mini~ problems

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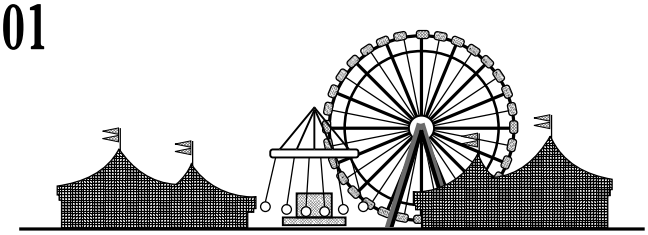
four winds maths

01



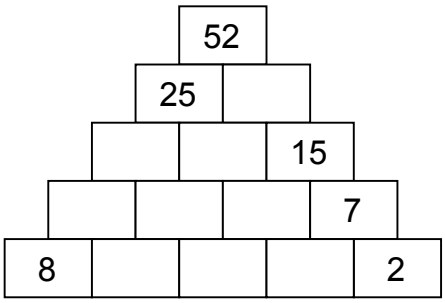
fair shares

One afternoon at the end of term, four teenagers set off to visit the fair. Abigail had £7 to spend, Bella had £4, Charlotte had £5 and Daisy had £8. On the way to the fair, they met Emma, who agreed to join them. There were now five teenagers in the group and guess what? The average sum of money among the teenagers had gone up by exactly £1.

How much money did Emma bring along?

02

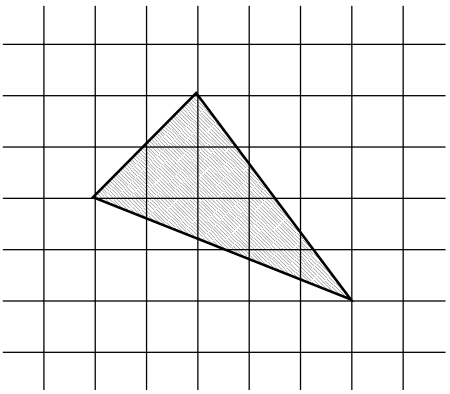
you're my number wall !



You probably already know how a number wall works : where you have a brick resting on two others, the number in the brick is simply the sum of the two numbers immediately beneath it. In other words, you add any two adjacent numbers to get the number above.

Copy the number wall above and work out what numbers to put in the empty bricks.

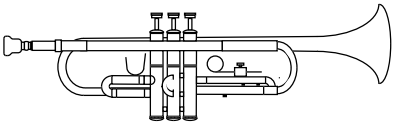
03



triangle area

Carefully copy the triangle above onto squared paper. Then use any method you like to work out its exact area. Imagine the squares of the grid are 1cm squares; this means you should give your answer as so many square centimetres.

04



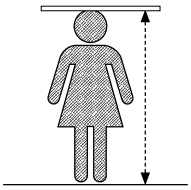
the trumpet major

Guy is an officer in the army. He is also a trumpeter in the regimental band. His friends Simon, Ralph and Peter play three different instruments. These instruments are : cornet, french horn and tuba. Here are some facts about the three friends :

- Either Simon or Ralph plays cornet
- Peter doesn't play the french horn
- Ralph doesn't play tuba or cornet

Who plays the cornet? Who plays the french horn? Who plays the tuba?

05



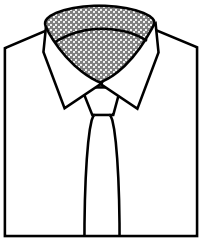
feet and inches

Long ago, a child in London, say, would measure his / her height in feet and inches. As you might know, 1 foot was the same as 12 inches.

On 25th October 1957, Mrs Locke wrote down in her diary the heights of her three children. They were : Annabelle 3 feet 2 inches, Sebastian 4 feet 6 inches and finally, Quentin 4 feet 10 inches.

What was the average height of the three Locke children on that date?

06



The Shirt Store

The Shirt Store sells shirts and ties, as well as handkerchiefs and other things. On Monday 42 customers visited The Store. After looking at what was on offer, 32 customers bought new shirts and 7 of these also bought ties. Sadly, 4 customers bought neither a shirt nor a tie.

How many customers bought just a tie?

07

canned music

When Lukas looks in the shop window, he sees just the pair of headphones he wants. The price? £400 plus tax. Yes, there's a tax on most things you buy and for the cans (headphones) the tax is 18%. This means Lukas has to add 18% to the basic price in order to find exactly what he has to pay. Even though they're noise-cancelling headphones, the final price is too much for poor Lukas and so he just goes home.

What is the final price of these cans?

08

mini sudoku

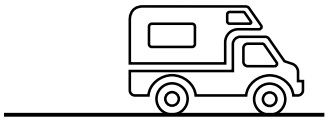
3	1	2	
2	4		1
1		4	2
	2	1	3

You might already have come across 'sudoku'. Here we've got a mini version of the original. The idea is to fill all the individual squares of the grid using only the numerals 1 to 4 and with no numeral appearing twice in any row or column or in any of the smaller 2 x 2 squares.

	2		
3			
		4	
	3	2	

Copy each of the two sudoku squares and see if you can complete them. When you've finished, each row, each column and each smaller 2 x 2 square should contain each of the numerals 1 to 4.

09



measure for measure

Roughly speaking, these speeds are the same :	20 mph	=	32 km/hr
	15 mph	=	24 km/hr
	35 mph	=	56 km/hr
	75 mph	=	120 km/hr

As you know, *mph* means 'miles per hour' and *km/hr* means 'kilometres per hour'. And there's a simple rule for changing the number on the left to the number on the right.

(a) Can you spot the simple rule for changing mph to km/hr? If so, can you put this simple rule into words?

(b) A train is travelling at 100 mph. What's that in km/hr? And if a car's speedometer shows 72 km/hr, what's that in mph?

10

milking time

Farmer Brown has a fine herd of dairy cows who live happily on his hillside twelve acre field. When it's time for milking, the cows must walk down to the milking shed. Moving at a steady 1.5 km/hr, it takes a typical cow 20 minutes to walk from the field gate down to the shed.

How many metres is this journey?

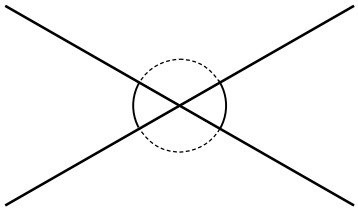
11

the brothers Karamazov

Four brothers (Arkady, Boris, Mikhail and Nikolai) live with their parents in a large old house on the edge of Leningrad (that's in Russia). Boris is 2 years older than Arkady; Mikhail is 2 years older than Boris; and Nikolai is 2 years older than Mikhail. The ages of the four brothers add up to 32.

How old is Arkady?

12

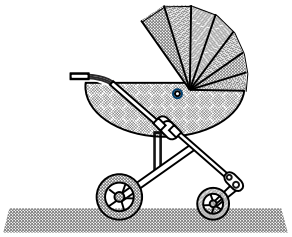


crossed lines

Two lines cross each other, as in the diagram above. As you can see, this makes two different angles; the larger angle is exactly double the size of the smaller angle.

What is the size of the smaller angle?

13



problem parents

My parents only ever had one child – me! As well as this, they now have just one grandchild, called Peter (he's in the pram pictured above). But as a matter of fact, I am not Peter's father.

What relation am I to Peter?

14

sums & diffs 1

	□	2
+	9	8
1	4	□

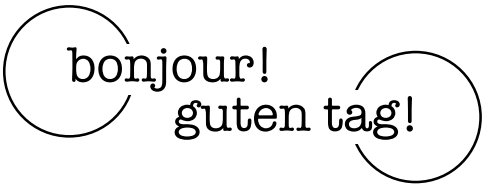
	5	4
+	7	□
□	3	3

	6	□
+	8	3
1	□	2

	□	3
+	9	□
1	2	0

Work out what numbers must go in the empty boxes.

15

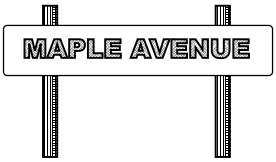


language barrier

There are 25 pupils in the 5th Form at Carte Blanche School. Each pupil in this form must learn either one or two foreign languages, chosen from French and German. At present there are 15 pupils learning French and there are 12 pupils learning German.

How many 5th Form pupils are learning both of these languages?

16

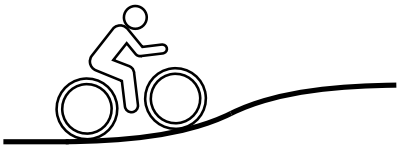


Sophie's cousins

Sophie is 12 years old and she has three cousins, Annabelle, Beatrice and Cressida. The cousins live in Maple Avenue, just round the corner from Sophie, and they're all younger than she is. One day, thinking about her cousins, Sophie notices an interesting fact : the cousins live at house number 20 Maple Avenue and their ages add up to 20. What's more, each cousin's age is a prime number!

What are the ages of the three cousins?

17

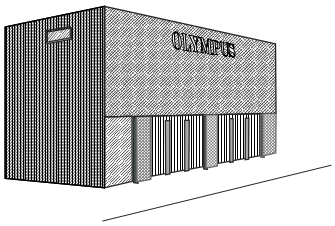


too far to Falmouth!

One fine Spring morning Amol set off to cycle to his uncle's house in Falmouth. By the time Amol was only half-way there, he was tired and hungry – and so he stopped to eat his packed lunch and to have a rest. After lunch he set off again and cycled a further 6 km before giving up; this was a pity as at that point he was only 4 km from his destination. Amol turned around and cycled home. Another day perhaps . . .

How far altogether did Amol cycle that Spring day?

18

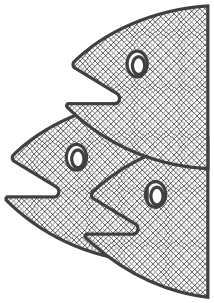


the big screen

Every time you visit the Olympus Cinema in Clapham, they stamp your membership card. As soon as you have 10 stamps on your card, they give you a free ticket. Chanu is mad about films and he's really overjoyed when his uncle gives him a Cinema Gift Card. This card covers the cost of 1000 visits to the Olympus Cinema. What a great gift!

Think carefully and then work out how many free cinema visits in total Chanu will get as a result of his uncle's generous gift.

19



fish supper

Barnes Water is a large lake in South London; it's very popular with anglers, who often spend Saturday, Sunday or even whole weekends there. Last Sunday was a fine day at Barnes and 26 keen anglers turned up for a day's fishing. By the end of the day 4 of the anglers had caught no fish at all; 17 anglers had caught pike – and 6 of these had caught roach as well.

How many anglers had caught just roach and nothing else?

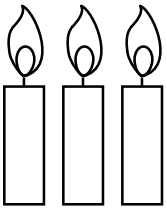
20 FIBONACCI

how to begin?

Fibonacci (by the way, you pronounce it like this : *fib-on-archie*) was a famous mathematician. Born in Italy, in Pisa (the place with the leaning tower), he lived from about 1170 to about 1250. One thing he's well known for is the 'Fibonacci Sequence'. You might already have heard of this. The idea is simple enough : you get each new term of the sequence by adding together the previous two terms. The most famous Fibonacci sequence is: 1, 1, 2, 3, 5, 8, 13 . . . and so on. This begins with 1, 1 but you can choose any two numbers to start.

What two numbers could you start with if you wanted the 6th term to be 20?

21



double birthday

Stephen and Stephanie are twins (but obviously not identical twins!). Their birthday is on the 20th September each year. As a matter of fact, in 2021 the 20th September was on a Monday.

On which day of the week was the 5th November that year? (Try to work this out without looking at a 2021 calendar!)

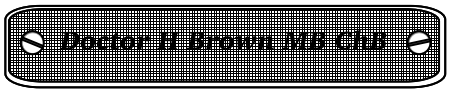
22

the seven ages of man

As you might know, William Shakespeare was famous for writing plays and poems. In one well-known speech, he describes the seven different stages which, he says, make up every human life. The speech is known as the 'Seven Ages of Man'.

Of course, Shakespeare didn't think of the seven stages of life as being exactly equal in length. But suppose they were equal . . . Well, Rahul's uncle has just died, aged exactly 62 years and 5 months. Suppose you had to divide his life into seven equal parts. How long (in years and months) would each part be?

23



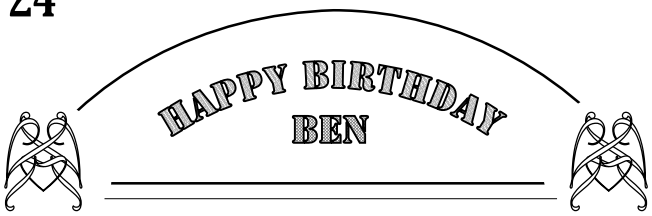
parking

Outside Dr Brown's surgery there are three cars parked: one is a sports car, one is a family saloon and the other is an estate car. One car belongs to Dr Brown, one car belongs to Mr Smith and one car belongs to Mrs Green. Here's some useful info :

- the sports car is blue
- the red car is a family saloon
- one of the cars is yellow
- Dr Brown's car is not yellow and it's not a sports car
- Mrs Green's car is not blue

Who owns the sports car?

24



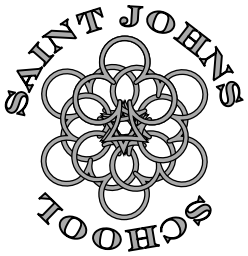
birthday boy

Today is Ben's birthday. After counting the candles on his birthday cake, young Ben surprises his parents by saying,"You know, in five years' time my age will be exactly double what it was last year."

So how old is Ben today?

ps All the ages in this question are whole numbers.

25

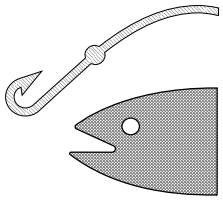


sibling rivalry

As part of a class survey, the 20 children in Form 3 at St John's School were asked whether they had any brothers or sisters. 7 children said they had brothers and sisters, 6 children said that they had just brothers – and 4 children said they had just sisters.

How many children had no brothers or sisters?

