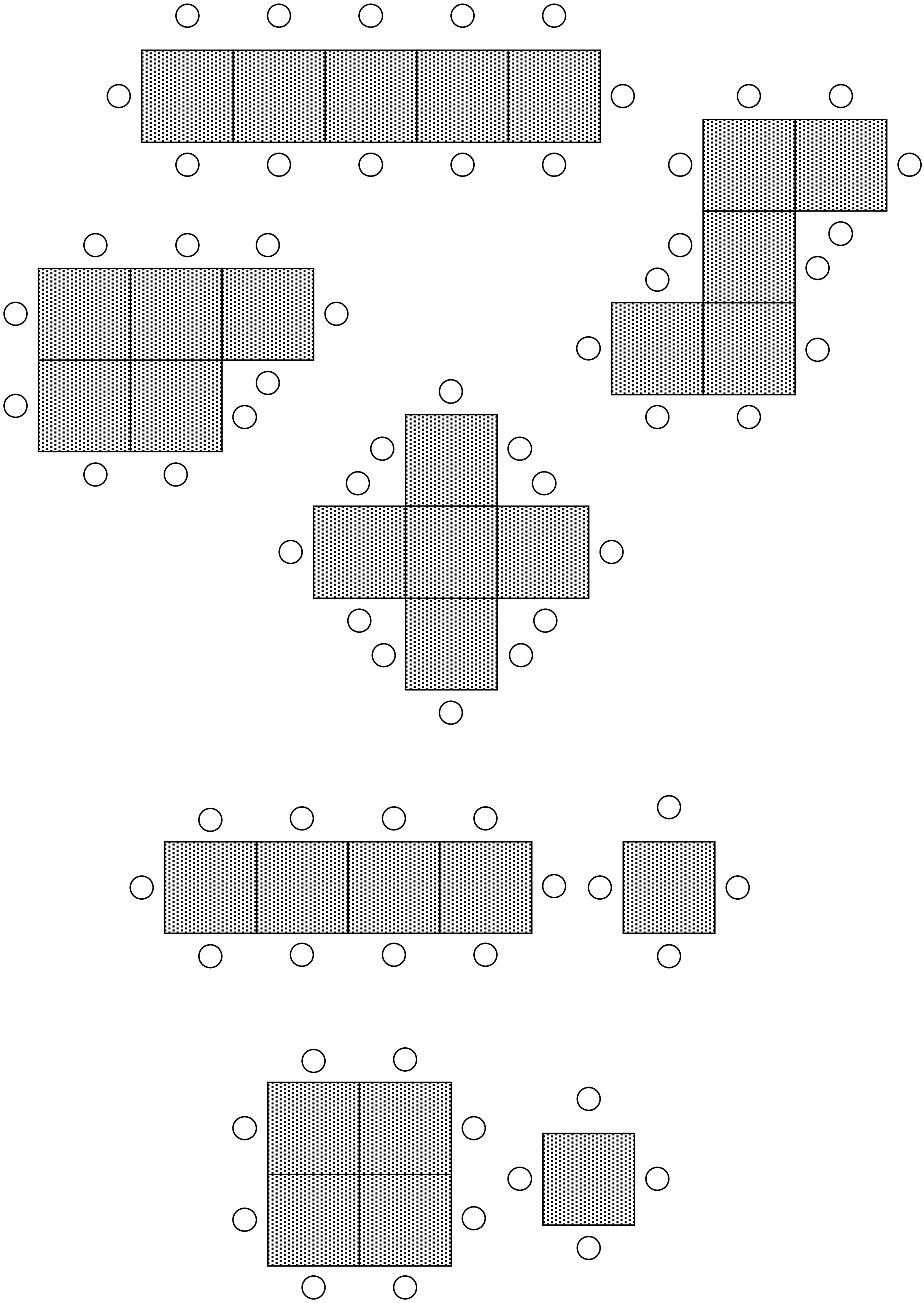




4 tables



5 tables



5 tables