26



fishing competition

This year's Devon Trout Fishing Contest attracted more or less the usual number of contestants. Although more than half of the contestants were men, there were also quite a few female anglers. The contest lasted six hours and this year the weather was good. Five entrants caught 4 trout each, eight entrants caught 3 trout each, nine entrants caught 2 trout each, six entrants caught just 1 trout each and, finally, six entrants caught no trout at all.

What was the average number of trout caught by this year's contestants?

27

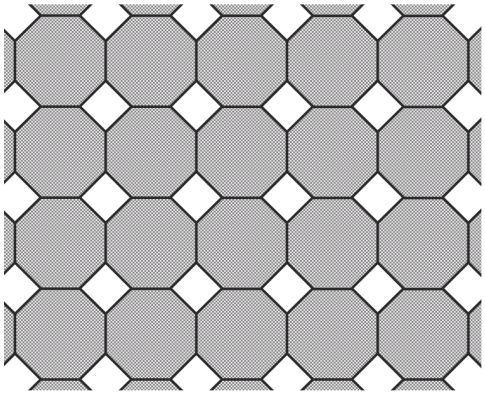
in the queue

There are four people waiting in the queue at the Post Office. There's a teenager, an old lady, a mother with a pram and a middle-aged man. Here are some facts about where they stand:

- the teenager is two places behind the middle-aged man
- the old lady is just in front of the middle-aged man

Where is the mother in the queue?

28

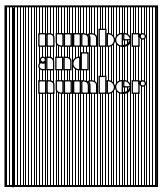


octagon floor tiles

The diagram above shows just a part of the courtyard of the famous Hermitage Museum in Saint Petersburg, Russia. This courtyard is totally paved with octagon and square tiles, arranged exactly in the pattern you can see in this section. Each square tile has an area of 4 square metres and each octagon tile has an area of 21 square metres.

Roughly what percentage of the area of the courtyard is covered by square tiles?

29

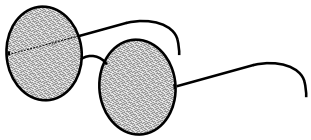


number & number

One day in the early 1960s, Mr Pascal (the maths teacher) was visiting his local bookshop when he noticed a new arrival on the shelves. The book was called 'Number & Number' and its price was 7 shillings. He knew that his Year 8 pupils would really love the book and straight away he bought 16 copies.

Work out how much the 16 books would cost. Give your answer in pounds and shillings. (20 shillings were exactly the same as 1 pound in those days.)

30



30 degrees in my shades . . .

Last year, May 24th was the hottest day of the year in Central London. At noon, the temperature reached a sweltering 30º. A survey of visitors to Hyde Park revealed the following information :

- wearing sunglasses but no hat 35%
- wearing a hat but not sunglasses 15%
- wearing sunglasses and a hat 15%

What was the percentage of visitors wearing neither sunglasses nor a hat?

31

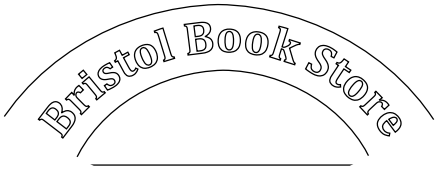
mini sudoku

	1	4	
	4		2
1			

	2	4	
	3	1	
2			4
3			1

Copy and complete these two 4 x 4 sudoku squares so that each of the numerals 1 to 4 appears once and once only on every row, in every column and in each of the smaller 2 x 2 squares.

32



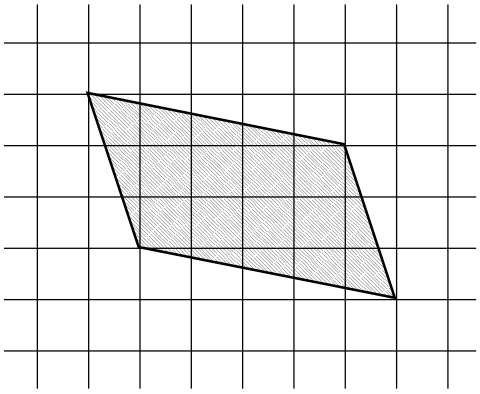
cover story

Each weekday the Bristol Book Store employs one student to help with sales. Here's some information about John, Kate, Lucy and Peter, the four students who work there at present :

- John works on Wednesday, the day before Lucy
- Kate works the day after Peter but the day before John
- Peter works the day after Lucy

1 Who works twice during the week?
2 On which day does Kate work?

33



parallelogram area

Carefully copy the parallelogram above onto squared paper. Then use any method you like to work out its exact area. Imagine the squares of the grid are 1cm squares; this means you should give your answer as so many square centimetres.

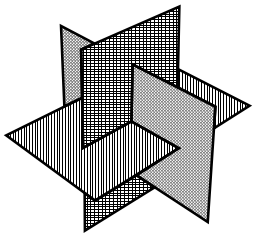
34

Ice Station Zebra

Zainab is a young scientist, just one of a small group based in the Arctic and studying climate change. Zainab has a number of responsibilities but one thing she has to do is to measure the outside termperature each day at midday. In the first week of January, her seven readings were as follows : 4º, 5º, 2º, -2º, -5º, -1º, 4º.

What is the mean (average) of these seven readings?

35

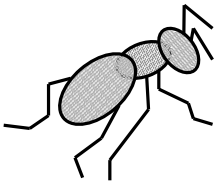


three dimensions

After opening a recent parcel, Liu had three rather attractive pieces of strong card left over from the packaging. One card was blue all over, one was green and one was yellow; each card measured 12cm x 6cm. Liu took the three cards and carefully, using a sharp craft knife, cut a 6cm slit lengthways in the centre of each one. Then she assembled the three cards to make the shape you can see above.

How many places are there in Liu's shape where you can look at three colours coming together in one vertex (corner)?

36

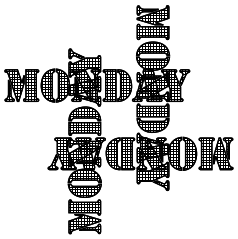


a bad year for ants

Biologists in Delaware, USA, have for some time been studying a local ant colony. The year 2022 began with all the ants in good health. In January a bear discovered the colony and ate 25% of the ants. Another 120 ants perished in March after catching a deadly virus. A terrible sandstorm struck the area in July, after which only 1/5 of the remaining ants were still alive and sadly three-quarters of these ants died in a forest fire in November.

After this no more ants died in 2022, and the year ended with just 9 ants left alive. How many ants had started the year?

37



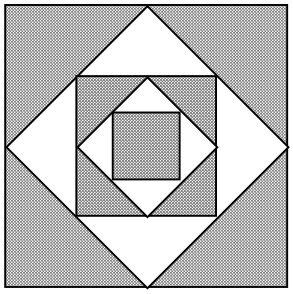
monday, monday . . .

Jenny has five different lessons on monday morning, three before break-time and two afterwards. Here are some facts about Jenny's monday morning :

- PE comes straight after English
- maths comes between break and history
- French is one of the first three lessons of the day
- the lesson just before break is not PE

Try to work out what Jenny's timetable must be and then write a list of the five lessons in order.

38

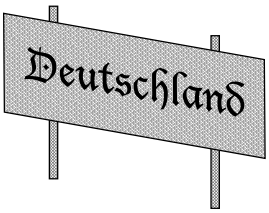


squares within squares

Look at the diagram above. At the centre lies a small square, which has an area of exactly 1cm². This square lies inside another square, which lies inside another square and so on . . .

Make a copy of the diagram; you might well find this easier to do if you're using eg 1cm square grid paper and if you make the side-length of the large outer square, say, 8cm or 16cm. Next, use any method you like to answer this question : What's the total area of all the shaded parts?

39

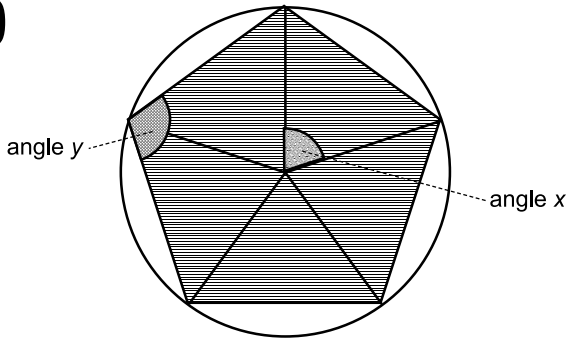


twins age problem

Kurt and Horst are twin boys who live in Lippstadt, a large town in Germany. They have a sister called Zelda who is exactly seven years older than they are. If you add together the ages of these three childen, you get 25.

How old is Zelda?

40



angles in a pentagon

Here you can see a regular pentagon (regular = equal sides, equal angles); it's drawn inside a circle. Lines are drawn from the centre of the circle to the vertices (corners) of the pentagon.

Make a rough sketch of the diagram above and then use what you know about angles to find what angle x and angle y must be.

41

mini sudoku

		3	
	4		
	2		4

2			
		3	
	4	2	

Copy and complete these two 4 x 4 sudoku squares so that each of the numerals 1 to 4 appears once and once only in every row, in every column and in each of the smaller 2 x 2 squares.

42

MEAN AVERAGE

mean sentences

The mean is a kind of average. You probably know how to work out the mean of a set of numbers : you just add up the numbers and then divide this total by how many numbers you've got. For example, if you have five people and their ages total 60, you divide 60 by 5, giving mean = 5.

What's the mean number of letters in a word in this sentence? Using a different number of letters in each word, make up a sentence of five words with a mean of five letters per word.

43

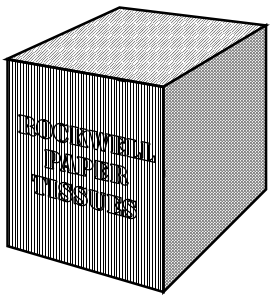
sums & diffs 2

$$\begin{array}{r} 7\,\square\,7 \\ +\ \square\,9\,9 \\ \hline 1\,7\,0\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9\,6\,8 \\ +\ 8\,\square\,7 \\ \hline \square\,8\,6\,5 \\ \hline \end{array}$$

$$\begin{array}{r} 5\,7\,\square \\ -\ \square\,7\,6 \\ \hline 2\,9\,8 \\ \hline \end{array} \qquad \begin{array}{r} 8\,9\,1 \\ -\ 4\,\square\,7 \\ \hline \square\,9\,4 \\ \hline \end{array}$$

Work out what numbers must go in the empty boxes.

44

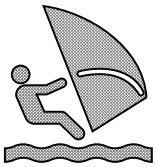


atishoo!

Rockwell paper tissues are the best you can buy, especially if you have a cold! Each box measures 10cm x 25cm x 20cm. The tissues are supplied to supermarkets in large cartons. The inside of each large carton measures 1m x 1m x 1m and of course each one contains many boxes of Rockwell tissues, all carefully packed.

Can one of the large cartons be packed with tissue boxes so that there's no space left over? And if so, exactly how many boxes of tissues are there in each carton ?

45

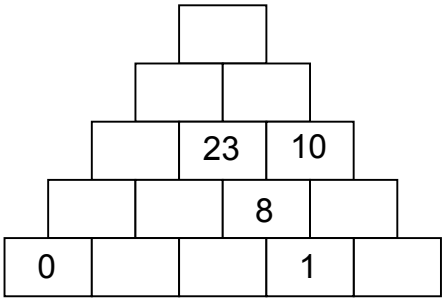


average windsurfing

Henry has a large wooden hut next to the beach at Woolacombe in North Devon. He makes a living by hiring out surfboards to holiday visitors. Business is fairly quiet in early Spring but Henry still gets a few customers. Last week he was closed on Monday but over the next four days (that's Tuesday to Friday) he hired out an average of 6 surfboards a day. Saturday was sunny and more people came along; so for the five days Tuesday to Saturday his average went up to 8 surfboards a day.

How many surfboard customers did Henry have on the Saturday?

46

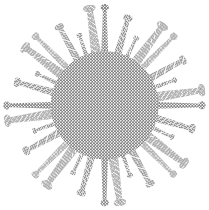


another number wall

You've already seen how number walls work : the sum of two adjacent numbers gives you the number immediately above.

Copy this number wall and work out what numbers should go in the empty spaces.

47



left out in the cold

Professor Sim and his team spend their lives investigating viruses. A while ago, they had four samples of a virus called 'Gamma E2' and they measured how long each sample could stay alive in the cold outdoors. The survival times were these :

3 weeks & 6 days

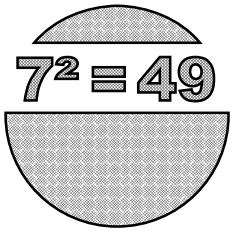
4 weeks & 3 days

3 weeks & 5 days

4 weeks & 4 days

What was the average survival time, in weeks and days, of these four samples?

48

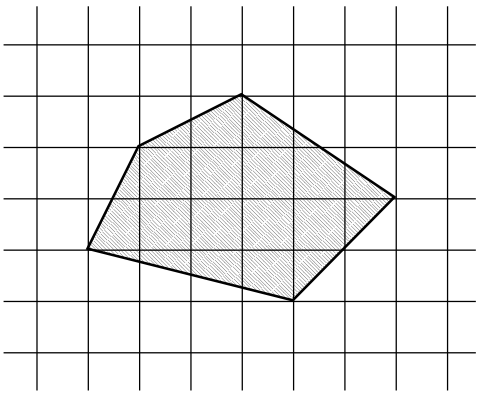


unsquare number

Mr Pascal, the maths master, had been teaching his class about square numbers. Now they were working out the squares of various numbers. Amol gave 1372 as his answer for the square of one number but Jane said straight away, 'That can't be a square number!' And she said this without doing any working-out!

How did Jane know straight away that 1372 could not be a square number ?

49



irregular pentagon

Carefully copy the pentagon above onto squared paper. Then use any method you like to find its exact area. Imagine the squares of the grid are 1cm squares; this means you should give your answer as so many square centimetres.

50

$a * b = 2a - b$

binary operations 1

Often in maths you come across a rule for combining two numbers to produce a single number as the answer. Such a rule is called a 'binary operation'. Addition and multiplication are two examples. There's another binary operation shown above. The rule it describes is just '*to combine numbers a and b, just double a and then subtract b; this gives you your answer*'. For example, $3 * 4 = 2$ $6 * 5 = 7$

Now answer these, using the same rule :

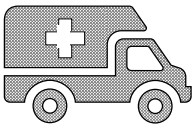
(1) what's $7 * 4$*?* (2) *what's* $3 * 3$*?*

(3) if $\square * 5 = 3$, *what must* $\square$ *be?*

(4) if $\bigcirc * \bigcirc = 6$, *what must* $\bigcirc$ *be?*

(5) what's $12 * (5 * 1)$*?*

51

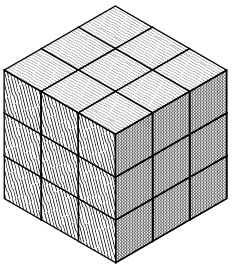


ostriches and camels

At the Karima animal sanctuary in Sudan, Africa, old and injured ostriches and camels are looked after by the dedicated team of carers. Monday was a particularly rainy day and so all the animals were kitted out in both rain-hats and wellingtons before taking their usual walk. Altogether, 11 rain-hats were issued (that's 1 rain-hat each for all animals) and 30 wellingtons (that's 2 wellingtons each for the ostriches and 4 wellingtons each for the camels).

How many camels were in the group?

52

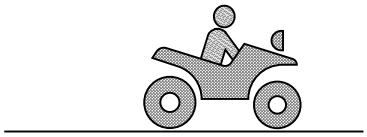


bad glue

Amok gets 27 small wooden cubes and glues them together to make a larger cube. It's pictured in the diagram above. He then paints the outside of this larger cube, using bright red paint. Next day he finds that the cube has fallen apart and once more he has 27 small cubes.

How many of the small cubes Amok is left with have red paint on them somewhere?

53

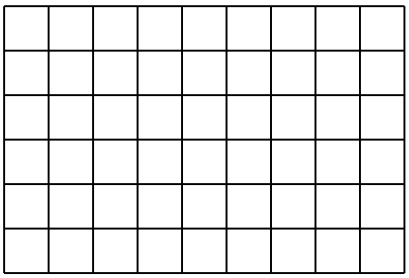


quad-bike climb

Gareth is riding his quad-bike from the farmhouse to the top sheep field, a total journey of 4 km. But it's not all plain sailing! The first part of the journey is along the lane, a perfectly level stretch. The second part of the journey is across a field and this stretch is all uphill. Travelling on the level, Gareth's speed is 20 km/hr but going uphill it's only 10 km/hr.

If Gareth's whole journey, from farmhouse to top sheep field, is exactly 15 mins, how many kilometres long is the straight part?

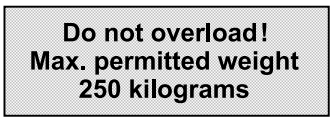
54



grid challenge

Can you find a way of arranging six 3 x 2 rectangles and two 4 x 3 rectangles on a 9 x 6 grid, like the one pictured above?

55

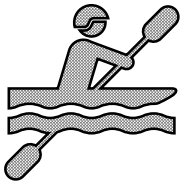


lift-off!

The public library in Bolton is a large brick building with books etc on several floors. There's a wide spiral stairway to take you from floor to floor; or, if you prefer, you can take the lift. This lift is a small one; it has a sign reading, '*Do not overload! Maximum permitted weight 250 kilograms*'. Soon after opening time on Monday, three adults got into the lift on the ground floor; their average weight was 53kg. The lift stopped on the second floor to admit one more adult; now the average weight of those in the lift went up to 60kg.

What did the fourth passenger weigh? And was the lift overloaded with four on board?

56



time will tell

It always takes Vanessa 3 hours exactly to canoe from her home, on the banks of the River Tamar, up to her grandmother's house, a few kilometres further upriver. On Saturday morning, Vanessa set off at 08:30 to canoe to her grandmother's house. When she was three-quarters of the way there, it began to rain heavily.

What time was it when it began to rain?

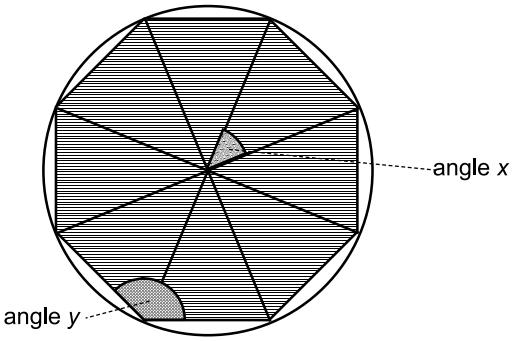
57

girls' triumph

It's almost holiday time and Mr Pascal's maths class must take the end-of-term test. They don't find it too difficult but when the results come out, there's a surprise, at least for the boys. Out of a possible 40 marks, the 12 girls in the class have an average score of 30, whereas the 8 boys in the class have an average score of 25.

What was the average score for the class as a whole?

58

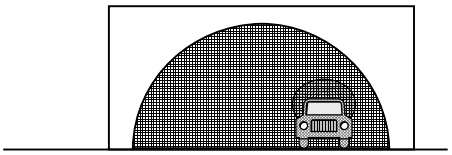


angles in an octagon

Here you can see a regular octagon (regular = 8 equal sides, 8 equal angles); it's drawn inside a circle. Lines are drawn from the centre of the circle to the vertices (corners) of the octagon.

Make a rough sketch of the diagram above and then use what you know about angles to find what angle x and angle y must be.

59

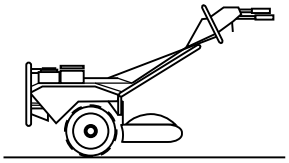


tyred out

One morning in 2021, a tanker, travelling through one of the Mersey Tunnels and carrying corrosive chemicals, hit a side wall and sprang a leak. During the following half-hour, 25 vehicles passed through this same stretch of road. Some of these were cars and the others were motorbikes. Sadly, each one of these vehicles had its tyres completely ruined, making a total of 90 spoiled tyres.

How many of the 25 vehicles were cars and how many were motorbikes?

60



rotavator . . .

Perhaps you know what a rotavator is? It's just a piece of garden equipment designed to break up, churn and aerate the soil. But look at the word 'rotavator' . . . can you see anything special about it? Yes, its letters are in the same order in each direction. It's what we call a 'palindrome'.

1 *How many years this century will be palindromes? What about next century?*

2 *Think of a 24-hour digital clock. Some times are palindromes, like 05:50 or 12:21 for example. How many of these are there?*

---

1 fair shares

The average we're talking about in this question is the *mean*.That's the average where you add up a number of quantities and then divide by however many quantities you've added. For example, if you have a group of ten people and their ages add up to 150, then the mean (average) age of the group is just 150 ÷ 10, or in other words, 15. Easy!

There are different ways of doing this question but one way is this: To begin with, you've got four teenagers and altogether their spending money adds up to a total of £24; this means of course that the group's average amount of spending money was £24 ÷ 4, that's to say, £6. When Emma joined the group, this average went up by £1; so now the average was £7. How can we get to the amount Emma must have had? Well, one thing we can do is to look at the 'average mean' calculation in a different way. With a group of five teenagers having an average of £7 each, the total for the group must have been 5 x £7, or in other words, £35. What made the total for the group jump from £24 to £35? There's only one answer : it's the amount Emma brought along. So this amount must have been £11.

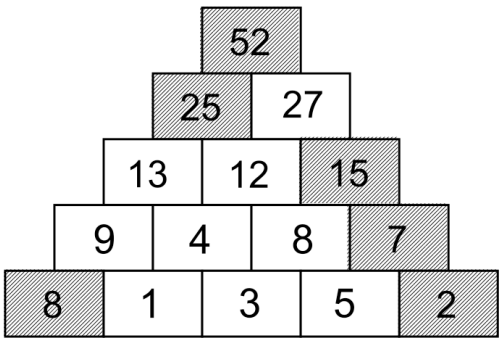
answer : £11

---

2 you're my number wall !

By gradually working through the number wall (using one subtraction after another), you should get this final picture of how things must be :

answer :



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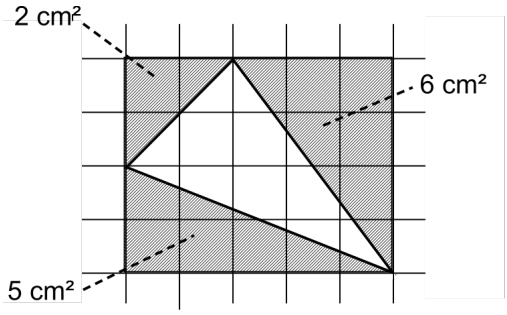
3 **triangle area**

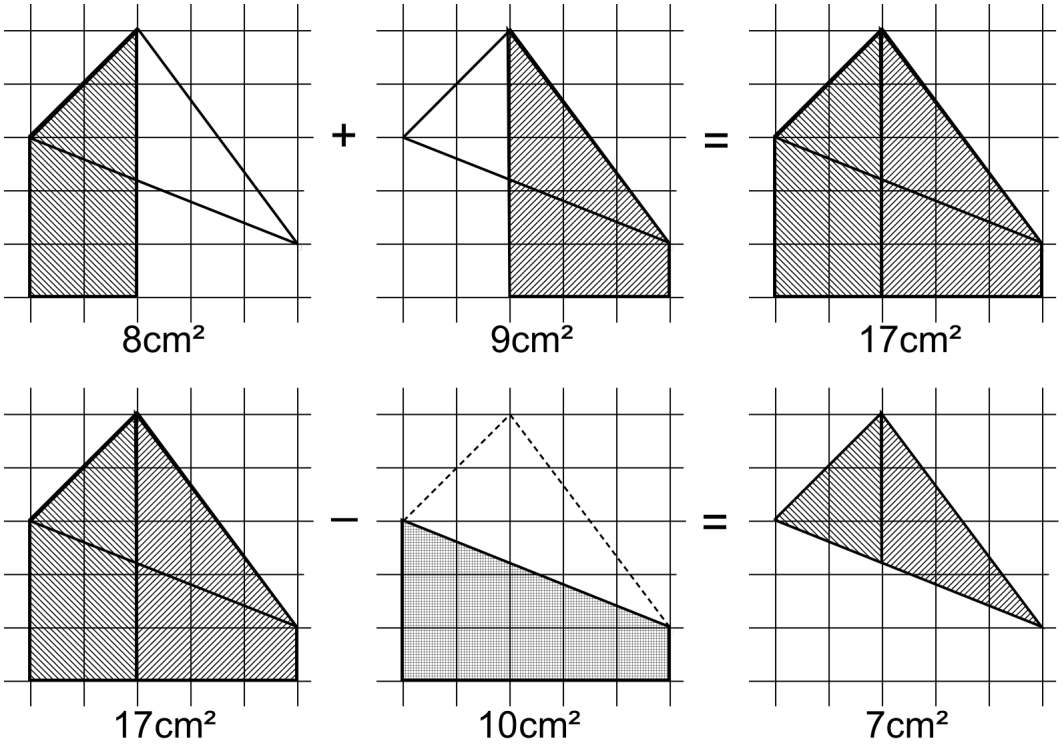
Perhaps the easiest way here is to draw a rectangle around the triangle, as in the diagram below. As you can see, the rectangle has an area of 20cm^2. Next, subtract from this the combined areas of the three corner triangles (13cm^2). This gives you 20 – 13 = 7cm^2 for the answer.



answer : 7cm²

If you know how to work out the area of a trapezium, there's another way of solving this problem, which you might prefer. Here's how it works . . .

You combine two trapezia to make one larger one. Then you take away another trapezium from beneath them – and what's left is the original triangle! The following diagrams show it all clearly :



answer : 7cm^2

4 the trumpet major

There's no one correct way of solving logic problems like this. You might well have solved this one by reasoning it out in your own way and if so, then well done! But if you don't know where to start, there's always the 'table' method – it's not a particularly exciting approach but it is quite straightforward. Here's how it goes :

First of all, make a table listing the friends down one side and the instruments along the other side (or you can put things the other way round if you prefer). Next, work your way through the facts you're given and put ticks or crosses in the appropriate place in the table.

1 Either Simon or Ralph plays cornet

– which means that Peter doesn't play cornet :

	cornet	french horn	tuba
Simon			
Ralph			
Peter	X		

* definitely don't put ticks for Simon and Ralph under cornet ! (can you explain why not?)

2 Peter doesn't play the french horn

– and now you have two crosses along Peter's row, so he must play the tuba, so that's a tick you can put in . . . which also means that neither Simon nor Ralph plays the tuba, so that's two more crosses to add :

	cornet	french horn	tuba
Simon			X
Ralph			X
Peter	X	X	✓

3 Ralph doesn't play tuba or cornet

– there's already a cross under tuba, so just add another cross under cornet.

	cornet	french horn	tuba
Simon			X
Ralph	X		X
Peter	X	X	✓

4 – which means you can put a tick for Ralph under french horn. This leaves only one instrument not claimed and only one person who can play it, Simon! So put a tick there. And if you want to make the table look complete, put a cross in the remaining empty square . . .

	cornet	french horn	tuba
Simon	✓	X	X
Ralph	X	✓	X
Peter	X	X	✓

answer : Simon plays cornet, Ralph plays french horn and Peter plays tuba

5 **feet and inches**

Don't be put off if you're not familiar with the old Imperial System of weights and measures – all you need to know is that feet and inches were units of length and that 12 inches was the same as 1 foot.

How to tackle the problem? Well, with luck you realised that the sensible thing to do would be to convert all the heights to the same units. And obviously it's easier to do this if we choose inches. So now we can write the heights as : Annabelle 38 inches (that's 3x12 plus 2), Sebastian 54 inches (that's 4x12 plus 6) and Quentin 58 inches (that's 4x12 plus 10). To find the average of these three heights, we just add them and divide the total by 3 :

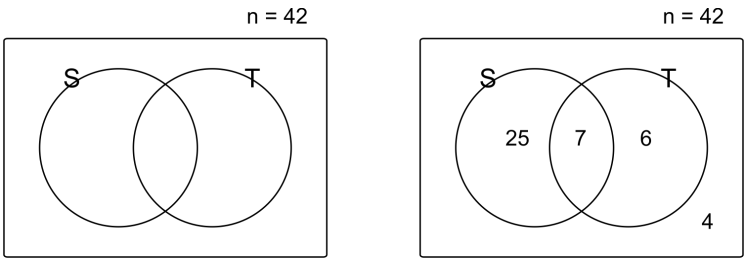
$$38 + 54 + 58 = 150 \text{ and } 150 \div 3 = 50$$

– and so we have our answer, that's to say, 50 inches. If we're feeling smart, we can of course write this in feet and inches : 4 feet 2 inches

answer : average height = 50 inches or 4 feet 2 inches

6 **shirt store**

A Venn Diagram makes things easier in this sort of question. The rectangle represents all the customers who visited the store on Monday, and we write 'n = 42' to show that the number of customers was 42. Set S is the set of customers who bought new shirts and set T is for customers who bought ties. Next, we enter the information we're given, taking care to put the numbers into the right part of the diagram :



As a general rule with these questions, if you have a number for the intersection set, put that in first. In our problem this is the overlap between S and T, and so in here we put 7 (because 7 customers bought shirts *and* ties). 32 customers in all bought shirts and so (after subtacting the 7 we've already entered in the intersection part of this set) we put 25 in the relevant part of set S. 4 customers didn't buy a shirt or a tie and so we put 4 inside the rectangle but outside the set circles. If we add up the numbers we've already entered, we get 36. This leaves just 6 customers and so we can put a 6 in the remaining blank part of the diagram (the part for those who bought a tie only).

answer : 6 customers bought just a tie.

7 **canned music**

Here are two easy ways of going about this one :

First approach. I'm sure you'll agree straight away that 18% of £100 is £18. So 18% of £400 must be four times this, or in other words, £72.

Second approach. Even if we can't say straight off what 18% of £400 must be, we can definitely say what 10% is . . . 10% of £400 = £40. Double this and you have 20% of £400 = £80. But we don't want 20%, we want 18%; so, we have to take off 2%. We've already worked out that 20% of £400 = £80, so 2%, as one-tenth of this, must be £8. Taking this £8 away from our £80 gives us the tax, £72. Now just add this to the original price : £400 + £72 = £472

answer = £472

---

8 **mini sudoku**

As sudoku problems go, these two are pretty easy. Answers :

3	1	2	**4**
2	4	**3**	1
1	**3**	4	2
4	2	1	3

1	2	**3**	**4**
3	**4**	**1**	**2**
2	**1**	4	**3**
4	3	2	**1**

* It's worth noticing that there's a kind of symmetry in the first completed sudoku square. Perhaps we could describe this as a *rotational symmetry*, around the centre of the square, of the rows and columns of numbers.

---

9 **measure for measure**

answers : (a) **Divide by 5, then multiply by 8** (b) **160 km/hr / 45 mph**

---

10 **milking time**

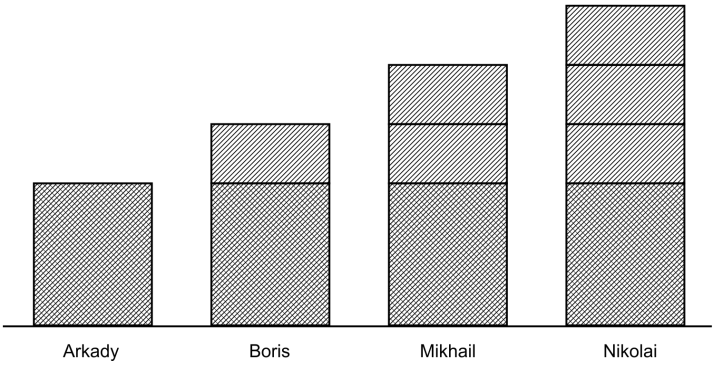
We're told that a typical cow moves at a speed of 1.5 km/hr. This is just the same as 1500 metres per hour. At this speed, our cow's jouney takes 20 minutes, which is one-third of an hour. So the journey must be one-third of 1500 metres, or in other words, 500 metres.

answer : 500 metres

---

11 the brothers Karamazov

Like most problems, this one can be done in different ways – and without using algebra! One way is to use a diagram :



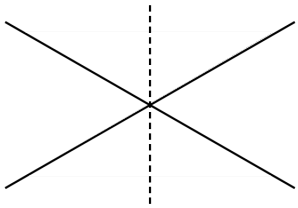
Can you see how this works? We've shown Arkady's age by the darker shaded rectangle. Boris's age has this rectangle plus a different one to stand for the 2 extra years he has over Arkady. Mikhail's age looks the same as Arkadi's age but with two of these 2-year rectangles on top. And Nikolai has the same basic rectangle plus three of the 2-year rectangles. Make sure you really understand how this all works.

What else do we know? Well, we're told that all these ages add up to 32. This means we have four of the basic rectangles plus six of the 2-year rectangles adding up to 32. In other words, four basic rectangles plus 12 comes to 32, which means that the four basic rectangles on their own must add up to 20. So the basic rectangle is worth 5 years and this rectangle, if you remember, stands for Arkadi's age.

answer : Arkadi is 5 years old

12 crossed lines

We have two crossed lines which give us two large angles and two smaller ones. Obviously angles across from each other are equal – and we're told that the larger angle is double the size of the smaller angle.



By simply adding a vertical line to the diagram we can cut the larger angle in half; we now have six equal angles. (Make sure you understand why this is.) One revolution is 360º, so each small angle must be one-sixth of this, that's to say 60º.

answer : 60º

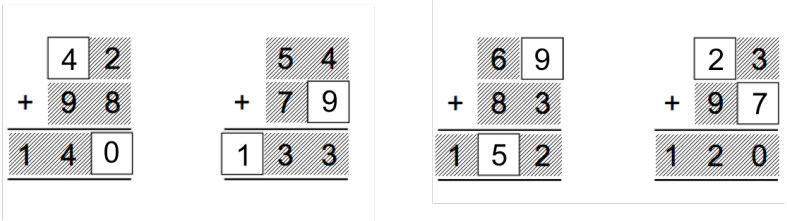
13 problem parents

As my parents had only one child (me) and they now have just one grandchild (Peter), I must be one of Peter's parents. Which means that if I'm not Peter's father, there's only one thing I can be . . .

answer : I am Peter's mother

14 sums & diffs 1

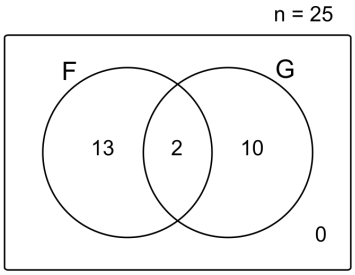
answer :



15 language barrier

There are 0 pupils who don't learn either French or German. So the total of all those learning French or German or both must be 25. However, if we add the 15 learning French to the 12 learning German, we get 27. This means there must be 2 pupils learning both languages.

This is how the completed Venn diagram looks :



answer : 2 pupils learn both languages

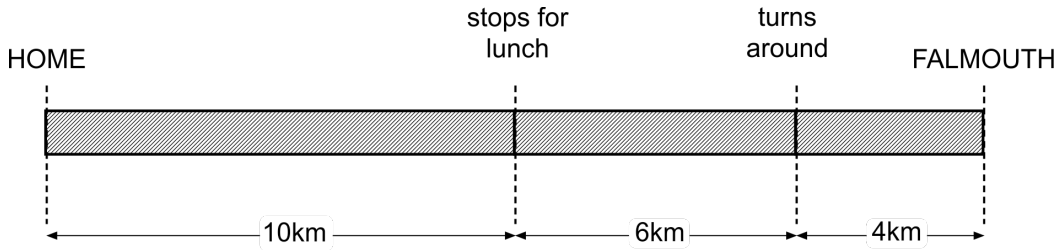
16 Sophie's cousins

We're looking for three prime numbers which add up to 20. The smallest prime number is 2, so we could start there. Are there two other prime numbers which add up to 18? Well, yes . . . we could have 5 and 13 . . . or we could have 7 and 11. Then we could try 3 as our smallest prime and see how many pairs of other primes add up to 17. Now we meet a problem! You can't find any. Why is that? Well, unless you include 2, you'll have three odd numbers – and three odd numbers can't add up to 20 (an even number). As no number can be greater than 12, there's only one group of three primes which will do . . .

answer : the cousins' ages are 2, 7 and 11

17 too far to Falmouth

This is not too hard a problem but perhaps a diagram will make it even easier :



Can you see how this diagram represents Amol's journey? Where do the distances come from? Well, we're told that after lunch, Amol cycles for 6km before stopping to turn around; and at this point he's just 4km from Falmouth. So from his lunch-stop to Falmouth must be 6km + 4km = 10km. Since the lunch-stop was exactly half-way along the road from home to Falmouth, the total distance from home to Falmouth must be 20km.

Of course, Amol did not cycle all the way to Falmouth; he turned back at a certain point 4km short of his destination. So the distance he cycled must be :

10km + 6km + 6km + 10km = 32km

answer = that day, Amol cycled 32km in all

18 The big screen

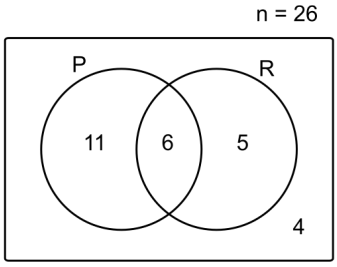
Well, the answer is definitely not 1000! Remember, every time Chanu visits the Olympus Cinema, they stamp his membership card. So the uncle's gift is worth not just 1000 visits to the cinema but also 1000 stamps on Chanu's membership card. For every 10 stamps on your card, you get a free ticket – which means that Chanu's 1000 visits will entitle him to 100 free tickets! When Chanu uses these 100 free tickets, he'll get his card stamped a further 100 times . . . and this will entitle him to 10 more free visits! These 10 visits will result in 10 more stamps on his card and of course 1 more free visit. So :

total number of free visits in all = 1000 + 100 + 10 +1 = 1,111

answer = 1,111 free cinema visits in total

19 fish supper

answer :



5 anglers caught only roach

20 where to begin?

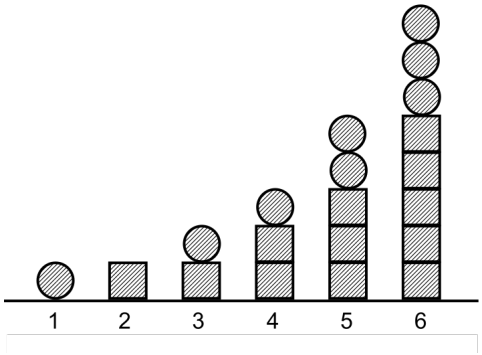
We're probably going to be looking at quite small numbers for the first pair. So, you might for example try 2,3 or perhaps 3,2 as your starting pair. Then this is how things would work out :

2 3 5 8 13 21 and 3 2 5 7 12 19

Neither of these is quite right (one sequence ends up just a little too high and the other sequence ends up a little too low), so you might well give up this approach here. Or, you could look at what comes just before the last number – and with 20 as your 'starting point' (endpoint really!), you could try working backwards. Why not try 12 as the next-to-last number? Then you can say that an 8 must come before the 12 (do you see why?) and that a 4 must come before the 8. Carrying on backwards like this, another 4 must come before this last one (that's to say, in position 2 in the sequence) and finally a 0 must be there in position 1. So now we have at least one answer – start the Fibonacci sequence with 0,4 and you'll get 20 for your sixth term :

0 4 4 8 12 20

Another way of looking at this problem is to use diagrams. You might find this a little hard to follow at first but it's worth trying hard to understand it. It's a good method! Suppose we use a circle and a square to stand for the first and second members of our Fibonacci sequence :



1st term = 1 circle

2nd term = 1 square

3rd term = 1 circle plus 1 square
(sum of the first two terms)

4th term = 1 circle plus 2 squares
(sum of the 2nd and 3rd terms)

and so on . . .

Following through in this way, the 6th term must be made up of 3 circles and 5 squares . . . Make sure you understand why, whatever the two starting numbers are, the 6th term of the sequence must always be made like this.

How does this help us to solve our problem? Well, we want the 6th term to equal 20 and we know that 6th term = 3 circles + 5 squares. So what we're looking for is a pair of numbers where 3 times the first numbers plus 5 times the second number equals 20.

We could just hunt randomly for numbers which will work for us in this way but it's better to be systematic . . . Why not just see how things work out if for the first number we try 0, then 1, then 2, then 3 and so on . . .

0	3 x 0	20	☞
1	3 x 1	17	
2	3 x 2	14	
3	3 x 3	11	
4	3 x 4	8	
5	3 x 5	5	☞

The first line will work for us, as 3 x 0 leaves 20, which we can make up as 5 x 4. So that's one starting pair, namely 0 and 4.

The last suggestion will also work for us, as 3 x 5 leaves 5, which we can write as 5 x 1. This gives us another starting pair, namely 5 and 1.

CHECK : 0 4 4 8 12 20

5 1 6 7 13 20

* Notice this method gave us two different answers, rather than just one.

answer : the first two terms could be 0,4 or 5,1

---

21 double birthday

First of all, we need to remember just how many days there are in each of the months we're dealing with. Probably you've been taught the rhyme

'Thirty days hath September, April, June and November;
All the rest have thirty-one, saving February alone,
And that has twenty-eight days clear
And twenty-nine in each leap-year.'

So the key facts are : September has 30 days and October has 31 days. Armed with this information, you can now carefully plot how things progress as we go forward from 20th September to 5th November :

20 sept	monday
27 sept	monday
30 sept	thursday
01 oct	friday
29 oct	friday
31 oct	sunday
01 nov	monday
05 nov	friday

answer : in 2021, 5th November was on a Friday

ps You weren't supposed to look at a calendar to find the answer but now you have the answer, here for your interest is a calendar extract showing the Autumn months from the year 2021 :

September							October							November						
S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S
			1	2	3	4						1	2		1	2	3	4	5	6
5	6	7	8	9	10	11	3	4	5	6	7	8	9	7	8	9	10	11	12	13
12	13	14	15	16	17	18	10	11	12	13	14	15	16	14	15	16	17	18	19	20
19	20	21	22	23	24	25	17	18	19	20	21	22	23	21	22	23	24	25	26	27
26	27	28	29	30			24	25	26	27	28	29	30	28	29	30				
							31													

22 the seven ages of man

The main thing here is to keep a clear head in dealing with the different units. There are 12 months in a year and it's much easier to do the calculations you need here if you keep everything in the same units, in this case months :

62 years = 744 months, and so

62 years 5 months = 749 months

749 ÷ 7 = 107

– and so we have our answer : each stage is 107 months. Since the original information was in years and months, rather than just months, it's a good idea to give our answer in years and months. To change 107 months into years and months, we just need to divide by 12 :

107 ÷ 12 = 8 rem 11

– which is to say that each stage is exactly 8 years and 11 months. Notice we've used 'whole number division' (division with remainders) rather than 'exact division' (division which gives an exact answer). Exact division wouldn't give you a wrong answer but we were asked to give our answer in years and months. And 8 years 11 months does seem more in keeping with the question than something like 8.917 years.

answer : 8 years 11 months

23 parking

This is not a hard logic problem and you might have found your own way to the answer. But with a problem like this, if you can't think how to get started, you can always draw a table to summarize the possibilities and then start crossing out the ones which don't match what you've been told. Here's how to apply the table method to this problem :

leaving out the colours for the moment, let's put the different types of car along the top and the three people along the side

	sports car	saloon	estate
Dr Brown			
Mr Smith			
Mrs Green			

next, we know that the sports car is blue and the saloon car is red, so the estate car must be yellow – so let's put these colours straight onto our table

	blue sports car	red saloon	yellow estate
Dr Brown			
Mr Smith			
Mrs Green			

We're told that Dr Brown's car is not yellow and it's not a sports car – which tells us straight away that he owns the red saloon. So that's a cross, a tick and another cross to put in along Dr Brown's row . . . and of course we can put in two crosses to show that neither Mr Smith nor Mrs Green owns the red saloon car

	blue sports car	red saloon	yellow estate
Dr Brown	✗	✓	✗
Mr Smith		✗	
Mrs Green		✗	

Finally, we're told that Mrs Green's car is not blue. So she must own the yellow estate! Let's put a tick in there. That just leaves Mr Smith for the blue sports car, so let's put a tick in that box.

	blue sports car	red saloon	yellow estate
Dr Brown	✗	✓	✗
Mr Smith	✓	✗	
Mrs Green		✗	✓

And so we have our answer : Mr Smith owns the sports car.

Some problem-solvers are happy to leave things there . . . but many don't feel happy until they've filled in the remaining boxes of the table. So, for completeness :

As the yellow estate car belongs to Mrs Green, it can't belong to Mr Smith, so let's put in a cross to show that. In the same way, Mrs Green can't be the owner of the blue sports car, so let's put a cross to show that. At last we're all complete!

	blue sports car	red saloon	yellow estate
Dr Brown	✗	✓	✗
Mr Smith	✓	✗	✗
Mrs Green	✗	✗	✓

answer : Mr Smith owns the sports car

---

24 birthday boy

Like most problems, this one can be done in different ways – and without using algebra! Obviously, you can try different numbers to see how things work out or you might think of a way of getting directly to the answer. But here's one approach :

We're told that in 5 years' time Ben's age will be double what it was last year. We're also told that all the ages in the question are whole numbers – which means that all the ages in five years' time must be even numbers (since each of them is double a whole number). From the wording of the question, we can make a reasonable guess that Ben is presently at least 3 years old and probably no more than 11 years old.

So, with these thngs in mind, let's look at a set of ages for Ben 5 years from now ranging from 8 to 16 . . . and let's put the corresponding 'now' ages for Ben alongside, together with the corresponding 'last year' ages :

This is the set of results we get : (Make sure you see how it works). A quick glance is enough to show you that Ben must be 7 years old today . . . because this means he will be 12 next year and must have been 6 last year.

last year	now	in 5 yrs time
10	11	16
8	9	14
6	7	12
4	5	10
2	3	8

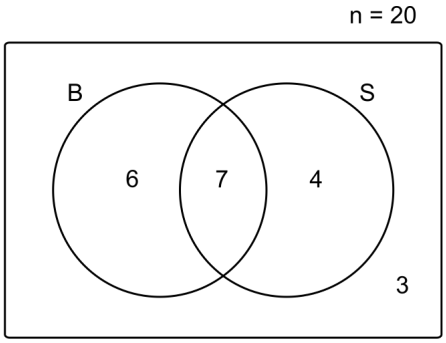
answer : Ben is 7 years old today

25 sibling rivalry

A Venn Diagram gives you the clearest picture of what's going on. Our outer rectangle stands for all the children in Form 3 and we put n=20 next to it, to show that this set has 20 members. Next, two overlapping circles, labelled B and S, stand for the sets of children with brothers and children with sisters. As ever, the part where the two circles overlap stands for the set of children who have both a brother and a sister. We are given numbers for these three sets (7 had brothers and sisters, 6 had just brothers and 4 had just sisters) and so we can put these numbers into the diagram.

Adding these three numbers together gives us 17. However, Form 3 has a total of 20 children in it, so there must be 3 children who have neither brothers nor sisters. Our '3' goes inside the rectangle but of course outside the two circles :

This is how the completed Venn diagram looks :



answer : there are 3 children with no brothers and no sisters

26 fishing competition

You don't have to draw tables or diagrams to solve problems like this one but many people find it makes life easier. Let's begin by making a simple table to summarize the information we've been given :

4	3	2	1	0
5	8	9	6	6
20	24	18	6	0

The heading at the top shows the number of trout a competitor might have caught eg 4 trout, 3 trout and so on. The middle row of the table shows how

many competitors actually caught that number of trout eg 5 anglers caught 4 trout each, 8 anglers caught 3 trout each and so on. The bottom row shows the totals of trout caught for each type of competitor eg all those catching 5 trout each made for a total of 20 trout caught and so on. You get the idea.

Working carefully from the table, we get the following figures :

total number of trout caught = 68

total number of contestants = 34

. . . which means average number of trout caught = 68/34 = 2

answer : the average number of trout caught was 2

---

27 In the queue

There's only one answer to this (rather easy) question :

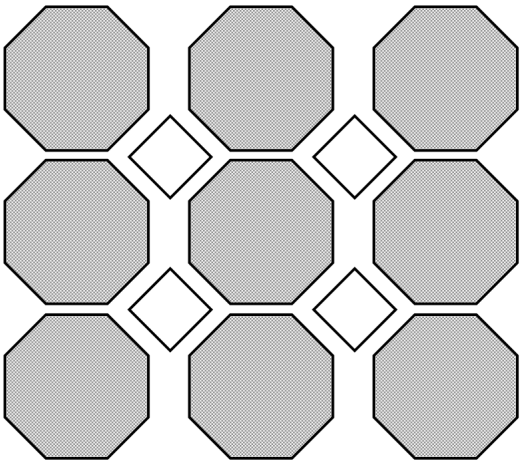
	teenager	mother with pram	middle-aged man	old lady
answer :	4	3	2	1

---

28 octagon floor tiles

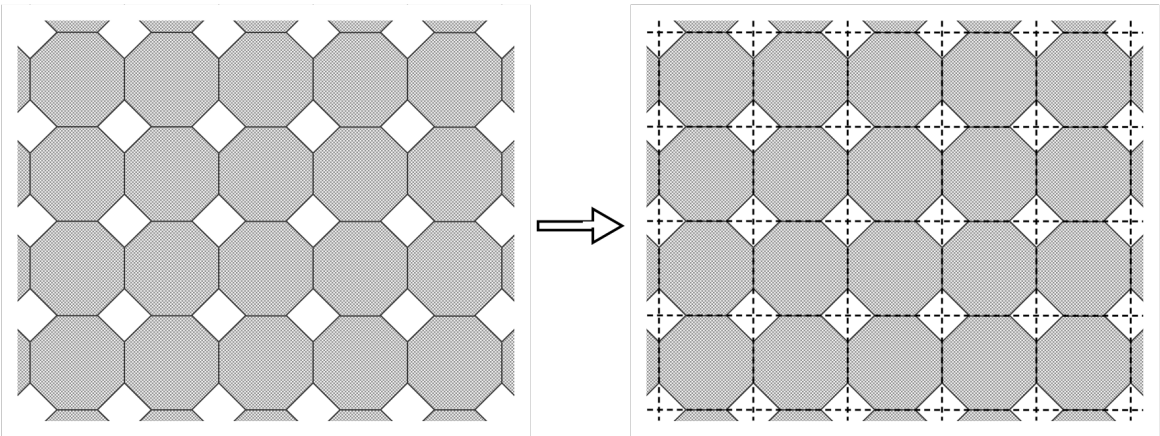
This is another problem where you can find different ways of getting to the answer. Finding the area of an octagon is not straightforward – but there are ways of getting round this. One way is to look at the tiling in a different way . . .

It's natural to think of this tiling in terms of the octagons and squares it's made up of :

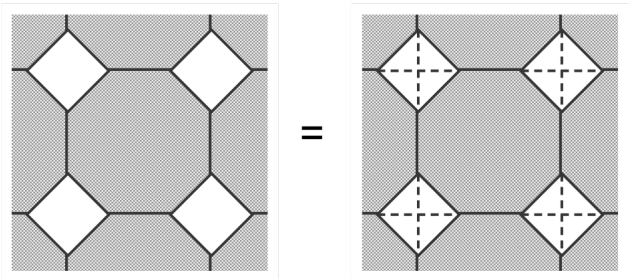


– but if you do think of the whole courtyard as a mixture of octagons and squares, then you probably move on to trying to solve the problem in a certain way. This way would be by estimating how many octagons and how many small squares you would have in a certain area, and then working out the total area of each. If you carry out this approach carefully, then the answer you get won't be wrong . . . it's just that there is an easier way :

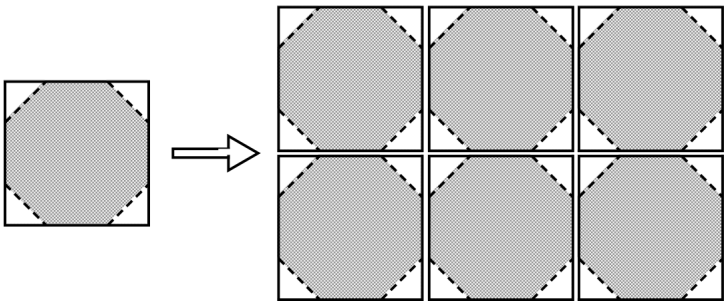
Suppose we take a section of the courtyard; the section shown in the question will do well enough. And suppose we draw dotted lines across it, like this :



Doing this has given us a different way of looking at the tiling. We can forget about the original octagons and squares and think instead of a tiling made up simply of squares, each square with a right-angled triangle cut out of the corner, like this :



Can you see how we're now thinking of the whole courtyard tiling as if it's made up simply of squares? The diagram below shows the idea :

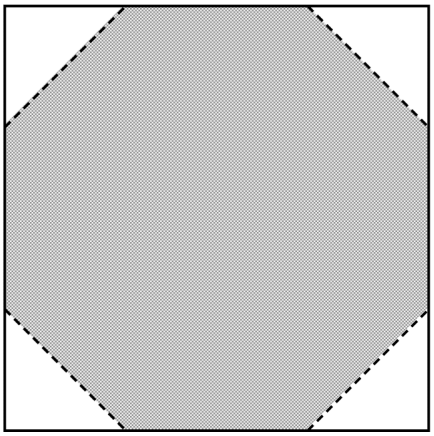


Let's now look a little more closely at one of these square 'tiles'. The shaded part (that's to say the original octagon) we're told has an area of 21 square metres. What about the triangular corners? Well, you can probably see that four of these triangles make up one of the original squares and we know these each had an area of 4 square metres. So obviously each of the small right-angled triangles has an area of just 1 square metre.

What does this mean for the area of our new square tiling units? I hope you can see that each of these squares must have an area of 25 square metres (that's 21 from the original octagon part and 4 from the 4 triangular corners) :

total white area
= 4 squ metres

shaded area
= 21 squ metres



area of whole
square
= 25 squ metres

So for each of our new square tiling units, we can say :

total white area = 4 squ metres and area of whole square = 25 squ metres

– which means that the white area is $^4/_{25}$ of the whole square

And since the whole floor can be made up from these squares, it follows that the white area (sum of the original small squares) forms $^4/_{25}$ of the whole area. What is $^4/_{25}$ as a percentage? Well, $^4/_{25}$ of 100 is 16, so our answer is 16%.

answer : 16% of the floor area is covered by square tiles

** Remember, this answer is an approximation – we can't be sure exactly how things work out at the edges, for example, or whether the courtyard is itself a perfect rectangle. But if it's a large enough courtyard (and we do know that it's impressively large), then the 16% figure will be pretty accurate.*

** We should also point out that many people find this problem one of the hardest in the whole set. It's worth going over the above answer a few times to try to understand it. But don't feel too bad if you still find it difficult to get your head round it – you won't be alone!*

29 number & number

This is a fairly straightforward problem. You might not be familiar with the old UK currency system of pounds, shillings and pence but that doesn't matter. All you need to know is that in that system, 20 shillings = 1 pound. So,

16 books at 7 shillings each = 16 x 7 = 112 shillings

We need our answer in pounds and shillings, so how many pounds and shillings do we get from 112 shillings? All we need to do is to divide 112 by 20, using of course whole number division (division with remainders) :

112 ÷ 20 = 5 rem 12

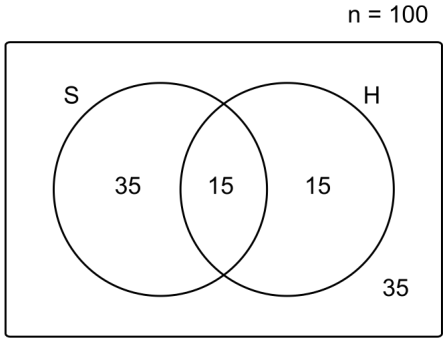
– which in currency terms means that we have 5 pounds and 12 shillings

answer : 5 pounds 12 shillings (£5 12s)

30 30 degrees in my shades . . .

This is an easy enough overlapping sets problem. Perhaps the one extra difficulty is deciding what number to give to the universal set (the set of everything involved in the question). A little thought will tell you that this number should be 100 (to stand for 100%). Now add together the 35, 15 and 15 you're given and you'll see that 35 is the missing number you need :

This is how the completed Venn diagram looks :



answer : 35% wore neither sunglasses nor a hat

31 mini sudoku

answer :

4	3	2	1
2	1	4	3
3	4	1	2
1	2	3	4

1	2	4	3
4	3	1	2
2	1	3	4
3	4	2	1

32 cover story

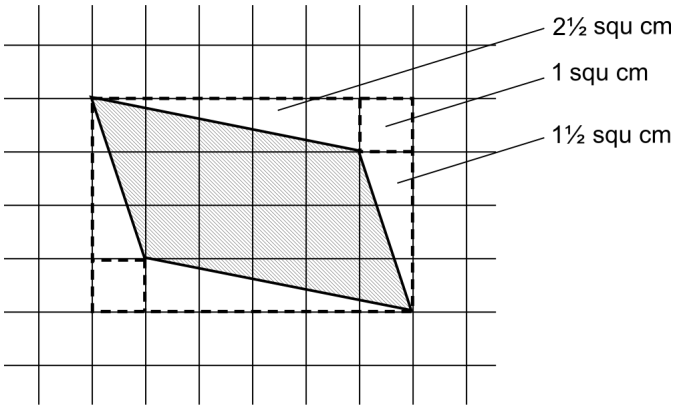
This is a really easy logic problem. You might be able to see the answer straight away but if not, you can always draw up a simple table like the one below. Taking the facts you're given one by one, you can use ticks to mark the days when each of the students is at work.

	MON	TUE	WED	THU	FRI
John			✓		
Kate		✓			
Lucy				✓	
Peter	✓				✓

answers : Peter works twice during the week / Kate works on Tuesday

33 parallel area

Perhaps the easiest way of solving this problem is to draw a rectangle around the parallelogram, like this :



The rectangle has an area of 24 squ cm. You can divide the area around the rectangle into four triangles and two squares. The total area of these six shapes is 10 squ cm. Subtract this from the area of the surrounding rectangle and you get the area of the parallelogram.

answer : area of parallelogram = 14 squ cm

34 Ice Station Zebra

This one's not too hard – that's if you're comfortable wth negative numbers! There are different kinds of average but you know how the mean of seven numbers is claculated: you just add up your seven numbers and then divide your total by 7. Easy enough!

So, we need the sum : 4 + 5 + 2 + (-2) + (-5) + (-1) + 4

Of course, adding -2 for example is just the same as taking 2 away . . .

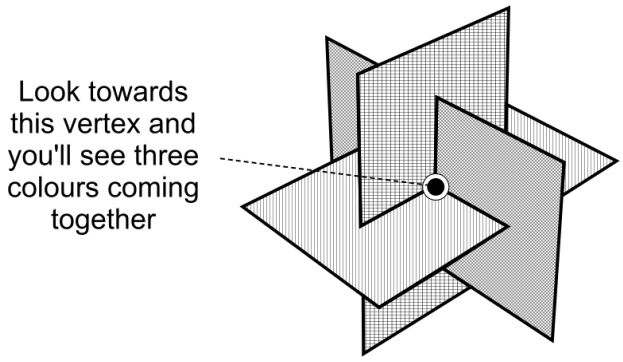
. . . which means our sum = 4 + 5 + 2 – 2 – 5 – 1 + 4 = 7

Dividing this total by 7 gives us 1 exactly and so that's our mean!

answer : the mean temperature for the seven days was exactly 1º

35 three dimensions

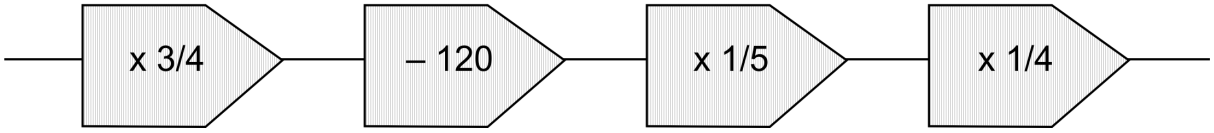
The diagram shows just one place on the shape where three colours can be seen coming together. Perhaps you can see that by the sheer symmetry of the shape, there must be exactly eight places just like this :



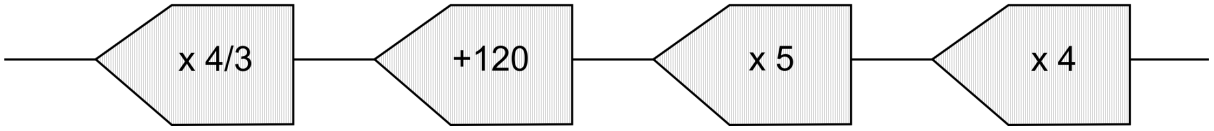
answer : there are eight corners where three colours meet

36 a bad year for ants

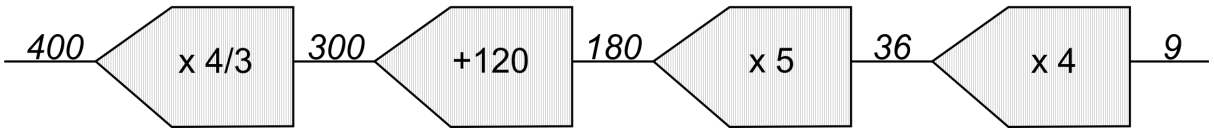
Obviously the question needs you to do so some working backwards, to see what the colony was like at the begnning of the year. One way of doing this is to use a mapping diagram to show what happened to the number of ants at each stage as things went forward – and then to reverse the diagram. So, to begin with, here's the basic mapping diagram :



Next, here's the same sequence of events but in reverse :

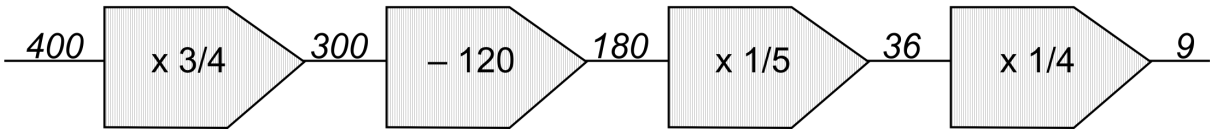


Now all we have to do is to put in the starting figure (that's 9 of course) and follow through what happens to this number after each separate mapping :



– and now we can see the complete picture. Carrying out the whole sorry sequence of separate mappings in reverse shows us that 9 ants eventually leads to 400 ants. This means that if we carry out the whole sequence of mappings in the correct order, 400 ants at the beginning will lead to just 9 at the end. If this isn't totally clear to you, do take the trouble to read it through a few times until you're sure you understand it. It's worth the effort!

If you like, you can now check this answer by carrying out the original sequence of mappings, beginning with 400 ants. You'll see that it works!



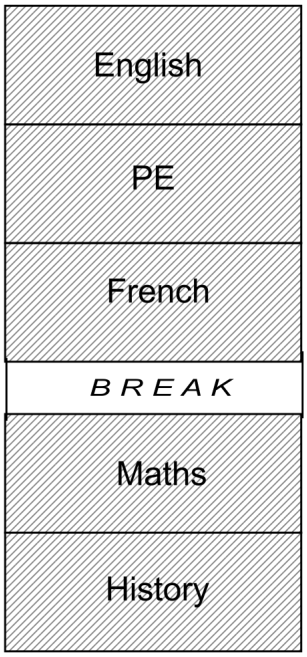
* This method of reversing a composite mapping is quite important. Make sure you really understand how it works.

answer : there were 400 ants at the start of the year

37 monday, monday . . .

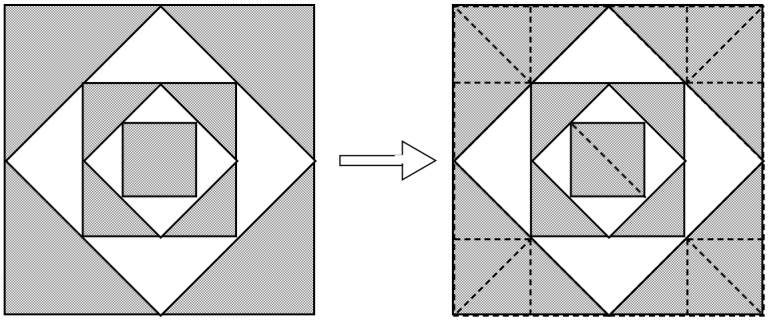
answer :

This problem shouldn't have caused you too much trouble. Whichever method you used to solve it, your answer should of course be as shown here :



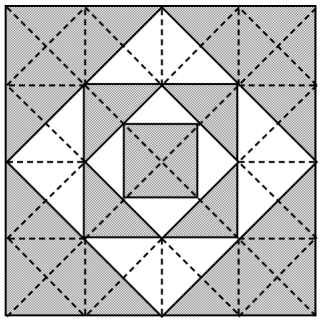
38 squares within squares

You need to keep a clear head for this one! One thing you can do is to divide up the shaded areas into a number of equal right-angled triangles; this certainly makes things easier when it comes to finding the total :



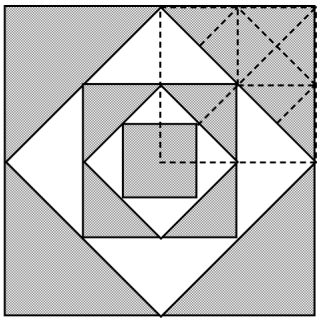
Using the dotted lines, we have now clearly marked out 22 shaded triangles. As you can see in the centre square, 2 of these triangles make up an area of 1cm² . . . and so the total shaded area must be 11cm².

A different approach, which some seem to prefer, is to divide the whole diagram into 64 very small, but all equal, right-angled triangles.

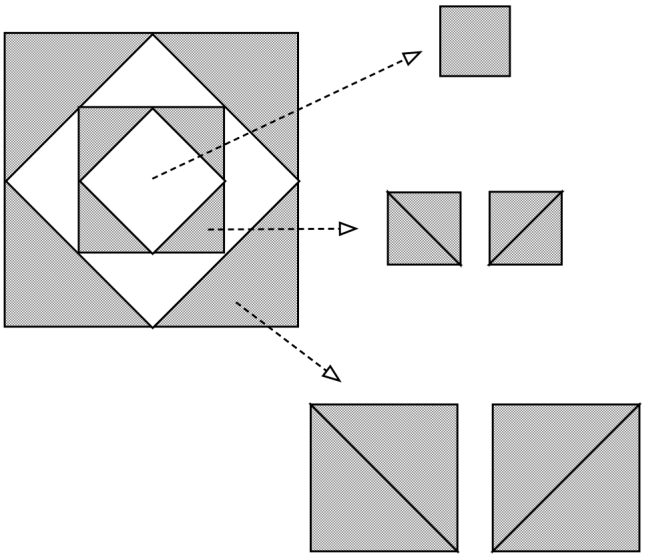


As you can see, there are 64 of these small triangles. How many of them are shaded? Well, it's probably quicker to count how many aren't shaded and subtract this number from 64. This gives us 64 – 20 = 44 small triangles. It takes 4 of these triangles to make up an area of 1cm² . . . and so once again we find the total shaded area = 11cm².

** You might already have noticed that whichever of these two methods you prefer, the smart approach to doing the counting up would be to look at just one-quarter of the diagram and then multiply our answer by 4 . . . something to remember for next time!*



Another, rather different approach is to break up the diagram in the following way : First of all, take out the small centre square; then take out the four triangles which form a kind of 'inner circle' and re-combine them into two squares, each having the same size as the small centre square; and finally, take the four outer triangles and re-combine them into two larger squares, each having the area of four times the small centre square. The diagram below shows you the method :



So now what have we got in terms of area? Well, we have 1 small square (the one from the centre) plus 2 small squares (made from small triangles) plus 8 small squares (made from larger triangles) . . . and that's a total of 11 squares, giving us altogether a final area of 11cm²

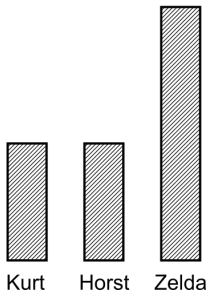
answer : total shaded area = 11cm^2

39 twins age problem

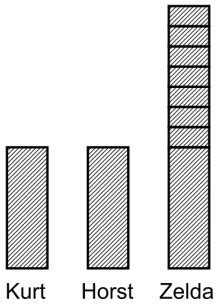
You've probably found a way to work this one out. Of course, some of those who are fluent in algebra will tell you straight away that, 'algebra's the way to do these problems!' However, it is perfectly possible to do problems like this without algebra. If by using your own imagination you've found a way to the answer which you find clear enough and which you can explain to someone else, then that's the best approach as far as you're concerned.

Some pupils prefer to use diagrams wherever possible when they're problem-solving and one way of doing this is to use rectangles to stand for the ages of the three children :

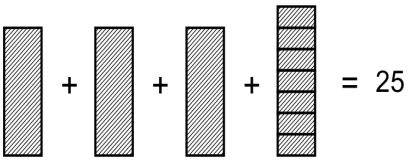
We can picture the ages of the three children like this :



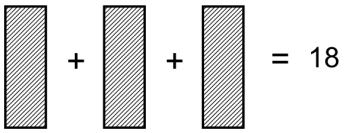
The diagram above shows that the two boys are equal in age and that Zelda is older than they are. In fact,though, we do know a little more about Zelda's age : we know that she's seven years older than the boys are. To picture this :



Remembering that the rectangles stand for numbers (years in our problem), we can rearrange them to show the fact that the three ages add up to 25 :

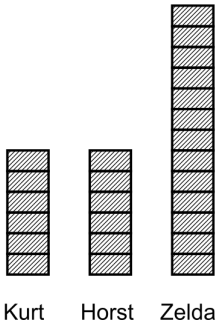


The seven small rectangles on the left stand for seven years. So, let’s re-draw this last diagram to show this :



Finally everything is clear: the larger rectangles must each be equal to 6 years. In terms of our problem, this means that Kurt and Horst are each 6 years old – and that Zelda is (6 + 7 = 13) years old.

If you like, you can now draw a final diagram to show exactly how the 25 years total is made up among the three children :

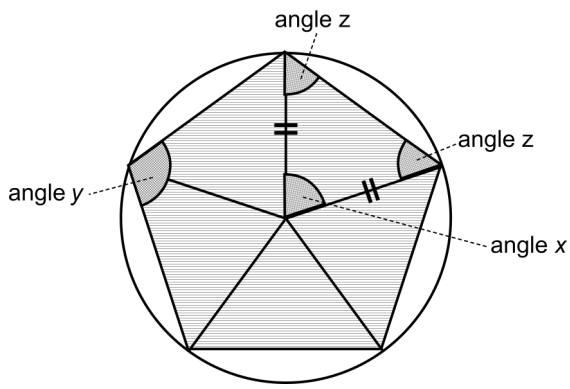


answer : Zelda is 13 years old

40 angles in a pentagon

To solve problems like this, you need to know a few facts about straight lines, angles, triangles and circles, eg / the angles on a straight line add up to 180º / the three angles of any triangle add up to 180º / in an isoscles triangle, the two equal sides meet in one angle – and the other two angles are equal / the angles in one complete revolution add up to 360º / all the radii in a circle are the same length / . . .

Now to the problem : First of all, we've re-drawn the diagram to put more detail into one of the triangles. Because it's an isosceles triangle (two sides are radii and so are equal), the two angles we've labelled *z* must be equal.



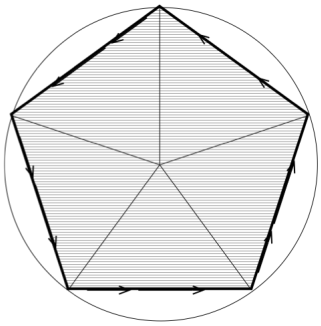
Next, look at angle *x* (at the centre), you'll see that it's one of five equal angles. Since one full revolution = 360º, angle *x* must be equal to 360º ÷ 5 = 72º.

Now look at the triangle we're focusing on : since the angles of a triangle always add up to 180º, the two angles labelled *z* must together add up to 180º – 72º, that's to say, 108º. So now we know that angle *z* = 54º. But as you can see, angle *y* is exactly double angle *z*. So angle *y* = 108º.

answer : angle *x* = 72º / angle *y* = 108º

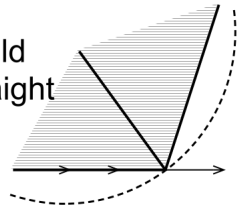
* There's a different way of going about this problem, which you might find more attractive. Think of a regular pentagon drawn inside a circle, as in our question, and imagine an ant starting off along one of the sides and making his way around the pentagon, keeping exactly to the the pathway marked out by the sides and moving in an anti-clockwise direction. See the diagrams on the following page :

Here's a plan of the ant's journey :

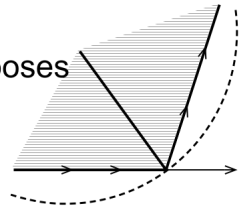


At each corner the ant could go straight on – but instead he turns left – and each time it's a turn through the same angle :

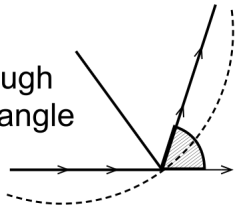
he could go straight on



but he chooses to turn left

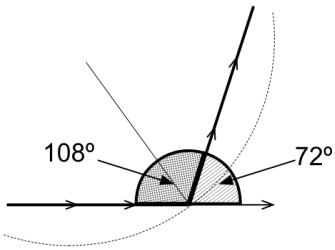


turns through a certain angle



Perhaps you'll agree that by the time the ant gets back to where he started, he'll be facing in the same direction but he will have turned altogether through an angle of 360º. To turn through 360º in five equal stages is the same as turning through 72º at each stage (360 ÷ 5 = 72). To find angle y, we just subtract the 72º from 180º, giving us 108º, as before :

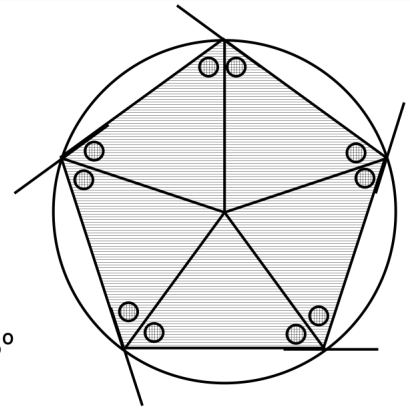
To find angle y, we just subtract 72º from 180º, giving us 108º, as before :

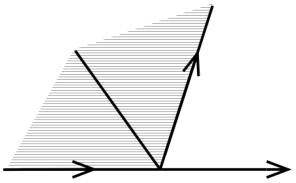


In short :

5 left turns = 360º,
so each turn = 72º

internal angle of pentagon = 108º





41 mini sudoku

answer :

2	1	4	3
4	3	1	2
1	2	3	4
3	4	2	1

4	1	3	2
2	3	4	1
1	4	2	3
3	2	1	4

42 mean sentences

$$\text{mean} = \frac{\text{total number of letters}}{\text{total number of words}}$$

$$= \frac{48}{12} = 4$$

answers : mean = 4 *example :* This is a difficult challenge !

43 sums & diffs 2

answers :

$$\begin{array}{r} 707 \\ +\ 999 \\ \hline 1706 \end{array} \qquad \begin{array}{r} 968 \\ +\ 897 \\ \hline 1865 \end{array} \qquad \begin{array}{r} 574 \\ -\ 276 \\ \hline 298 \end{array} \qquad \begin{array}{r} 891 \\ -\ 497 \\ \hline 394 \end{array}$$

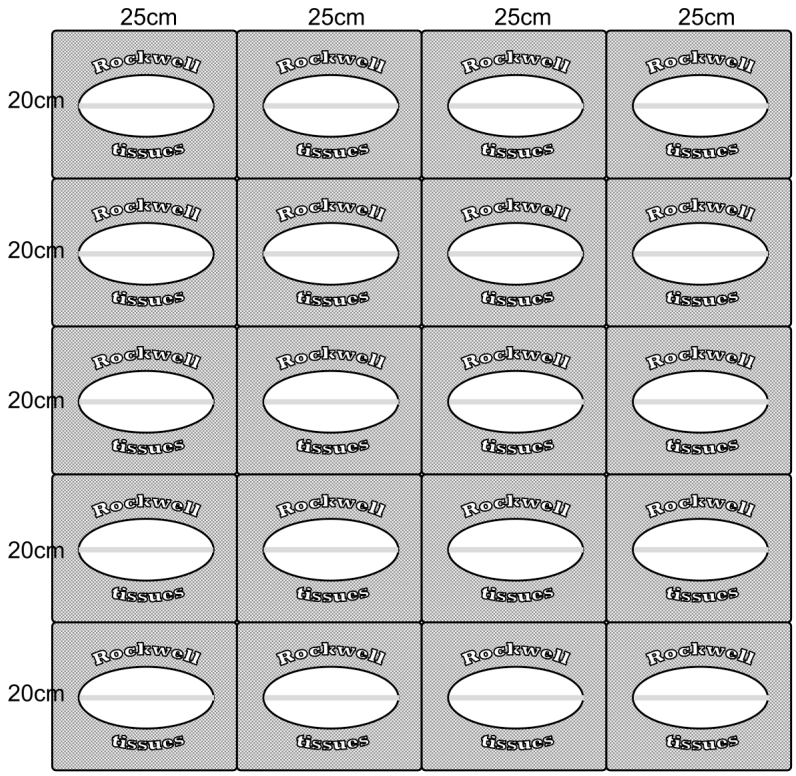
* Quite a few problem-solvers say that they find the subtraction ones much more difficult. But – if you do get stuck on one of these, it's sometimes useful to remember that the bottom two lines must add up to the top line . . .

44 atishoo!

First of all, we need to remember the simple fact that 1metre is the same as 100cm. So the large cartons must measure 100cm x 100cm x 100cm. And we know that each tissue box is 25cm wide x 20cm front-to-back x 10cm deep. Let's think first about filling just one layer at the base of a large Rockwell carton :

The large carton is 100cm x 100cm across its base. Looking at this base from above, the question is : how can we pack it with tissue boxes? Well, we can lie 4 boxes in one direction (since each box is 25cm wide) and we can lie 5 boxes in the other direction (since each box is 20 cm from front to back) . . .

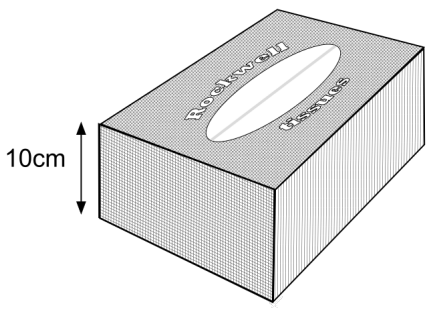
. . . and that's a total of 20 tissue boxes in just one layer of the carton.



And of course, this layer will be just 10cm deep. So how many of these 10cm layers can we fit into a carton which is 1metre (100cm) deep? This is not a hard thing to work out :

It's pretty obvious that you'll fit exactly 10 layers into the carton – and then it will be full! And that means that, with 20 tissue boxes per layer, you'll have filled the large carton with 10 x 20 = 200 boxes.

So that's it! In every large carton of Rockwell Tissues, there are 200 separate boxes.



answer : there are 200 boxes of tissues in every carton

45 average windsurfing

Let's summarise the information we've been given :

tuesday wednesday thursday friday	4 days mean = 6 per day	tuesday wednesday thursday friday saturday	5 days mean = 8 per day

And this is all fine . . . but there's something else we know. Look at the summary on the left, covering the days tuesday to friday. We know that to get the average (mean) in the first place, someone must have divided a total by the number of days. What was this total? Well, to find it, surely all we have to do is to multiply the number of days by the mean, that's to say work out 4 x 6. Easy stuff! So the total of surfboards hired across these four days must have been 24. In the same way, we can easily work out that the number of surfboards hired in the days tuesday to saturday must have been 5 x 8 = 40. Now here's a summary of things, to include the totals we've worked out :

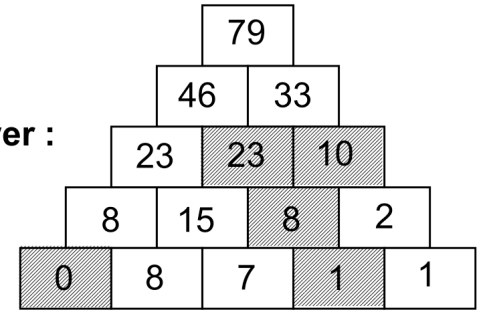
tuesday wednesday thursday friday	4 days mean = 6 per day TOTAL = 24	tuesday wednesday thursday friday saturday	5 days mean = 8 per day TOTAL = 40

The rest is not too difficult . . . if the total of boards hired by the end of friday was 24 and the total of boards hired by the end of saturday was 40, then it's pretty clear that 16 boards must have been hired on the Saturday (simple subtraction gives you 40 – 24 = 16). And that's it!

answer : Henry had 16 surfboard customers on Saturday

46 another number wall

answer :



47 left out in the cold

Nothing too difficult here, as long as you keep a clear head! You probably realised straight away that what makes this look like a problem is having the lifespans given in a mixture of weeks and days. So, the first thing to do is to re-write the data in terms of just one of the units, 'days' being the obvious one to choose. Here goes :

3 weeks & 6 days ⇒ 27 days

4 weeks & 3 days ⇒ 31 days

3 weeks & 5 days ⇒ 26 days

4 weeks & 4 days ⇒ 32 days

Now it's just a matter of finding the average (and it's the mean average we're talking about) in the usual way. There are just four numbers to deal with :

total = 116 days

so, mean = 116 / 4 = 29 days

and, with 7 days to the week, 29 days = 4 weeks & 1 day

answer : average survival time = 4 weeks & 1 day

48 unsquare number

Sometimes you look at a problem and you can't see straight away what the thing is getting at. However, you can always do a little investigating . . . Let's begin here by looking at the squares of a few numbers :

numbers	corresponding square numbers
0, 10, 20, 30 . . .	0, 100, 400, 900 . . .
1, 11, 21, 31 . . .	1, 121, 441, 961 . . .
2, 12, 22, 32 . . .	4, 144, 484, 1024 . . .
3, 13, 23, 33 . . .	9, 169, 529, 1089 . . .
4, 14, 24, 34 . . .	16, 196, 576, 1156 . . .
5, 15, 25, 35 . . .	25, 225, 625, 1225 . . .
6, 16, 26, 36 . . .	36, 256, 676, 1296 . . .
7, 17, 27, 37 . . .	49, 289, 729, 1369 . . .
8, 18, 28, 38 . . .	64, 324, 784, 1444 . . .
9, 19, 29, 39 . . .	81, 361, 841, 1521 . . .

Look really carefully at this table. You can probably see why we've grouped the numbers and their squares in this particular way. Suppose for example we look at the fifth row of the table : the numbers on the left all end in 4 and we see that the squares of these numbers all end in 6! And the same sort of thing happens on every row of the table. (Ask yourself : why does this happen?)

One way of summarising what we've found here is this :

number ends in :		square number ends in :
0	→	0
1	→	1
2	→	4
3	→	9
4	→	6
5	→	5
6	→	6
7	→	9
8	→	4
9	→	1

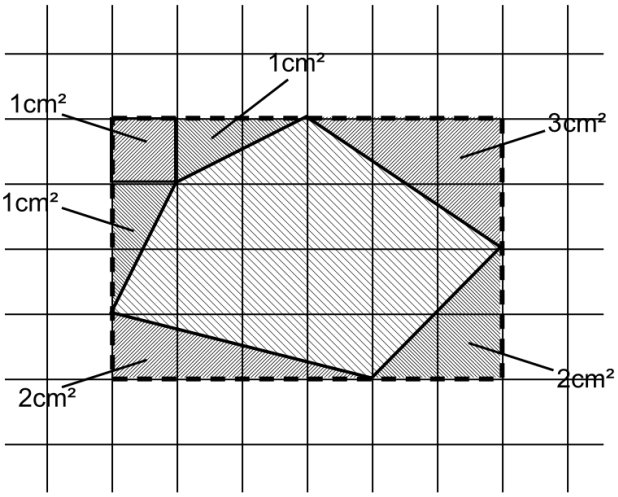
What do you notice about the results shown in this table? Well, for one thing, if you ignore the first line, you'll see that the endings of the numbers form a sort of palindrome . . . that's to say, the list from the beginning reads : 1, 4, 9, 6, 5 and so on – and the list from the end upwards reads : 1, 4, 9, 6, 5 and so on. In maths, it's always interesting when you come across symmetry. A more important thing, though, is that the square number endings are a select group; there's no 2, there's no 3, there's no 7 and there's no 8! And that's how Jane knew straight away that 1372 just couldn't be a square number.

answer : no square number ends in a 2

49 **irregular pentagon**

Probably the easiest way of soving problems like this is the simple 2-stage approach we've used before (eg question 3) :

1. Draw a rectangle around the shape and find its area
2. Add up the areas of all the separate regions which are inside the rectangle but not in the shape we're interested in



area of rectangle = 24cm²

total area of triangles plus small square = 10cm²

area of pentagon = 24 – 10 = 14cm²

answer : 14cm^2

50 **binary operations 1**

There are many operations in maths where you have a rule for combining two numbers to produce another number. Operations like this are called **binary operations**. For example, the four basic operations of arithmetic (addition, subtraction, multiplication and division) are all binary operations. In each of these, you start with two numbers and then combine them in a certain way to produce one single number as a resutl. And because we use these binary operations a lot, each one has its own special symbol. For example,

7 + 3 = 10 106 – 57 = 49 15 x 7 = 105 96 ÷ 16 = 6

These are the most famous binary operations. But there's nothing to stop us making up new binary operations of our own. We just have to explain clearly how to combine the two starting numbers to get to the result. And if we make up a new binary operation, it won't already have a symbol to stand for it – and so we can use any symbol we like, or even make up a new symbol.

So, when you look at this problem, don't be put off by the new idea. And don't be put off by just seeing a new symbol. The questions themselves are not really all that hard . . .

1. $7 * 4 = (2 \times 7) - 4 = 14 - 4 = 10$

answer = 10

2. $3 * 3 = (2 \times 3) - 3 = 6 - 3 = 3$

answer = 3

3. Put another way, this question is just asking : What number, when you double it and subtract 5, leaves you with 3 ? With luck you might well have spotted that the number we're after is 4.

answer = 4

4. Once again, you can easily put this question into words : What number, when you double it and then subtract the number itself, gives you 6 ? Not surprisingly, perhaps, the answer is 6.

answer = 6

5. When you're faced with working out something like $12 * (5 * 1)$, the general rule is 'brackets first', that's to say, first work out what the expression in the bracket comes to. So . . .

$$5 * 1 = 10 - 1 = 9$$

– which means we can write :

$$12 * (5 * 1) = 12 * 9 = 24 - 9 = 15$$

answer = 15

---

51 ostriches and camels

People who have learned algebra will find this an easy enough problem. But you don't have to know algebra to solve it . . .

To begin with, we know for definite that there were 11 animals altogether on the walk. So now all we have to do is to find the balance of ostriches and camels within this group of 11. One easy way of going about this is to try some numbers and see how things work out :

Having no idea of the final answer, why not see what happens when the group is divided roughly half-and-half? So let's try 5 ostriches and 6 camels for a start. How many wellingtons would that need? Well, the ostriches would need 10 wellingtons (5 x 2) and the camels would need 24 wellingtons (6 x 4). Altogether that's 34 wellingtons, 4 more than the 30 the group actually needed.

So we obniously need to try some different numbers. And this is where we can be smart. Instead of just guessing at random our next pair of numbers, let's stop and think for a minute. If our first estimate of 5 ostriches and 6 camels needed too many wellingtons, do we need more ostriches or fewer ostriches in our next estimate? A camel needs more wellingtons than an ostrich, so we should make sure our next estimate has fewer camels. Let's try 6 ostriches and 5 camels for our second estimate. If you work things out carefully, you'll see that this time we would need 32 wellingtons. Still too many!

This means our next estimate should have even fewer camels. Let's try 7 ostriches and 4 camels. Success! With this combination, we would need exactly 30 wellingtons. So now we have our answer.

answer : there were 4 camels in the group

***** Sometimes with a problem like this, where we need to try out different numbers, it's a good idea to set out the results we get in a table. This is how we might have done this for our particular problem :

ostriches	**camels**	wellingtons total
5 (10)	**6** (24)	(34)
6 (12)	**5** (20)	(32)
7 (14)	**4** (16)	(30)
8 (16)	**3** (12)	(28)
9 (18)	**2** (8)	(26)

52 **bad glue**

There are different ways of going about this problem but whichever approach you choose, the important thing is to be systematic and careful. We don't want to miss out any cubes – and we don't want to count any cubes twice!

One approach is to divide the outside cubes into different groups. For example, when the large cube falls apart, leaving 27 small cubes, there will be : cubes with just 1 red face (there would have been one of these cubes in the centre of each face of the original large cube), cubes with 2 red faces (centre edge pieces in the original cube) and cubes with 3 red faces (corner pieces in the original large cube). Let's count :

1 red face	6 cubes
2 red faces	12 cubes
3 red faces	8 cubes

This gives us a total of 26 cubes with paint on them somewhere.

Another approach is to think of how the 27 small cubes are arranged to form the large cube. Basically, there's one small cube in the dead centre and then a number of cubes around this one. All the cubes except the single cube in the dead centre will have red paint on them somewhere. How many are there of these cubes? The answer must of course be 26 (that's 27 cubes minus 1).

answer : 26 cubes have paint on them somewhere

---

53 quad-bike climb

This is another of those problems where those fluent in algebra might well know how to set up equations which lead directly to the answer. But you can solve this problem without algebra. We know that the whole journey is 4km and we know that this is split into two parts, a level stretch and an uphill stretch. One thing we can do is to try out some different ways of making this split. We could have, for example, 1km level combined with 3km uphill, or 2km level combined with 2km uphill, or 3km level combined with 1km uphill. Then we can see how long is the total journey time for these different combinations. You get the idea . . .

It's easy to get the numbers mixed up when dealing with questions of speed, so we need to be careful.

Let's try the first combination above, that's to say 1km travelling on the level, followed by 3km travelling uphill. Gareth's speed on the level is 20 km / hour, so 1km would take him just 1/20 of an hour, that's to say 3 minutes. His speed going uphill is only 10 km/ hour, or 1km every 6 minutes, so 3km would take him 18 minutes. So now we can say that for this combination, the total journey time = 21 mins

We can carry on like this for other combinations, working out carefully the times for each part of the journey. Here's a summary of the results we get for the three combinations we've listed above :

level stretch 20 km/hr	uphill stretch 10 km/hr	total time
1km = 3 mins	3 km = 18 mins	21 mins
2 km = 6 mins	2 km = 12 mins	18 mins
3 km = 9 mins	1 km = 6 mins	15 mins

As you can see, our third combination hits the jackpot. A journey made up of 3km travelling on the level followed by 1km travelling uphill takes Gareth and his quad-bike exactly 15 minutes.

answer : the straight part of the journey is 3km

---

54 **grid challenge**

We can give the answer to this question straight away. The answer is simply : *no, you can't!*

Why do we know this answer so readily? Well, there are two different sizes of rectangle and you've been told how many you've got of each size. If you work out the total area which this collection of rectangles adds up to, you'll find it's greater than the area of the grid (which we've been told is 9 x 6, or 54 squares). Here's the working-out :

area of grid = 9 x 6 = 54

type of rectange	how many	total area
3 x 2	6	36
4 x 3	2	24
		60

total area of all tiles = 60

answer : no, you can't!

55 lift-off

To begin with, we have three passengers with average weight 53kg. So that's a total weight of 3 x 53, or 159kg. Now someone new enters the lift and the average weight changes to 60kg. The total weight changes too. In fact, the new total weight is 4 x 60, or 240kg.

Obviously, the increase in total weight in the lift is caused by the addition of the new passenger. This increase is 240 – 159, or 81kg. So that's the weight of the new passenger.

Does the new passenger cause the lift to become overloaded? Not quite, because the maximum pernitted load is 250kg and even with the fourth passenger included, the total weight in the lift is only 240kg. Here's a summary of our working-out :

3 x 53 = 159
4 x 60 = 240
240 – 159 = 81

answers : the fourth passenger weighed 81kg / the lift was not overloaded

---

56 time will tell

All we need to calculate to solve this problem is : what's three-quarters of 3 hours? Let's begin :

If you're quite confident with fractions, you can just work out ¾ of 3. One way of doing this is to note first of all that ¼ of 3 is ¾. Now we just need to multiply this by 3 to find ¾ of 3. Our answer is $^{9}/_{4}$, or 2¼ if you prefer. And of course that's 2¼ hours, remember.

Many people prefer to do the calculations with minutes rather than hours. ¼ of 3 hours is the same as ¼ of 180 minutes, and that comes to 45 minutes. To find ¾ of 180 minutes, we just multiply by 3. Our answer is 3 x 45 minutes, or 135 minutes. Changing this back into hours gives us 2 hours 15 minutes or 2¼ hours.

Whichever way you've worked it out, the answer above tells you that it bagan to rain exactly 2¼ hours after Vanessa began her journey. So we now need to add this to the 08:30 starting time. Adding 2 hours to this gets you to 10:30 and adding the extra ¼ hour or 15 minutes gets you to 10:45

answer : it began to rain at 10:45

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57 girls' triumph

This is another of those problems involving averages where you need to go about things differently from the usuual process of just finding averages (that's to say, dividing a total of all scores by the number of scores you have).

If we think about how we usually calculate averages and look at this in a different way, we can see that multiplying the number of scores you have by the average score will give you the total of all scores.

In this problem, we can use this fact to find the separate overall totals of girls' scores and boys' scores :

girls 12 x 30 = 360

boys 8 x 25 = 200

obviously the total of all scores for the class is 760 and so all we need to do is divide this by 20 to get the average for the class :

whole class : 20 pupils, total score = 560

class average = 560 ÷ 20 = 28

answer : the average score for the class was 28

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58 angles in an octagon

The answer to question 40 ('angles in a pentagon') explains in detail two different ways of finding both the angle at the centre and the internal angle of a regular pentagon. You can use exactly the same ideas to solve this question, which is also about a regular polygon, in this case a regular octagon. Only the numbers are different.

answers : angle x = 45º, angle y = 135º

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59 tyred out

Here we have another of those problems where you can home in on the answer by making an initial guess and then working out the results this would give, followed by making a better guess and seeing how this works out – and so on until finally you make a guess which gives you exactly the results you need.

In this problem we know that a car has 4 tyres and that a motorbike has 2 tyres and we know that there were 25 of these vehicles in all.

What we don't know is how many there were of each type of vehicle. So, a good initial guess is probably to split the number of vehicles into two roughly equal halves, say 12 cars and 13 motorbikes. This combination gives you a total of 74 tyres. This is too low and so we need (think about it) more cars in our mix. Let's try 16 cars and 9 motorbikes; this gives a total of 82 tyres . . . and so we can go on. The table below shows one set of guesses which eventually give you the combination you're after :

cars	motorbikes	total number of tyres
12 48 tyres	**13** 26 tyres	74
16 64 tyres	**9** 18 tyres	82
18 72 tyres	**7** 14 tyres	86
20 80 tyres	**5** 10 tyres	90

answer : there were 20 cars and 5 motorbikes

Another (similar) way of getting to the answer is not to make an initial 'guess' but just to work steadily through all the possible combinations until finally you reach the combination which works. We know that each combination has 25 vehicles. So, suppose we begin with 25 cars plus 0 motorbikes : this gives us 100 tyres. Too many! Next, we try 24 cars pus 1 motorbike : this gives us 98 tyres. Still too many! And so we continue in this way until we reach the magic combination . . .

cars	car tyres	bikes	bike tyres	total number of tyres
25	→ 100	**0**	→ 0	= 100
24	→ 96	**1**	→ 2	= 98
23	→ 92	**2**	→ 4	= 96
22	→ 88	**3**	→ 6	= 94
21	→ 84	**4**	→ 8	= 92
20	→ 80	**5**	→ 10	= 90

. . . which of course is as before. This method is like the first method – it just homes in on the answer in a different way. Obviously this approach could involve you in a longer process but as you will already have noticed, there's a pattern in the final column. This pattern shows clearly that each time you change the mix by adding 1 motorbike (and by taking away 1 car), the number of tyres goes down by 2. This means that if you have a problem like this and the table you're building up feels as if it will go on for ever, you can cut things short. Just calculate how many steps you'll need to get to the answer you need and then work out what combination that will give you. As for the current problem, once again we can write :

answer : there were 20 cars and 5 motorbikes

* special note : Ignore anyone who tells you that algebra is the 'proper way' or perhaps the only way to solve problems like this. The methods used here are perfectly good ones. And our first method isn't just wild guessing or 'trial-and-error'; our 'guesses' are intelligently chosen . . .

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60 rotavator

The years part of the problem is easy – there was just one palindrome this century (2002) and there will be just one palindrome next century (2112). As for the digital times, be careful not to include times like 19:91 or 24:42 !

answers : year palindromes are 2002 and 2112

digital clock palindromes are

00:00	10:01	20:02
01:10	11:11	21:12
02:20	12:21	22:22
03:30	13:31	23:32
04:40	14:41	
05:50	15:51	